THE QUANTUM LUMINIFEROUS AETHER

D. J. Larson

Version 2 – March 14, 2023

Abstract. A two-component, quantum luminiferous aether is proposed to exist. Simple postulates are hypothesized for the aetherial quantum pressure, tension and flows. Some physical laws and assignments are proposed. In its nominal state, each aether component is an internally-attached solid. A delta-force is shown to result from sources of detached-aether, and a gamma-force is shown to result from sources of energy or mass. By analyzing an elemental aetherial quantum in the presence of the forces, physical laws and assignments, rigorous derivations are shown to lead to Maxwell's Equations, the Lorentz Force Equation and Newton's Law of Universal Gravitation. The derivations also reveal two field-masses. The first field-mass is identified as dark matter. The second field-mass is shown to contribute to orbital motions and it also results in significant negative-mass within and surrounding dense stellar objects. The theory is shown to successfully meet the classic tests of General Relativity: calculations for the advance of the perihelions, the Shapiro effect, the gravitational redshift and the bending of light in gravitational fields all agree with experiment. Gravitational waves are understood. A new approach for analyzing the hydrostatic equilibrium of white dwarfs and neutron stars is proposed. The theory has no singularity; instead of black-holes, speculative alternative models for super-massive objects are proposed and evaluated.

Key Words: Aether, Electrodynamics, Maxwell's Equations, Lorentz Force Equation, Newton's Law of Universal Gravitation, General Relativity, Perihelion Advance, Light Bending, Shapiro Effect, Dark Matter, White Dwarfs, Neutron Stars, Black Holes

Part A. Introduction.

A.1 – The Historical Setting for This Work

A.1.1. The Classical Aether. The classical approach to physics involved two critical assumptions: 1) space and time are absolute; and 2) an objective reality exists. The goal of classical physics was to hypothesize physical models of the assumed objective reality, rigorously derive mathematical equations resulting from those models, and then test those equations for veracity. If the tests showed agreement with the equations, the underlying model was accepted; if any one test showed disagreement with the equations, then the underlying model needed to be modified or abandoned. By repeating the process of modeling, derivation, and testing, the classical physicists believed that we were obtaining an ever-closer approximation to nature's underlying reality.

Between the 17th and 19th centuries numerous theories of light were put forth that involved a substance called "the aether" that was postulated to exist throughout space and that participated in light propagation. In the 19th century Maxwell developed a set of equations that showed light to be a wave moving at speed c. Under Newtonian theory (which assumed Galilean relativity) there can only be one frame of reference wherein the speed can attain any specific value, and therefore it was assumed that Maxwell's equations were the description of nature as seen by observers at rest with the aether.

Under Galilean relativity, observers moving with respect to the aether should see a velocity of light that is the vector addition of the aetherial velocity of light with their own aetherial velocity, and this led to a proposal to measure the earth's velocity through the aether. The experiment was performed by Michelson and Morley[1], with the surprising result that no motion through the aether was found. The Michelson-Morley null result could be explained by a proposal that physical objects shrink in their direction of motion through the aether. Other experiments also showed evidence that clocks ran slow when they traveled through the aether.

Lorentz, Larmor, Fitzgerald and Poincare all contributed to the development of the Lorentz Aether Theory[2, 3], wherein "true" time and space could only be measured at rest with respect to the aether. For observers moving through the aether, "fictitious" temporal and spatial measurements would be made, as their instruments would be affected by their motion. The equations that transform from one set of moving observations to another have come to be known as the Lorentz Transformation, and all experiments done to date are consistent with those equations.

While the Lorentz Aether Theory was compatible with all experimental results as well as with Maxwell's Equations, a satisfactory derivation of Maxwell's Equations from aetherial assumptions and postulates was never completed. Many attempts were made at such a derivation, including several by Maxwell himself, but as of 1905 there was growing disillusionment with the entire Lorentz Aether Theory, primarily due to the ad hoc nature of the length contraction and time dilation proposals.

A.1.2. Special Relativity Replaces Absolute Space and Time. Einstein's Special Relativity[4] was a radical departure from classical physical theory. Poincare had proposed that the principle of relativity and a constant speed of light can be used to derive the Lorentz Transformation, and hence such a theory was consistent with all experimental results. But Poincare did not take the crucial step of calling into question the axioms of absolute space and time. It was Einstein who made the critical break with classical thought, following the philosophy of Hume and Mach. Hume and Mach made the philosophical point that all we can truly trust are our observations, and that while underlying beliefs (such as physical models and axioms) might usefully be in agreement with the observations, that does not mean that the underlying beliefs can themselves be trusted. Only observation can be trusted. Einstein extended the work of Hume and Mach to observe that time and space might not be absolute. Einstein interpreted the Lorentz Transformations not as relating “fictitious” time and space to “real” time and space, but rather that all reference frames are equally “real”.

The approach of Einstein was to replace the classical approach of using postulated physical models within an absolute space and time by an approach that uses postulated principles that hold equally well in all frames of reference. Einstein’s approach has the advantage of intellectual simplicity, and since it led to the same Lorentz Transformation as did the more cumbersome and ad hoc Lorentz Aether Theory, the latter was eventually abandoned. Serious work on the aether ceased.

A.1.3. Problems with Relativity. While relativity results in a simple formulation for physics, it is not compatible with quantum mechanics. One of the earliest papers on this incompatibility was by Einstein, Podolski and Rosen (EPR)[5]. Bell extended the EPR arguments to testable conditions[6], and Aspect, Dalibard and Roger’s (ADR) tests of Bell’s theorem[7] showed the essential correctness of quantum mechanics. In order to save relativity in light of the ADR results, ideas such as multiple universes or an abandonment of realism have been considered. Relativity also leads to difficulties in understanding simple quantum mechanical results such as the two-slit experiment. All of the problems of understanding quantum mechanics can be easily eliminated if we assume an instantaneous collapse of a finite wave function, but since relativity has relative simultaneity, instantaneous collapse over a distance is not allowed. The problem is often called “the problem of quantum mechanics”, but it could equally be called “the problem of realism” or “the problem of relativity”. Given the experimental evidence, physics is faced with the following choice: Relativity, quantum mechanics, and realism: pick two. Since both relativity and quantum mechanics are in agreement with all experimental data, the position most often taken is to call realism into question. But if we give up realism, the next philosophical question becomes what are we replacing realism with? Are we going to just simply map data to increasingly complex empirical formulas and stop asking what lies beneath those equations?

Also relevant to philosophy is that the presently prevailing relativistic paradigm simply accepts Maxwell's Equations and the Lorentz Force Equation as "nature's laws"; no attempt is made to find any underlying basis for those critical fundamental equations. And those equations have a rather large amount of complexity, with many interacting terms. If two simple empirical ad hoc proposals (length contraction and time dilation) are considered enough to abandon absolute space and time in favor of relativity, then how can we blindly accept the far more complex ad hoc proposals buried within Maxwell's Equations and the Lorentz Force Equation?

A.1.4. A Return to Realism, Absolute Space and Time, and the Luminiferous Aether. In this work we will return to the classical assumptions of realism and an absolute space and time, and we propose a new physical model for the luminiferous aether. It will be shown that a physical model consisting of simple assumptions and postulates lead to a complete derivation of the known equations for electrodynamics, as Maxwell's Equations and the Lorentz force equation will be derived. With a few additional assumptions, a new equation for gravitation will also be derived.

A.2 – Preliminaries: Approach and Scope of this Work

A.2.1. Approach to Notation. In this work, forces and other vectors will be denoted in bold, such as $\mathbf{F_X}$, $\mathbf{F_Y}$, and $\mathbf{F_Z}$. Scalars, such as the magnitude of the tension, are denoted in normal weight font, such as T_0. Unit vectors in the x, y and z directions are **i**, **j** and **k**, respectively. Coming up with good notation is challenging due to the large number of entities involved in electromagnetism and gravity. Where possible, the contemporary notations are used for various quantities and this consumes most of the English alphabet and a significant amount of the Greek one. Hence, the liberal use of subscripts is employed herein to attempt to clarify many specific physical entities.

A.2.2. Verbosity. This work is done in a verbose mode, as it often includes many intermediate algebraic steps in order to help the reader avoid turning to pen and paper or a word processor in order to get from one equation to the next. Also, certain very similar treatments appear in a rather repetitive way for the sake of completeness. (Some of the repetitive material is relegated to Appendices F, G and H.) This results in a rather long work, but one that is both more complete and easier to follow than a terse work would be.

A.2.3. A Primary Analysis from the Aetherial Rest Frame. Since Einstein's Special Relativity and the Lorentz Aether Theory agree that transformation between frames results in the equations of electrodynamics retaining their form, we are free to choose the aether rest frame for our development from the standpoint of electrodynamics, and that is what is done below. We also choose the aether rest frame for our analysis of gravitation, and will discuss ramifications of that choice in Appendix I.

A.3 – A Theory Based Upon Reality of Fields – Continuum Analysis Within the Quantum

Relativity does not allow actions at a distance, because to do so conflicts with causality. To see this, simply consider two events that happen at points A_E and B_E simultaneously in one reference frame. In another reference frame the event at A_E will happen earlier than the one at B_E. In yet another reference frame the event at A_E will happen later than the one at B_E. If we consider a photon collapsing on a wall in the two-slit experiment, then the problem is that a collapse at point A_E must somehow preclude a collapse at point B_E, yet, in some reference frame B_E occurs before A_E, thus violating causality. To evade this logical difficulty, one explanation is to think of the quantum wavefunction as being some sort of magic mathematical guide for underlying point-like entities. However, as discussed above, when a mathematical quantum collapse occurs we have difficulties describing things with an assumption of an underlying realism. However, by setting relativity aside we can return to a simpler theoretical footing that allows for finite-sized entities leaving no problems with causality nor realism.

In this work, we assume a philosophy that quantum physics arises out of the individual nature of physical entities. That is, all physical entities can be divided into finite-sized packets (quanta) but those quanta cannot be further divided. It is well known (even celebrated) that experimental detection of small subsections of the quanta cannot be done without altering what we are trying to measure. Nonetheless, we can still assume (and do so here) that the physical quantum is itself made up of a continuous field and that we can analyze subsections of that field, and that we can do so down to any arbitrarily small size.

A.4 – The Aetherial Hypothesis

A.4.1. Basic Assumptions. It is assumed that Newton's Force Law, $\mathbf{F} = d\mathbf{p}/dt$ is correct, where $\mathbf{F}$ is the force, $\mathbf{p}$ is the momentum and t is time. The usual definitions are assumed, where the momentum is $\boldsymbol{\beta}\gamma mc$, $\gamma = (1 - \beta^2)^{-1/2}$, $\boldsymbol{\beta} = \mathbf{v}/c$, m is mass, $\mathbf{v}$ is the distance traveled per unit time, and c is the speed of light. Newton's Law that every action has an equal and opposite reaction is assumed to be valid. The Pauli exclusion rule and Schrödinger's Equation from standard quantum mechanics are assumed to be valid. And the relation $E = mc^2$ between mass and energy is also assumed to be valid.

A.4.2. The Assumed Physical Model. It is postulated that in the absence of sources and waves, a homogeneous, isotropic, continuous, fermionic, two-component solid-aether occupies space. One component of the aether is called positive-aether; the other is called negative-aether. In the nominal state, each solid-aether component is internally attached; this is called attached-aether. However, it is postulated that by applying sufficient energy, portions of aether can be pair-produced to form free sources of detached-aether. Hence, four types of aether are postulated to exist: positive-attached-aether, negative-attached-aether, positive-detached-aether and negative-detached-aether. In addition to the aether, other foreign substances are assumed to exist. Extrinsic-energy and extrinsic-mass are defined here as the energy and mass, respectively, of any foreign substance immersed into the aether.

In the assumed physical model, we see that the aetherial substances are assumed to be similar to other substances we are familiar with. Normal solids are homogeneous, isotropic, and continuous, and their electron shells and nuclei reveal a two-component nature. However, development of the aether model will also require several additional assumed aetherial attributes that will now be specified.

A.4.3. The Postulates.

The following postulates are proposed:

The Flow Postulate. When aether of one type flows relative to aether of another type, a flow force is generated that is proportional to the flow, and the force is aligned with the flow.

The Tension Postulate. In the absence of external effects, the tension within a quantum of attached-aether is proportional to the separation of any two parallel faces of its surrounding volume.

The Density Postulate. In any volume, the density of the positive-aether equals the density of the negative-aether minus an amount proportional to the extrinsic-energy within the volume.

A.4.4. The Small Disturbance Assumption. For this work we will make the assumption that the nominal density of the attached-aether is much greater than the density changes that result from detached-aether or extrinsic-energy.

A.5 – The Aetherial Laws and Assignments

Section A.4 lays out our starting hypothesis, which consists of some assumptions and postulates we will use as a starting point for our further derivations. In addition to this foundation, development of this work led to the discovery of several additional aetherial attributes, which we classify as either laws or assignments. We will see that there are laws for work due to displacement, flows laws, a law for the effect that extrinsic-energy has on the aether, and assignments for gravitational-mass and inertial-mass. Each of these laws and assignments will be introduced as it is needed.

Part B. Setting up the Analysis.

We assume that the physical quantum is made up of a continuous field and that we can analyze an arbitrarily small subsection of that field. In this Part B we will set up our study of the aether by introducing the quantum-cube, the analysis-cube, and displacement vectors for those cubes. We will also derive some fundamental properties of the quantum and analysis-cubes.

B.1. Displacement Vectors. Defining **P**, **N**, $\mathbf{P_G}$ and $\mathbf{N_G}$

We assume that in the absence of disturbances a two-component, attached-aether exists throughout all of space that is homogeneous and isotropic. In this rest condition it is possible to divide the attached-aether into small volume elements (cubes) and label each cube with the coordinates of its center, (x, y, z). The displacement and motion of the attached-aether can then be analyzed by looking at deviations from this nominal condition at each point over time. Both the negative-attached-aether and the positive-attached-aether can move independently. Here, the vector **P** (which is a function of x, y, z and t) is defined to be the displacement of a positive-attached-aether cube relative to its nominal position. The magnitude of **P** is the distance that the positive-attached-aether has moved away from its nominal position, while the direction of **P** is the direction it has moved away from its nominal position. For the negative-attached-aether, similar considerations apply, and the vector **N** is defined to be the displacement of a negative-attached-aether cube relative to its nominal position.

It will be seen below that there are two independent sources for aetherial motion. Those sources are detached-aether and extrinsic-energy (such as mass). For motion caused by detached-aether the vectors **P** and **N** are used (with no additional subscript). For motion caused by extrinsic-energy, the vectors $\mathbf{P_G}$ and $\mathbf{N_G}$ are used.

B.2. Mathematical Expression of the Density and Tension Postulates

We define ρ_{PA} as the positive-attached-aether density, ρ_{PD} as the positive-detached-aether density, ρ_{NA} as the negative-attached-aether density and ρ_{ND} as the negative-detached-aether density. With these definitions, in the absence of extrinsic-energy, the equality of the positive and negative aether density specified by the density postulate can be expressed as:

$$\rho_P = \rho_{PA} + \rho_{PD} = \rho_{NA} + \rho_{ND} = \rho_N \tag{1}$$

We consider the effects of extrinsic-energy, predominantly mass, in section D.

Consider next a cube containing one quantum of aether (an aetherial-quantum-cube) with each dimension of the cube equal to ξ_Q. With no external work having been done on the cube, and with $F_{T\xi}$ the magnitude of the force at the cube-face due to tension and $K_{T\xi 0}$ the nominal-tension-parameter for the quantum-cube, the tension postulate can be expressed mathematically as

$$F_{T\xi} = K_{T\xi 0}\xi_Q \tag{2}$$

B.3. Forces, Energy and Density of the Aetherial-Quantum-Cube

Eq. (2) is the magnitude of the inward force on a cube-face due to tension. To find the tension-energy stored in the cube, we form the work, $\int \mathbf{F} \cdot d\mathbf{x}$, that it takes to expand the cube-faces from $x = 0$ (where the force is zero) to $x = (+/–)\xi_Q/2$. To evaluate the work done to move the right-side-face to $+\xi_Q/2$ notice that the force will grow as $2K_{T\xi 0}x$ during the expansion. This is because the tension is determined by the total stretching of the cube, which is twice the amount that each face is displaced from 0. (As we expand the right face from $x = 0$ to $x = \xi_Q/2$, the left face expands from $x = 0$ to $x = –\xi_Q/2$ and the faces are always separated by 2x.) The work done on the right-side-face is thus $\int 2K_{T\xi 0}x\,dx = K_{T\xi 0}(\xi_Q/2)^2 = (1/4)K_{T\xi 0}\xi_Q^2$, where we have integrated from 0 to $\xi_Q/2$. Both the force to expand against the tension and the displacement of the right-side cube wall are in the positive x direction and hence the work is positive leading to an increase in stored energy. For the left-side-face the force and displacement are both in the negative direction, and it will contribute an equal amount to the stored energy, leaving the total stored tension-energy in one dimension of a single aetherial-quantum-cube as:

$$E_{T\xi} = (1/2)K_{T\xi 0}\xi_Q^2 \qquad (3)$$

Within the solid aether each aetherial-quantum-cube is surrounded by other aetherial-quantum-cubes. With the hypothesis of a fermionic aether, no two quanta will be able to occupy the same state, and the neighboring cubes will be modeled here as forming a three-dimensional square well surrounding the aetherial-quantum-cube of interest. Within a three-dimensional square well, the quantum-momentum **p** is determined by the De Broglie relationship, $\mathbf{p} = \hbar\mathbf{k}$ where $\hbar$ is Planck's constant h divided by 2π, $k = |\mathbf{k}| = 2\pi/\lambda$ is the wave number, and λ is the wavelength. In the ground state the wavelength will be twice the width of the well, or, $\lambda = 2\xi_Q$, leaving $p = |\mathbf{p}| = \hbar\pi/\xi_Q$ for each dimension. The quantum-energy is $E_{Q\xi} = p^2/2m = (p_x^2 + p_y^2 + p_z^2)/2m = 3\hbar^2\pi^2/2m\xi_Q^2$ where m is the mass of the quantum. In the symmetric situation that pertains prior to any disturbance, each dimension is the same as the others, and we can simplify our analysis to one dimension leaving

$$E_{Q\xi} = K_{Q\xi 0}/\xi_Q^2 \qquad (4)$$

In Eq. (4), for one dimension, $K_{Q\xi 0} = \hbar^2\pi^2/2m$. $K_{Q\xi 0}$ is the nominal quantum-force-parameter for the quantum-cube. Differentiating $E_{Q\xi}$ with respect to ξ_Q leaves the expression for the magnitude of the quantum-force $F_{Q\xi}$ acting on two opposing faces of the cube that arises from the quantum-pressure:

$$F_{Q\xi} = dE_{Q\xi}/dX_Q = 2K_{Q\xi 0}/\xi_Q^3 \qquad (5)$$

Positive signs are employed in Eqs. (2) through (5) because they involve magnitudes of the various quantities. Since $\mathbf{F}_{T\xi}$ arises from a tension, $\mathbf{F}_{T\xi}$ is directed inward. Since $\mathbf{F}_{Q\xi}$ arises from a pressure, $\mathbf{F}_{Q\xi}$ is directed outward. The physical picture is that of an elastic-entity being both pushed outward by its own quantum-pressure and simultaneously pulled inward by its own internal tension.

It is of some interest to consider Eq. (5) to be fundamental rather than Eq. (4) and to then determine the quantum-energy of the aetherial-quantum-cube. To do so, we evaluate the work done against the quantum-force as the aetherial-quantum-cube is compressed inward from infinity, since in this case the force of Eq. (5) is zero at infinity. For the right-side-face of a very long aetherial-quantum-box we integrate the force $-2K_{Q\xi0}/(2x)^3 = -K_{Q\xi0}/4x^3$ from $x = \infty$ to $x = \xi_Q/2$. We use 2x in the force because the distance from zero to the position of the right-face (x) is half of the full length of the aetherial-quantum-cube. Performing the integration as we compress the right-face results in $K_{Q\xi0}/8x^2$ which we need to evaluate from ∞ to $\xi_Q/2$, or $E = K_{Q\xi0}/8(\xi_Q/2)^2 = K_{Q\xi0}/2\xi_Q^2$. And since work against the left-face contributes the same amount of energy, the total quantum-energy is $E_{Q\xi} = K_{Q\xi0}/\xi_Q^2$, Eq. (4). To compress the box to a length of ξ_Q we must push inward, in the opposite direction of d**x**, which is why the integrand ($-K_{Q\xi0}/4x^3$) has a minus sign. (As discussed above, here we are focusing on just a single dimension. Had we instead used a very large aetherial-quantum-cube in three dimensions and compressed all six sides we would have ended up with three times the energy, which would have been absorbed into our definition of the constant $K_{Q\xi0}$. See the discussion surrounding Eq. (4).)

The total energy of one dimension of our aetherial-quantum-cube is the sum $E_\xi = E_{T\xi} + E_{Q\xi} = (1/2)K_{T\xi0}\xi_Q^2 + K_{Q\xi0}/\xi_Q^2$, the extremum of which can be found by setting

$$(dE_\xi/d\xi_Q)_0 = K_{T\xi0}\xi_Q - 2K_{Q\xi0}/\xi_Q^3 = 0 \qquad (6)$$

From Eq. (6) we see that $K_{T\xi 0}\xi_Q = 2K_{Q\xi 0}/\xi_Q^3$ at the extremum. Defining ξ_0 as the value obtained at the extremum, $\xi_0^4 = 2K_{Q\xi 0}/K_{T\xi 0}$, or,

$$\xi_0 = (2K_{Q\xi 0}/K_{T\xi 0})^{1/4} \quad (7)$$

To see whether the extremum is a minimum or a maximum, the second derivative is evaluated:

$$(d^2E_\xi/d\xi_Q^2)_0 = (K_{T\xi 0} + 6K_{Q\xi 0}/\xi_Q^4)_0 = K_{T\xi 0} + 6K_{Q\xi 0}/(2K_{Q\xi 0}/K_{T\xi 0}) = 4K_{T\xi 0} \quad (8)$$

Eq. (8) is positive, indicating an energy minimum. To achieve this energy minimum the aetherial-quantum-cube will have the length given by Eq. (7) for each dimension. Hence, with Q_ξ the amount of an aether-quantum (which we will see has units of charge) the attached-aether density ρ_0 in the absence of any disturbances is a constant:

$$\rho_0 = Q_\xi/\xi_0^3 = Q_\xi/(2K_{Q\xi 0}/K_{T\xi 0})^{3/4} = Q_\xi(K_{T\xi 0}/2K_{Q\xi 0})^{3/4} \quad (9)$$

It is the equilibrium between the inward force due to tension and the outward force due to quantum-pressure that leads to Eq. (9). However, the tension and pressure are continuous and bi-directional which could lead to some confusion. Consider two cubes, a left-cube and a right-cube, that share a common face. $\mathbf{F}_{T\xi}$ on the left face of the right-cube is inward, which is to the right. But $\mathbf{F}_{T\xi}$ on the left-cube is also inward for it, which is now to the left. The net force due to tension on that face is therefore zero, which might cause doubt about the derivation of Eq. (9). However, the quantum-pressure within the left-cube is directed toward the right, and it exactly balances and offsets the tension-force exerted by the left-cube. Therefore, the net force from neighboring cubes is zero, and the derivation of Eq. (9) need not include effects from neighboring cubes in the case treated in this section, where no external work has been done on the aether-quantum.

B.4. Forces, Energy and Density of the Sub-Quantum Analysis-Cube

When analyzing finite-sized objects, such as the aetherial-quantum-cubes, artifacts related to the finite size may arise. It is desirable for our purposes to eliminate those artifacts from our analysis.

Toward that aim we will now introduce the analysis-cube. The analysis-cube is a cube with each dimension having a nominal size

$X_Q = \xi_Q/n$ (10)

In Eq. (10) n is some value greater than one. By letting n tend toward infinity, we can analyze the aether in the continuum limit, achieving exact expressions for the values of fields. (See section A.3 above for our assumption on the validity of this approach.) The quantum effects manifest in the individual quantum; we will now investigate how those manifestations relate to the sub-quantum. As a first observation, we see that the volume of the analysis-cubes is $1/n^3$ times that of the aetherial-quantum-cubes. Hence, for homogenous and isotropic cubes, we arrive at the following energy relations:

$E_{T0} = E_{T\xi}/n^3 = (1/2)K_{T\xi0}\xi_Q^2/n^3 = (1/2)K_{T0}X_Q^2$ (11)

$E_{Q0} = E_{Q\xi}/n^3 = K_{Q\xi0}/\xi_Q^2 n^3 = K_{Q0}/X_Q^2$ (12)

In the above, E_{T0} is the tension-energy and E_{Q0} is the quantum-energy of the analysis-cube, K_{T0} is the tension-parameter and K_{Q0} is the quantum-force-parameter where all quantities without the subscript ξ refer to the analysis-cube entities. All quantities are for the case of an undisturbed aether. With Eq. (10), $X_Q = \xi_Q/n$, we find that $(1/2)K_{T\xi0}\xi_Q^2/n^3 = (1/2)K_{T0}X_Q^2 = (1/2)K_{T0}(\xi_Q/n)^2$ and $K_{Q\xi0}/\xi_Q^2 n^3 = K_{Q0}/X_Q^2 = K_{Q0}/(\xi_Q/n)^2$, which leads us to

$K_{T0} = K_{T\xi0}/n$ (13)

$K_{Q0} = K_{Q\xi0}/n^5$ (14)

We can now form the force relations by taking the appropriate derivatives of Eqs. (11) and (12):

$F_{T0} = |dE_T/dX_Q| = K_{T0}X_Q$ (15)

$F_{Q0} = |dE_Q/dX_Q| = 2K_{Q0}/X_Q^3$ (16)

F_{T0} is the total tension-force and F_{Q0} is the total quantum-force present on the faces of the analysis-cube in the undisturbed aether. Using Eq. (2), $F_{T\xi} = K_{T\xi0}\xi_Q$, along with Eqs. (10), (13) and (15) we see that $F_{T0} = K_{T0}X_Q = (K_{T\xi0}/n)(\xi_Q/n) = F_{T\xi}/n^2$. Now both F_{T0} and $F_{T\xi}$ are the total force on the cube-face. Forming the force per unit area we find

$$F_{T0}/X_Q^2 = (F_{T\xi}/n^2)/(\xi_Q/n)^2 = F_{T\xi}/\xi_Q^2 \tag{17}$$

Eq. (17) informs that the force per unit area due to the tension remains the same independent of the size of our analysis-cube, as it must. Using Eq. (5), $F_{Q\xi} = 2K_{Q\xi0}/\xi_Q^3$, along with Eqs. (10), (14) and (16) we see that $F_{Q0} = 2K_{Q0}/X_Q^3 = (2K_{Q\xi0}/n^5)/(\xi_Q/n)^3 = F_{Q\xi}/n^2$. With F_{Q0} and $F_{Q\xi}$ the total force on the cube-face we form the forces per unit area to find

$$F_{Q0}/X_Q^2 = (F_{Q\xi}/n^2)/(\xi_Q/n)^2 = F_{Q\xi}/\xi_Q^2 \tag{18}$$

Eq. (18) relates that the force per unit area due to the quantum-pressure remains the same independent of the size of our analysis-cube, as it must.

The total energy of one dimension of the analysis-cube is the sum $E = E_{T0} + E_{Q0} = (1/2)K_{T0}X_Q^2 + K_{Q0}/X_Q^2$, the extremum of which can be found by setting

$$(dE/dX_Q)_0 = K_{T0}X_Q - 2K_{Q0}/X_Q^3 = 0 \tag{19}$$

From Eq. (19) we see that $K_{T0}X_Q = 2K_{Q0}/X_Q^3$ at the extremum. Defining X_0 as the value obtained at the extremum we have the following useful relations

$$X_0^4 = 2K_{Q0}/K_{T0} \tag{20}$$

$$K_{T0}X_0 = 2K_{Q0}/X_0^3 \tag{21}$$

$$K_{T0}X_0^2 = 2K_{Q0}/X_0^2 \tag{22}$$

$$X_0 = (2K_{Q0}/K_{T0})^{1/4} \tag{23}$$

To see whether the extremum is a minimum or a maximum, the second derivative is evaluated:

$$(d^2E/dX_Q^2)_0 = (K_{T0} + 6K_{Q0}/X_Q^4)_0 = K_{T0} + 6K_{Q0}/(2K_{Q0}/K_{T0}) = 4K_{T0} \tag{24}$$

Eq. (24) is positive, indicating an energy minimum. To achieve this energy minimum the analysis-cube will have the length given by Eq. (23) for each dimension. And now, with $Q_A = Q_\xi/n^3$ the amount of aether in the analysis-cube, and using Eqs. (7), (13), (14) and (23) the attached-aether density is to seen to remain at the value ρ_0 given in Eq. (9) when there are no aetherial disturbances:

$\rho_0 = Q_A/X_0^3 = Q_A/(2K_{Q0}/K_{T0})^{3/4} = (Q_\xi/n^3)/[2(K_{Q\xi0}/n^5)/(K_{T\xi0}/n)]^{3/4}$

$= Q_\xi(K_{T\xi0}/2K_{Q\xi0})^{3/4} = Q_\xi/\xi_0^3$ (25)

B.5. Energy and Work Equations of the Arbitrary Analysis-Cube

When disturbances of detached-aether or extrinsic-energy enter a region of attached-aether, those disturbances will be seen below to compress or expand the attached-aether. In addition to expansion or compression, portions of the attached-aether will displace. The displacement occurs because expansion or compression of the attached-aether in some regions will result in a displacement of the attached-aether in other regions, as the regions push or pull on each other. The displacement will be shown to alter the force and energy equations.

We see above from Eqs. (11) and (15) that the tension-energy is $E_{T0} = (1/2)K_{T0}X_Q^2$ and that the tension-force on the walls is $F_{T0} = K_{T0}X_Q$. These expressions resulted from the special case of an undisturbed aether wherein the force parameter K_{T0} is a constant. However, we will see below that the force parameter may not remain constant, leading to the following expressions for the arbitrary case:

$F_T = K_T X_Q$ (26)

$E_T = (1/2)K_T X_Q^2$ (27)

E_T is the energy it takes to expand the analysis-cube from 0 to X_Q. We have used K_T rather than K_{T0} in Eqs. (26) and (27) because it will be seen later that K_T can vary from its undisturbed-aether value.

Similarly Eqs. (12) and (16) relate that the quantum-energy is $E_{Q0} = K_{Q0}/X_Q^2$ and that the quantum-force on the walls is $F_{Q0} = 2K_{Q0}/X_Q^3$. Each of those equations are for the case of the undisturbed aether, and the quantum-force parameter K_{Q0} is considered a constant. In general, the quantum-force parameter will not be a constant leaving us with

$$F_Q = 2K_Q/X_Q^3 \quad (28)$$

$$E_Q = K_Q/X_Q^2 \quad (29)$$

E_Q is the energy it takes to compress the analysis-cube from infinity to X_Q. We have used K_Q rather than K_{Q0} in Eqs. (28) and (29) because it will be seen later that K_Q can vary from its undisturbed-aether value. Notice that Eqs. (26) and (28) are for the force magnitude, and there is no direction nor sign.

When portions of attached-aether are displaced, work will be done. The tension and pressure fields may do work on the attached-aether, or, the attached-aether may do work on the tension and pressure fields. The work equation will of course be given by the integral of the force over the distance travelled, $W = \int F\, dx$.

For the displacement work we will treat the quantum-cube before treating the analysis-cube. On the face of a quantum-cube, the tension-force is given by Eq. (2), $F_{T\xi} = K_{T\xi 0}\xi_Q$; $F_{T\xi}$ is the tension force per unit area multiplied by the area of the cube face. Similarly, the quantum-force of Eq. (5), $F_{Q\xi} = 2K_{Q\xi 0}/\xi_Q^3$, is the quantum-pressure multiplied by the area of the cube face. As we displace, expand or compress an attached-aether quantum-cube within the tension and pressure fields, we propose:

The Aetherial Displacement-Work Law. When attached-aether is displaced, work is done on the aether by the tension and quantum-force fields that is proportional to the force, proportional to the distance of displacement, and proportional to the amount of aether displaced.

The aetherial displacement-work law is expressed mathematically for the aetherial-quantum as:

$$W_{TD\xi} = K_c \int F_{T\xi}\, dx \tag{30}$$

$$W_{QD\xi} = K_c \int F_{Q\xi}\, dx \tag{31}$$

In Eqs. (30) and (31) K_c is a coupling constant specific to the amount of aether in an aetherial-quantum. For the value $K_c = 1$ we have already used these equations for the expansion of the quantum-cube against the tension and for the compression of the quantum-cube against the quantum-pressure. But here we observe that the work done by displacing the quantum-cube through the tension and quantum-pressure fields may be some fraction of the work done on a cube face due to expansion and/or contraction, as the work done by stretching (or compressing) the cube may not be the same as the work done by displacing the entire cube through the fields. (In both cases, expansion/contraction and displacement, the work will be the force multiplied by the distance, but the force against displacement includes an additional coupling coefficient, K_c.)

For the displacement-work equations we notice that the aetherial displacement-work law states the work will also scale as the amount of aether moving through the fields. For aether at the nominal density, this leads to scaling Eqs. (30) and (31) by a factor $(X_0/\xi_0)^3$, which is the analysis-cube volume divided by the quantum-cube volume. Thus, the displacement-work equation for the tension work on the analysis-cube is $W_{TD}=(X_0/\xi_0)^3 W_{TD\xi}=(X_0/\xi_0)^3 K_c \int F_{T\xi}\, dx=(X_0/\xi_0)^3 K_c \int K_{T\xi}\xi_0\, dx$ $= (X_0/\xi_0)^2 K_c \int K_T\xi_0\, dx$ or

$$W_{TD} = (X_0/\xi_0) K_c \int K_T X_0\, dx \tag{32}$$

To arrive at Eq. (32) use is made of Eq. (2), $F_{T\xi} = K_{T\xi 0}\xi_Q$, Eq. (10), $X_Q = \xi_Q/n$, Eq. (13) $K_{T0} = K_{T\xi 0}/n$, $K_{T\xi} \approx K_{T\xi 0}$ and $X_Q \approx X_0$, so we have $(X_0/\xi_0) = (1/n)$ and $K_{T\xi 0} = nK_{T0}$, so $(X_0/\xi_0)K_{T\xi} = K_{T0}$. Later we will see that K_T can vary, so we replace K_{T0} with K_T in Eq. (32).

The displacement-work equation for the quantum-pressure work is $W_{QD} = (X_0/\xi_0)^3 W_{QD\xi} = (X_0/\xi_0)^3 K_c \int F_{Q\xi}\,dx = (X_0/\xi_0)^3 K_c \int 2K_{Q\xi 0}/\xi_0^3\,dx = (X_0/\xi_0)^6 K_c \int 2K_{Q\xi 0}/X_0^3\,dx$,

$$W_{QD} = (X_0/\xi_0) K_c \int 2K_Q/X_0^3\,dx \qquad (33)$$

To arrive at Eq. (33) use is made of Eq. (5), $F_{Q\xi} = 2K_{Q\xi 0}/\xi_Q^3$, Eq. (10), $X_Q = \xi_Q/n$, Eq. (14) $K_{Q0} = K_{Q\xi 0}/n^5$, and $X_Q \approx X_0$, as we have $(X_0/\xi_0) = (1/n)$ and $K_{Q\xi 0} = n^5 K_{Q0}$, so $(X_0/\xi_0)^5 K_{Q\xi 0} = K_{Q0}$. Later we will see that K_Q can vary, so we replace K_{Q0} with K_Q in Eq. (33).

Note that Eqs. (32) and (33) are both the magnitude of the work done. In what follows we will see that the displacement work can be either positive or negative, depending on the physics of the particular situation. We will include the appropriate sign as needed.

B.6. Arbitrary Analysis-Cubes and Analysis-Spheres and the More General Case.

In what follows we will often do an analysis of a constant-density individual cube or sphere. By letting our cubes or spheres tend toward zero size (and in the case of spheres, using different sized spheres all tending toward zero size) we assume that we can fill any arbitrary volume with any arbitrary density to any arbitrary accuracy. (Each such cube or sphere will have its own constant density, which may be different than that of other cubes or spheres.) By further assuming a principle of superposition of fields, the treatment of the constant-density individual cube or sphere is sufficient for the more general case.

Part C. Electromagnetism

In this section the aether will be analyzed in the absence of gravitational effects. Gravitation will be analyzed in section D below.

C.1. The Electromagnetic Aetherial Density Law

Consider adding a small amount of positive-detached-aether into an analysis-cube. (This is the case when one quantum of detached-aether occupies a volume of many attached aetherial-quantum-cubes.) In this case, Eq. (1) becomes $\rho_{PA} + \rho_{PD} = \rho_{NA}$, which can be met by ρ_{PA} and ρ_{NA} becoming different from their nominal value of ρ_0:

$$\rho_{PA} = \rho_0 - f\rho_{PD} \tag{34}$$

$$\rho_{NA} = \rho_0 + (1 - f)\rho_{PD} \tag{35}$$

In Eqs. (34) and (35) f is some arbitrary fraction, with $0 < f < 1$. To achieve the decrease in ρ_{PA}, the volume occupied by the positive-attached-aether must be expanded. (Since the amount of attached-aether is the same within the analysis-cube, the density can only be decreased if the analysis-cube occupies a larger volume.) And to increase ρ_{NA}, the negative-attached-aether must be compressed. Prior to expansion or compression each analysis-cube contains $Q_0 = \rho_0 X_0^3$ of attached-aether. For the positive case, after expansion Q_0 occupies a larger volume V at the lower density ρ_{PA}: $Q_0 = \rho_{PA}V$. Hence, $V = Q_0/\rho_{PA} = \rho_0 X_0^3/\rho_{PA} = \rho_0 X_0^3/(\rho_0 - f\rho_{PD}) = X_0^3/(1 - f\rho_{PD}/\rho_0)$. Assuming $\rho_{PD} << \rho_0$, $V = (1 + f\rho_{PD}/\rho_0)X_0^3$, and due to symmetry in the expansion, each dimension of the positive analysis-cube increases to $(1 + f\rho_{PD}/\rho_0)^{1/3}X_0 \approx (1 + f\rho_{PD}/3\rho_0)X_0$. (Here, and throughout this work, we make use of the binomial theorem.) With $C_1 = \rho_{PD}X_0/3\rho_0$, the change in each dimension is

$$\delta X_{PA} = f\rho_{PD}X_0/3\rho_0 = fC_1 \tag{36}$$

For the negative-attached-aether, achieving $\rho_{NA} = \rho_0 + (1 - f)\rho_{PD}$ requires that the negative-analysis-cube be compressed. The volume will become $V = Q_0/\rho_{NA} = \rho_0 X_0^3/\rho_{NA} = \rho_0 X_0^3/[\rho_0 + (1\text{–}f)\rho_{PD}] = X_0^3/[1 + (1\text{–}f)\rho_{PD}/\rho_0] \approx [1 - (1\text{–}f)\rho_{PD}/\rho_0]X_0^3$ and each dimension of the negative analysis-cube decreases to $[1 - (1\text{–}f)\rho_{PD}/\rho_0]^{1/3}X_0 \approx [1 - (1\text{–}f)\rho_{PD}/3\rho_0]X_0$. The change in each dimension is

$$\delta X_{NA} = -(1-f)\rho_{PD}X_0/3\rho_0 = -(1-f)C_1 \qquad (37)$$

Eq. (23) relates that both the positive and negative analysis-cubes have an equilibrium value of $X_0 = (2K_{Q0}/K_{T0})^{1/4}$ before any expansion or compression, and Eqs. (19) and (24) relate that dE/dX is zero and $d^2E/dX^2 = 4K_{T0}$ at $X = X_0$. This leads to an expression for the energy within both the positive and negative analysis-cubes for small changes δX_A near X_0 of

$$E = E_0 + 2K_{T0}(\delta X_A)^2 \qquad (38)$$

Eq. (38) arises since the first term in the Taylor expansion of E is zero near E_0 and the second term is $2K_{T0}(\delta X_A)^2$. By taking the first and second derivatives of Eq. (38) with respect to δX_A and evaluating at $\delta X_A = 0$, we arrive at Eqs. (19) and (24), verifying our choice of the constant $2K_{T0}$. After immersion of detached-positive-aether the total energy change of the positive analysis-cube is $\Delta E_P = 2K_{T0}(\delta X_{PA})^2 = 2K_{T0}f^2C_1^2$, while for the negative cube $\Delta E_N = 2K_{T0}(\delta X_{NA})^2 = 2K_{T0}(1\text{–}f)^2C_1^2$. The total change in energy in the region occupying these cubes is $\Delta E = \Delta E_P + \Delta E_N = 2K_{T0}C_1^2[f^2 + (1\text{–}f)^2] = 2K_{T0}C_1^2[2f^2 + 1 - 2f]$. Differentiating ΔE with respect to f yields $d(\Delta E)/df = 2K_{T0}C_1^2[4f - 2]$, which has an extremum at f = 1/2, and taking the second derivative $d^2(\Delta E)/df^2 = 8K_{T0}C_1^2$, shows that this extremum is a minimum. Therefore f = 1/2 will manifest itself in nature. With f = 1/2 Eqs. (34) and (35) become:

$$\rho_{PA} = \rho_0 - \rho_{PD}/2 \qquad (39)$$

$$\rho_{NA} = \rho_0 + \rho_{PD}/2 \qquad (40)$$

Applying the same analysis to a region immersed in negative-detached-aether and combining the two cases leads to the more general expressions:

$$\rho_{PA} = \rho_0 - \rho_{PD}/2 + \rho_{ND}/2 \tag{41}$$

$$\rho_{NA} = \rho_0 - \rho_{ND}/2 + \rho_{PD}/2 \tag{42}$$

Stated in words:

The Electromagnetic Aetherial Density Law. When no detached-aether and no extrinsic-mass is present in a volume, the attached-aether densities are each equal to a constant value ρ_0. When a small amount of detached-aether is present and no extrinsic-energy is present, the density of the attached-aether of like (unlike) kind is reduced (increased) by one half the density of the detached-aether present.

The derivation of $f = 1/2$ has made use of the expressions $dE/dX_Q = 0$ and $d^2E/dX_Q^2 > 0$, and these expressions were derived above assuming no external work has been done on the cube of analysis. However, the derivation will still hold in the case where external work has been applied. Eqs. (72) and (77) below show that the only change is that E_0 of Eq. (38) will also include a term related to the cube displacement to lowest order in small quantities.

Note also that the total aether-density is the sum the attached and detached parts:

$$\rho_P = \rho_{PA} + \rho_{PD} = \rho_0 + \rho_{PD}/2 + \rho_{ND}/2 \tag{43}$$

$$\rho_N = \rho_{NA} + \rho_{ND} = \rho_0 + \rho_{PD}/2 + \rho_{ND}/2 \tag{44}$$

We see that the presence of a small amount of detached-aether, whether negative or positive, increases the total density of each kind of aether equally.

C.2 – Electrodynamic Density Variations in the Attached-Aether due to Detached-Aether. Poisson's Equation.

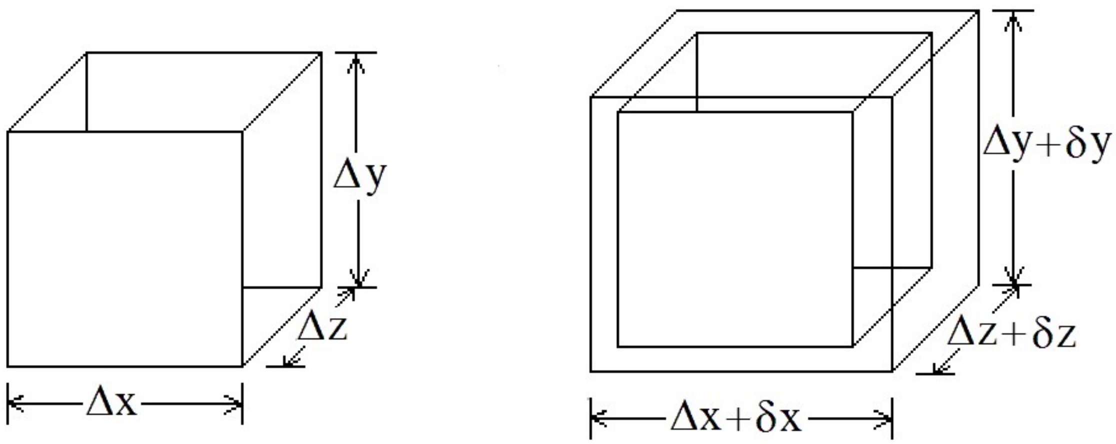

Figure 1. Analysis-cube of undistorted aether (left). Analysis-cube of expanded attached-aether showing original cube within (right).

Figure 1 shows two aetherial-analysis-cubes. On the left is an undistorted cube of positive-attached-aether, which is Δx wide by Δy high by Δz deep. ($\Delta x = \Delta y = \Delta z = X_0$.) In the undistorted state, the density of the positive-attached-aether, ρ_{PA}, is equal to the nominal density, ρ_0. The density of anything is the amount of it divided by the volume it occupies. Hence, on the cube on the left, the amount of positive-attached-aether within the volume is $\rho_0 \Delta x \Delta y \Delta z$.

If an amount of positive-detached-aether is added to the cube on the left, the size of the volume required to contain it will increase. (Eq. (39) informs that the density of the positive-attached-aether will decrease by half the density of the injected positive-detached-aether. That density decrease requires an increase in volume of the positive-attached-aether.) The required expanded volume is shown on the right side of Fig. 1. In that second cube, the width is increased by an amount δx, while the height and depth are increased by δy and δz respectively. The amount of positive-attached-aether remains $\rho_0(\Delta x\Delta y\Delta z)$, but with the larger volume the positive-attached-aether density is now:

$$\rho_{PA} = \rho_0(\Delta x\Delta y\Delta z)/[(\Delta x + \delta x)(\Delta y + \delta y)(\Delta z + \delta z)]$$

$$\approx \rho_0(\Delta x\Delta y\Delta z)/(\Delta x\Delta y\Delta z + \delta x\Delta y\Delta z + \delta y\Delta x\Delta z + \delta z\Delta x\Delta y)$$

$$= \rho_0/(1 + \delta x/\Delta x + \delta y/\Delta y + \delta z/\Delta z) \approx \rho_0(1 - \delta x/\Delta x - \delta y/\Delta y - \delta z/\Delta z) \quad (45)$$

It is assumed that the nominal density of the positive-attached-aether is far greater than the density of the positive-detached-aether, and so the δ quantities are very much smaller than the Δ quantities in Eq. (45). This allows the approximations made in Eq. (45).

To further refine Eq. (45), recall that the vector **P** is the displacement of the positive-attached-aether with respect to its equilibrium position. Hence, the x component of **P** at x is the displacement of the center of the cube shown in Fig. 1, and the x component of **P** at x+Δx/2 is the displacement of the right yz face of that cube. Therefore $\delta x/2 = P_x(x + \Delta x/2, y, z, t) - P_x(x, y, z, t)$. Dividing each side by $\Delta x/2$ leaves $\delta x/\Delta x = [P_x(x + \Delta x/2, y, z, t) - P_x(x, y, z, t)]/\Delta x/2 = \partial P_x/\partial x$, where the last equality is in the limit when we shrink our analysis-cube to zero size. Repeating the derivation for y and z will lead to similar expressions. Hence, Eq. (45) can be re-expressed as $\rho_{PA} = \rho_0(1 - \delta x/\Delta x - \delta y/\Delta y - \delta z/\Delta z) = \rho_0(1 - \partial P_x/\partial x - \partial P_y/\partial y - \partial P_z/\partial z)$, or,

$$\rho_{PA} = \rho_0(1 - \nabla \cdot \mathbf{P}) \quad (46)$$

In Eq. (46) and throughout this work, ∇ has its usual definition, $\nabla = \mathbf{i}\partial/\partial x + \mathbf{j}\partial/\partial y + \mathbf{k}\partial/\partial z$.

Eq. (46) can be manipulated by bringing $\rho_0\nabla \cdot \mathbf{P}$ to the left-hand side, bringing $-\rho_{PA}$ to the right-hand side and dividing through by ρ_0 to get $\nabla \cdot \mathbf{P} = (\rho_0 - \rho_{PA})/\rho_0$. Then utilizing Eq. (41) from above:

$$\nabla \cdot \mathbf{P} = (\rho_0 - \rho_{PA})/\rho_0 = (\rho_{PD} - \rho_{ND})/2\rho_0 \tag{47}$$

(While Figure 1 shows the situation where the cube expands due to the presence of some detached-positive-aether, the cube could instead compress by the presence of some negative-detached-aether. The derivation immersing ρ_{ND} is similar to that for ρ_{PD}. Eq. (47) includes both effects.)

We can decompose **P** into its longitudinal ($\mathbf{P_L}$) and transverse ($\mathbf{P_T}$) components, where $\mathbf{P_L}$ is defined as having zero curl ($\nabla \times \mathbf{P_L} = 0$) and $\mathbf{P_T}$ is defined as having zero divergence ($\nabla \cdot \mathbf{P_T} = 0$).

(This is the Helmholtz decomposition.) Since the longitudinal component of the vector field has zero curl it can be expressed as the gradient operator applied to a scalar field:

$$\mathbf{P_L} = \nabla\Psi_P \tag{48}$$

Ψ_P is just a function that has a single scalar value at every point in space and time.

Since the divergence of the transverse component of **P** is zero, the divergence of **P** is equal to the divergence of its longitudinal portion alone, $\nabla \cdot \mathbf{P} = \nabla \cdot \mathbf{P_L}$. And now, with $\mathbf{P_L} = \nabla\Psi_P$:

$$\nabla \cdot \mathbf{P} = \nabla \cdot \mathbf{P_L} = \nabla \cdot \nabla\Psi_P = \nabla^2\Psi_P \tag{49}$$

Next, combine Eqs. (47) and (49) to yield:

$$\nabla^2\Psi_P = \nabla \cdot \mathbf{P_L} = \nabla \cdot \mathbf{P} = (\rho_{PD} - \rho_{ND})/2\rho_0 \tag{50}$$

A similar derivation (replace P by N) can be applied to the negative-aether to arrive at:

$$\mathbf{N_L} = \nabla\Psi_N \tag{51}$$

and

$$\nabla^2\Psi_N = \nabla \cdot \mathbf{N_L} = \nabla \cdot \mathbf{N} = (\rho_{ND} - \rho_{PD})/2\rho_0 \tag{52}$$

Subtract Eq. (51) from Eq. (48):

$$\mathbf{P_L} - \mathbf{N_L} = \nabla\Psi_P - \nabla\Psi_N \tag{53}$$

Subtract Eq. (52) from Eq. (50):

$$\nabla^2(\Psi_P - \Psi_N) = (\rho_{PD} - \rho_{ND})/\rho_0 \tag{54}$$

Now define ϕ by $\phi = -(\Psi_P - \Psi_N)\rho_0/\varepsilon_0$, where ε_0 is the permittivity of free space and define $\rho_D = (\rho_{PD} - \rho_{ND})$. We have $(\Psi_P - \Psi_N) = -\varepsilon_0\phi/\rho_0$ and hence $\nabla^2(\Psi_P - \Psi_N) = -\varepsilon_0\nabla^2\phi/\rho_0 = (\rho_{PD} - \rho_{ND})/\rho_0 = \rho_D/\rho_0$. Hence we see that Eq. (54) is Poisson's Equation:

$$\nabla^2\phi = -\rho_D/\varepsilon_0 \tag{55}$$

With this definition for ϕ we can also rewrite Eq. (53) as:

$$\mathbf{P_L} - \mathbf{N_L} = \nabla(\Psi_P - \Psi_N) = -\varepsilon_0\nabla\phi/\rho_0 \tag{56}$$

Dimensional analysis. Both ρ_D and ρ_0 refer to aetherial densities. Eq. (55) informs us that aetherial density is a charge density, and so the dimensions of ρ_D and ρ_0 are $C/m^3 = A\ s\ m^{-3}$. From Eq. (56) $\varepsilon_0\nabla\phi/\rho_0$ is a length. ε_0 has dimensions $m^{-3}\ kg^{-1}\ s^4\ A^2$. ∇ has dimensions m^{-1}. Hence, from Eq. (55) ϕ has dimensions of $m^2(A\ s\ m^{-3})/(m^{-3}\ kg^{-1}\ s^4\ A^2) = m^2\ A^{-1}\ s^{-3}\ kg = kg\ m^2\ C^{-1}\ s^{-2}$ = J/C or Volts.

It is useful to observe that by comparing Eqs. (50) and (52):

$$\nabla \cdot \mathbf{N_L} = -\nabla \cdot \mathbf{P_L} \tag{57}$$

Eq. (57) shows that the divergence of $\mathbf{N_L}$ is equal to the negative of the divergence of $\mathbf{P_L}$. At this point it is relevant to recall the physics of the situation. Referring back to Fig. 1 and the analysis that is used to derive the equations above, we see that the presence of a density of positive-detached-aether within a cube of positive-attached-aether pushes the boundary of the cube outward. Since the situation is symmetric, the effect is the same in each dimension. That same density of positive-detached-aether within a cube of negative-attached-aether pulls the boundary of the negative-attached-aether cube inward. The opposite effect is achieved by the presence of negative-detached-aether within the cubes of attached-aether. In each case, the presence of detached-aether leads to displacement of the negative-attached-aether that is always equal and opposite to the displacement of the positive-attached-aether. Since injection of detached-aether is the only physical cause for the longitudinal displacements $\mathbf{P_L}$ and $\mathbf{N_L}$ of the attached-aether, we arrive at:

$$\mathbf{N_L} = -\mathbf{P_L} \tag{58}$$

We see that the presence of detached-aether forces a purely longitudinal separation between the aetherial components, and that the separation is achieved with the negative-attached-aether displacement equal and opposite to the positive-attached-aether displacement.

C.3 – The Detached-Aether-Immersion Force (The Delta-Force)

C.3.1. Aetherial Forces in an Isolated Analysis-Cube. Recalling Eqs. (15) and (16), $F_{T0} = K_{T0}X_Q$ and $F_{Q0} = 2K_{Q0}/X_Q^3$, respectively, we see that an increase in the size of the analysis-cube from X_0 to $X_Q = X_0+\delta X_P$ increases the inward tension-force $\mathbf{F_T}$ on the sides of the cube and simultaneously decreases the outward quantum-force $\mathbf{F_Q}$ on the sides of the cube. Were these forces not otherwise compensated for, the cube would return to its original size, X_0. To maintain the larger size X_Q the presence in the cube of positive-detached-aether must therefore result in a detached-aether-immersion force $\mathbf{F_\delta}$ (the delta-force) that acts on the walls of the positive-attached-aether analysis-cube that is equal and opposite to the sum of the longitudinal-tension-force and the quantum-pressure force,

$$\mathbf{F_\delta} = -\mathbf{F_{TL}} - \mathbf{F_Q}. \tag{59}$$

C.3.2. A Spherical Attached-Aether Region Containing Like-Kind Detached-Aether. Consider now the injection of a sphere of positive-detached-aether into the positive-attached-aether. In this case, with $\rho_{ND} = 0$, Eq. (50) yields $\nabla \cdot \mathbf{P} = \rho_{PD}/2\rho_0$, which has the solutions

$$\mathbf{P_{IN}} = (\rho_{PD}/6\rho_0)(x\mathbf{i} + y\mathbf{j} + z\mathbf{k}) = (\rho_{PD}/6\rho_0)\mathbf{r} \qquad (r < R_0) \qquad (60)$$

$$\mathbf{P_{OUT}} = P_0R_0^2\hat{\mathbf{r}}/r^2 \qquad (r > R_0) \qquad (61)$$

In Eqs. (60) and (61) R_0 is the radius of the sphere and $P_0 = (\rho_{PD}/6\rho_0)R_0$ is the magnitude of P at R_0. Verifying Eqs. (60) and (61) are solutions to Eq. (50): $\nabla \cdot \mathbf{P_{IN}} = (1/r^2)\partial(r^2[(\rho_{PD}/6\rho_0)r])/\partial r =$ $(1/r^2)\partial(r^3\rho_{PD}/6\rho_0)/\partial r = \rho_{PD}/2\rho_0$ and $\nabla \cdot \mathbf{P_{OUT}} = (1/r^2)\partial(r^2[P_0R_0/r^2])/\partial r = (1/r^2)\partial P_0R_0/\partial r = 0$.

Notice that immersion of detached-aether into like-kind attached-aether will lead to an outward expansion of the like-kind attached-aether. Since tension is a force directed inward, the radially-outward motion of the attached-aether will do work against the tension. And since the quantum-pressure force is directed outward, the radially-outward motion of the attached-aether will do negative work against the quantum-pressure field.

C.3.3. Expansion (and Compression) Work Expressions in the Case of Uniform Spherical Sources. We will now consider expansion and compression work done on analysis-cubes for the case of a uniform spherical source. Slow injection of a uniform source into a spherical region will result in the expansion (or compression) of each analytical cube within that spherical region. Analysis-cubes a distance r from the center of the sphere will be displaced as well as expanded (or compressed). If there are n–2 analysis-cubes between cube-P and the central cube, and if each cube expands in size by δX, then the center of cube-P will displace by $(n–1)\delta X$, as it will move by $\delta X/2$ due to its own expansion, $\delta X/2$ due to the center-cube expansion, and δX due to the expansion of each of the n–2 analysis-cubes lying between cube-P and the center-cube. Once the source is completely injected, cube-P will move a distance P, and it, along with all other cubes inside the sphere, will expand (or compress) by an amount δX. During the injection we will use the variable x to define how far cube-P displaces due to the partially-injected source, and we will use the variable w to define how much cube-P expands or compresses due to the partially-injected source. When the cubes do not expand at all $w = 0$ and $x = 0$, and when they are fully expanded $w = \delta X$ and $x = P$, and since the movements will be linearly related (they are each proportional to the amount of source injected) we obtain

$$x = (P/\delta X)w \qquad (62)$$

The tension-force on a wall of an analytical cube is given by Eq. (26), $F_T = K_T X_Q$. The work done against the tension to compress or expand a cube by δX (in one dimension) is given by $W_{TC} = 2 \int K_T X_Q \, dw$, where the integral is evaluated from $w = 0$ to $w = \delta X/2$ since each wall moves a distance $\delta X/2$ and the 2 in front of the integral is because there are two walls. We also observe that $X_Q = X_0 + w$. This leaves the equation for the compression (or expansion) energy as $W_{TC} = 2 \int K_T(X_0+w) \, dw$. In what follows below we will use

$$W_{TC} = 2 \int F_T \, dw = 2 \int K_T X_0 \, dw \qquad \text{w is evaluated from 0 to } \delta X/2 \qquad (63)$$

In Eq. (63) we have dropped w from the $K_T(X_0+w)$ in the integrand. This is done because during integration $0 < w < \delta X/2$, and hence the w in the integrand will contribute to the result via a higher order of $\delta X/X_0$, and below we will only keep terms to the lowest non-vanishing order of $\delta X/X_0$. In using Eq. (63) the sign chosen for the work will be assigned depending on the physics of each situation.

The quantum-force on a wall of an analytical cube is given by Eq. (28), $F_Q = 2K_Q/X_Q^3$. The work done against the quantum-force to compress or expand a cube by δX (in one dimension) is given by $W_{QC} = 2 \int 2K_Q/X_Q^3$ dw, where the integral is again evaluated from w = 0 to w = $\delta X/2$, we again have a factor of 2 because there are two walls, and again, $X_Q = X_0 + w$. This leaves the equation for the compression (or expansion) energy as $W_{QC} = 2 \int 2K_Q/(X_0+w)^3$ dw. In what follows below we will use

$W_{QC} = 4 \int K_Q/X_0^3$ dw w is evaluated from 0 to $\delta X/2$ (64)

In Eq. (64) we have again dropped w from the X_0+w in the integrand as below we will keep terms only to the lowest non-vanishing order of $\delta X/X_0$. In using Eq. (64) the sign chosen for the work will be assigned depending on the physics of each situation.

Notice that Eqs. (63) and (64) are both of the general form

$W_C = 2 \int F$ dw w is evaluated from 0 to $\delta X/2$ (65)

C.3.4. The Tension-Force and Tension-Energy Inside a Spherical Attached-Aether Region Containing Like-Kind Detached-Aether. Eq. (60) informs us that adding like-kind detached-aether into a spherical region will cause the attached-aether cubes within that region to expand equally in each cartesian direction. Focusing first on the tension, Eq. (15) informs us that an expansion δX of an analysis-cube leads to a tension-force magnitude $F_{TP} = K_{T0}(X_0+\delta X) = K_{T0}X_0(1+\delta X/X_0)$ within that analysis-cube. (The subscript P is for **P**ositive-attached-aether.)

First consider cube-1, the analysis-cube at the center of the sphere. When it is expanded, the tension-force on its wall becomes $F_{TP1} = K_{T0}X_0(1+\delta X/X_0)$. Cube-1 is not displaced.

An immediately adjacent analysis-cube (cube-2) will also expand by δX but it will additionally be displaced. The displacement involves motion against the field, doing work on the field and increasing the tension-energy E_T. It will be shown below in section C.3.10 that displacement effects do not change the size of cube-2. With $E_T = (1/2)K_TX_Q^2$, and X_Q not changing due to displacement effects, we see that E_T changes but X_Q does not, and hence *K_T is no longer constant: the work done during displacement changes K_T.* K_T is the variable force parameter. K_{T0}, used above, is the constant force parameter of an aetherial cube in its nominal (undisturbed) state.

As we slowly add detached-aether into the spherical region, the work done against the tension-field by the displacement of any cube is given in Eq. (32), $W_{TD} = (X_0/\xi_0)K_c\int K_T X_0\, dx$. As a first-order approximation, we use the nominal tension-force of $F_T = K_{T0}X_0$ ($K_T = K_{T0}$ to first-order). Injection of detached-aether causes the center of cube-2 to move a distance $\delta X/2$ due to the expansion of cube-1 and a distance $\delta X/2$ due to its own expansion for a total displacement of δX. Prior to the detached-aether injection, the center of cube-2 is located at X_0. Performing the integral of Eq. (32) from X_0 to $X_0 + \delta X$ (the cube center moves from X_0 to $X_0 + \delta X$) the work done against the tension by the cube-2 displacement is $W_{TD2} = (X_0/\xi_0)K_c K_{T0} X_0 \delta X$.

Next, consider cube-3, which is adjacent to cube-2 and further away from the center of the detached-aether sphere. The work to displace cube-3 against the tension is again given by Eq. (32), but this time the displacement is $2\delta X$, with $\delta X/2$ coming from cubes 1 and 3, and δX coming from cube 2. Integrating Eq. (32) from $2X_0$ to $2X_0 + 2\delta X$, the work done on cube-3 during the displacement is $W_{TD3} = 2(X_0/\xi_0)K_c K_{T0} X_0 \delta X$.

And now, consider cube-J, where J is some large integer with the center of cube-J separated from the center of the detached-aether-sphere by $(J–1)X_0$. The energy to displace cube-J is again given by Eq. (32), but this time the displacement is $(J–1)\delta X$, with $\delta X/2$ coming from each of cubes 1 and J, and an additional δX coming from each of the cubes between cube-1 and cube-J. Integrating Eq. (32) from 0 to $(J–1)\delta X$, the work done against the tension on cube-J from the displacement is $W_{TDJ} = (J–1)(X_0/\xi_0)K_c K_{T0} X_0 \delta X$ to first order.

Continuing to set aside the energy change due to expansion so we can focus on the effect of displacement, the tension-energy of cube-J is $E_T = E_{T0} + W_{TDJ} = (1/2)K_{T0}X_0^2 + K_cK_{T0}X_0^2(J–1)\delta X/\xi_0$. ($E_{T0} = (1/2)K_{T0}X_0^2$.) With $E_T = (1/2)K_TX_0^2$, we arrive at the expression $K_T = K_{T0}(1 + 2K_c(J–1)\delta X/\xi_0)$. The tension-force remains $F_T = K_TX_0$, but now K_T includes the next order correction.

As detached-aether is slowly injected, the center of the Jth cube will move from $(J–1)X_0$ to $(J–1)X_0 + (J–1)\delta X$. The motion will thus cover a distance $(J–1)\delta X$. At the beginning of the motion F_{TP} is of course just $K_{T0}X_0$, and it is at the end that $F_{TP} = K_TX_0 = K_{T0}X_0[1 + 2K_c(J–1)\delta X/\xi_0]$. With x now defined as the deviation of the cube center from its nominal center (which varies from 0 to $(J–1)\delta X$ during the detached-aether injection) and adding a third subscript P for the effect of **P**ositive detached-aether, the full second-order expression for the tension-force in the analysis-cube is

$$F_{TPP} = K_TX_0 = K_{T0}X_0[1 + 2K_cx/\xi_0] \quad (66)$$

Eq. (66) is seen to be the nominal value of $K_{T0}X_0$ when there is no injected detached-aether (and $x = 0$), and $K_{T0}X_0[1 + 2K_c(J–1)\delta X/\xi_0]$ when we reach full injection (and $x = (J–1)\delta X$). The linearity follows because δX (and hence $(J–1)\delta X$) is linear with the amount of detached-aether injected. With the second-order expression for the tension just derived, we can now calculate the second-order effect on the cube tension-energy due to displacement. Eq. (32) gives the work done on the field by the cube displacement as $W_{TDJ} = (X_0/\xi_0)K_c\int K_TX_0\, dx = K_c(X_0/\xi_0)K_{T0}X_0 \int [1+2K_cx/\xi_0]\, dx = K_c(X_0/\xi_0)K_{T0}X_0x + K_c^2(X_0/\xi_0)^2K_{T0}x^2 = K_cK_{T0}X_0(X_0/\xi_0)(J–1)\delta X + K_c^2K_{T0}(X_0/\xi_0)^2(J–1)^2\delta X^2$. (For the displacement, the integral is evaluated between zero and its final displacement $(J–1)\delta X$.) Now

$(J-1)\delta X$ is the distance the cube moves from its nominal position, $P = (J-1)\delta X$, and hence $W_{TDJ} = K_cK_{T0}X_0(X_0/\xi_0)P + K_c^2K_{T0}(X_0/\xi_0)^2P^2$.

The expansion energy is now calculated using Eq. (63), $W_{TC} = 2\int K_TX_0\, dw$. From Eq. (66), $K_TX_0 = K_{T0}X_0[1 + 2K_cx/\xi_0]$. We now use Eq. (62), $x = (P/\delta X)w$, and we obtain $K_TX_0 = K_{T0}X_0[1+2K_c(P/\delta X)w/\xi_0]$. As specified in Eq. (63) we integrate from $w = 0$ to $w = \delta X/2$. Hence $W_{TC} = 2\int K_TX_0\, dw = 2K_{T0}X_0\int [1+2K_c(P/\delta X)w/\xi_0]\, dw = 2K_{T0}X_0w + 2K_{T0}X_0K_c(P/\delta X)w^2/\xi_0 = K_{T0}X_0\delta X + K_{T0}X_0K_cP\delta X/2\xi_0$. Recall that Eq. (63) is for one dimension only, and that inside the sphere the expansion will be the same in all three dimensions. Hence, $W_{TC3} = 3K_{T0}X_0\delta X + 3K_{T0}X_0K_cP\delta X/2\xi_0$. The total tension-energy of an arbitrary analysis-cube inside the spherical region is the undisturbed energy plus the displacement energy plus the expansion energy, $E_{TPPI} = (1/2)K_{T0}X_0^2 + K_cK_{T0}X_0(X_0/\xi_0)P + K_c^2K_{T0}(X_0/\xi_0)^2P^2 + 3K_{T0}X_0\delta X + 3K_{T0}X_0K_cP\delta X/2\xi_0$, or, $E_{TPPI} = K_{T0}X_0^2[(1/2) + K_cP/\xi_0 + K_c^2(P/\xi_0)^2 + 3\delta X/X_0 + 3K_cP\delta X/2X_0\xi_0]$. Here we have derived the force for positive P. For negative P (or **P** at any angle) the work done is still positive, since such a cube is still expanding outward against the tension. Hence we will replace P by its absolute value, $|\mathbf{P}|$:

$$E_{TPPI} = K_{T0}X_0^2[(1/2) + K_c|\mathbf{P}|/\xi_0 + K_c^2(|\mathbf{P}|/\xi_0)^2 + 3\delta X/X_0 + 3K_c|\mathbf{P}|\delta X/2X_0\xi_0] \tag{67}$$

In Eq. (67) and the above paragraph, the subscript TPPI refers to Tension of the Positive-attached-aether due to immersed Positive-detached-aether in the region Inside of the sphere.

C.3.5. The Quantum-Force and Quantum-Energy Inside a Spherical Attached-Aether Region Containing Like-Kind Detached-Aether. When slowly adding detached-aether into a spherical volume, the expansion of the attached-aether will lead to a decreased quantum-force in cube-1. From Eq. (16) we see that the quantum-force magnitude will be $F_{QP1} = 2K_{Q0}/(X_0+\delta X)^3$ within cube-1 once the detached-aether is fully immersed. An immediately adjacent cube (cube-

2) will also expand by δX but it will additionally be displaced. The displacement will involve motion in the direction of the quantum-force, and in this case negative work will be done on the field, reducing the quantum-energy E_Q. We show in section C.3.10 that displacement alone will not change the size of cube-2. With $E_Q = K_Q/X_Q^2$, and with X_Q not changing due to displacement, we see that E_Q changes but X_Q does not, and hence *K_Q is no longer constant: the energy due to displacement changes K_Q*. K_Q is the variable quantum-force parameter; K_{Q0}, used in Eq. (12), is the nominal force parameter of an undisturbed aetherial cube. (K_{Q0} is a constant.)

As we slowly add detached-aether into a spherical region, the displacement of any cube within that region is aided by the quantum-force-field work done, $W_{QD} = (X_0/\xi_0)K_c\int 2K_Q/X_0^3\, dx$, as given in Eq. (33). As a first-order approximation, we use a quantum-force of $F_Q = F_{Q0} = 2K_{Q0}/X_0^3$. Injection of detached-aether causes the center of cube-2 to move a distance $\delta X/2$ due to the expansion of cube-1 and a distance $\delta X/2$ due to its own expansion for a total displacement of δX. Prior to the detached-aether injection, the center of cube-2 is located at X_0. Performing the integral of Eq. (33) from X_0 to $X_0 + \delta X$ (the cube center moves from X_0 to $X_0 + \delta X$) the work done during the displacement on the quantum-force-field is $W_{QD2} = -(X_0/\xi_0)[2K_cK_{Q0}/X_0^3]\delta X$. In this case, W_{QD} will be negative, since the displacement is in the direction of the quantum-force, and the quantum-energy is reduced.

The energy freed as cube-J is displaced is again given by Eq. (33), but this time the displacement is $(J-1)\delta X$. The work done against the quantum-force-field as cube-J is displaced is $W_{DQJ} = -(X_0/\xi_0)[2(J-1)K_cK_{Q0}/X_0^3]\delta X$ to first order. Continuing to set aside the energy change due to expansion so we can focus on the effect of displacement, the quantum-energy of cube-J is $E_Q = E_{Q0} + W_{QDJ} = K_{Q0}/X_0^2 - (X_0/\xi_0)[2(J-1)K_cK_{Q0}/X_0^3]\delta X = (K_{Q0}/X_0^2)(1 - 2K_c(J-1)\delta X/\xi_0)$. With $E_Q =$

K_Q/X_0^2, we arrive at the expression $K_Q = K_{Q0}(1 - 2K_c(J-1)\delta X/\xi_0)$. The quantum-force remains $F_Q = 2K_Q/X_0^3$, but now K_Q includes the next order correction.

As detached-aether is slowly injected, the center of the Jth cube will move from $(J-1)X_0$ to $(J-1)X_0 + (J-1)\delta X$. The motion will thus cover a distance $(J-1)\delta X$. At the beginning of the motion F_{QP} is of course just $2K_{Q0}/X_0^3$, and it is at the end that $F_{QP} = 2K_Q/X_0^3 = (2K_{Q0}/X_0^3)[1 - 2K_c(J-1)\delta X/\xi_0]$. Again defining x as the deviation of the cube center from its nominal center (x varies from 0 to $(J-1)\delta X$ during the detached-aether injection) the full expression for the quantum-force in the analysis-cube is

$$F_{QPP} = (2K_{Q0}/X_0^3)[1 - 2K_c x/\xi_0] \qquad (68)$$

With the expression for the second-order quantum-force just derived, we can now include the second order effect on the cube quantum-energy. From Eq. (33), the work done on the quantum-force-field due to the cube displacement is $W_{QDJ} = -\int F_Q dx = -(K_c X_0/\xi_0)(2K_{Q0}/X_0^3) \int [1-2K_c x/\xi_0]dx = -(2K_cK_{Q0}/X_0^2\xi_0)x + (2K_c^2K_{Q0}/X_0^2\xi_0^2)x^2 = -(2K_cK_{Q0}/X_0^2\xi_0)(J-1)\delta X + (2K_{Q0}K_c^2/X_0^2\xi_0^2)(J-1)^2\delta X^2$. (For the displacement, the integral is evaluated between zero and $(J-1)\delta X$.) Now $(J-1)\delta X$ is the distance the cube moves from its nominal position, $P = (J-1)\delta X$, and hence $W_{QDJ} = -(2K_cK_{Q0}/X_0^2\xi_0)P + (2K_{Q0}K_c^2/X_0^2\xi_0^2)P^2$.

The expansion energy is now calculated using Eq. (64), $W_{QC} = -4 \int K_Q/X_0^3\, dw$ where we now use a negative sign since expansion lowers the quantum-energy of the cube. From Eq. (68), $F_{QPP} = (2K_{Q0}/X_0^3)[1 - 2K_c x/\xi_0]$. We now use Eq. (62), $x = (P/\delta X)w$, and we obtain $2K_Q/X_0^3 = (2K_{Q0}/X_0^3)[1-2K_c(P/\delta X)w/\xi_0]$. As specified in Eq. (64) we integrate from $w = 0$ to $w = \delta X/2$. Hence $W_{QC} = -4 \int K_Q/X_0^3\, dw = -4K_{Q0}/X_0^3 \int [1-2K_c(P/\delta X)w/\xi_0]\, dw = -(4K_{Q0}/X_0^3)w + (4K_{Q0}/X_0^3)K_c(P/\delta X)w^2/\xi_0 = -(2K_{Q0}/X_0^3)\delta X + (K_{Q0}/X_0^3)K_cP\delta X/\xi_0$. Recall that Eq. (64) is for one dimension only, and that inside the sphere the expansion will be the same in all three dimensions. Hence, $W_{QC3} = -(6K_{Q0}/X_0^3)\delta X + (3K_{Q0}/X_0^3)K_cP\delta X/\xi_0$. The total quantum-energy of an arbitrary analysis-cube inside the spherical region is the undisturbed energy plus the displacement energy plus the expansion energy, $E_{QPPI} = K_{Q0}/X_0^2 - (2K_cK_{Q0}/X_0^2\xi_0)P + (2K_{Q0}K_c^2/X_0^2\xi_0^2)P^2 - (6K_{Q0}/X_0^3)\delta X + (3K_{Q0}/X_0^3)K_cP\delta X/\xi_0$, or, $E_{QPPI} = (2K_{Q0}/X_0^2)[(1/2) - K_cP/\xi_0 + K_c^2P^2/\xi_0^2 - 3\delta X/X_0 + 3K_cP\delta X/2X_0\xi_0]$. Here we have derived the force for positive P. For negative P (or **P** at any angle) the work done is still negative, since such a cube is still expanding outward aided by the quantum-force. Hence we will replace P by its absolute value, $|\mathbf{P}|$:

$$E_{QPPI} = (2K_{Q0}/X_0^2)[(1/2) - K_c|\mathbf{P}|/\xi_0 + K_c^2|\mathbf{P}|^2/\xi_0^2 - 3\delta X/X_0 + 3K_c|\mathbf{P}|\delta X/2X_0\xi_0] \quad (69)$$

In Eq. (69) and the above paragraph, the subscript QPPI refers to Quantum-force of the Positive-attached-aether due to immersed Positive-detached-aether in the region Inside of the sphere.

C.3.6. The Delta-Force and Delta-Energy Fields Inside a Spherical Attached-Aether Region Containing Like-Kind Detached-Aether. Injection of detached-aether causes like-kind attached-aether to expand leading to the forces described in Eqs. (66) and (68), $F_{TPP} = K_{T0}X_0[1 + 2K_cx/\xi_0]$ and $F_{QPP} = (2K_{Q0}/X_0^3)[1 - 2K_cx/\xi_0]$, respectively. In order to achieve a force balance within the attached-aether, it has been proposed above that presence of detached-aether leads to a balancing force called the delta-force obeying Eq. (59), $\mathbf{F_\delta} = -\mathbf{F_{TL}} - \mathbf{F_Q}$. At this point recall that the tension is directed inward (toward the center of the sphere) while the quantum-force is directed outward to arrive at $\mathbf{F_\delta} = K_{T0}X_0[1 + 2K_cx/\xi_0]\hat{\mathbf{r}} - (2K_{Q0}/X_0^3)[1 - 2K_cx/\xi_0]\hat{\mathbf{r}}$. And now recall Eq. (21), $K_{T0}X_0 = 2K_{Q0}/X_0^3$, leaving $\mathbf{F_\delta} = K_{T0}X_0[1 + 2K_cx/\xi_0 - 1 + 2K_cx/\xi_0]\hat{\mathbf{r}}$, or,

$$\mathbf{F_{\delta PP}} = 4K_{T0}K_c(X_0/\xi_0)x\hat{\mathbf{r}} \quad (70)$$

Eq. (70) informs us of the total delta force needed to balance the difference between the tension and quantum forces, but it does not tell us how that balancing force arises. There are three possibilities for the delta-force: 1) it could be a tension; 2) it could be a quantum-force; or 3) it could have components of both a tension and a quantum-force. Since the tension exceeds the quantum-force for the like-kind-immersion case, the delta-force could be a negative tension, a positive quantum-force or some combination of a tension and a quantum-force summing to the net force field given in Eq. (70). We propose that the delta-force of Eq. (70) is a positive quantum-force; it is directed outward.

Since Eq. (70) has the form of a tension force, the work done on the delta-fields due to the cube displacement is calculated from Eq. (32) as $W_{\delta D} = (X_0/\xi_0)\, K_c \int F_{\delta PP}\, dx$, and substituting in Eq. (70) we obtain $W_{\delta D} = (X_0/\xi_0)\, K_c \int 4K_{T0}K_c(X_0/\xi_0)x\, dx = 2K_{T0}K_c^2(X_0/\xi_0)^2x^2 = 2K_{T0}K_c^2(X_0/\xi_0)^2(J-1)^2\delta X^2 = 2K_{T0}K_c^2(X_0/\xi_0)^2P^2$. (For the displacement, the integral is evaluated between zero and $(J-1)\delta X$ and we again note that $(J-1)\delta X = P$. For the sign of the energy see section C.3.10.)

The expansion energy is now calculated by using Eq. (65), $W_{\delta C} = 2 \int F_{\delta PP}\, dw$. We now use Eq. (62), $x = (P/\delta X)w$ and include a minus sign since expansion will reduce the quantum-energy to obtain $F_{\delta PP} = -4K_{T0}K_cX_0(P/\delta X)w/\xi_0$. As specified in Eq. (65) we integrate from $w = 0$ to $w = \delta X/2$. Hence $W_{\delta C} = -2 \int 4K_{T0}K_cX_0(P/\delta X)w/\xi_0\, dw = -4K_{T0}X_0K_c(P/\delta X)w^2/\xi_0 = -K_{T0}X_0K_cP\delta X/\xi_0$. Recall that Eq. (65) is for one dimension only, and that inside the sphere the expansion will be the same in all three dimensions. Hence, $W_{\delta C3} = -3K_{T0}X_0K_cP\delta X/\xi_0$. The total delta-energy of an arbitrary analysis-cube inside the spherical region is the displacement energy plus the expansion energy, $E_{\delta PPI} = 2K_c^2K_{T0}(X_0/\xi_0)^2P^2 - 3K_{T0}X_0K_cP\delta X/\xi_0$. The calculation will hold for any angle so we can replace P by $|\mathbf{P}|$, and therefore summing the displacement and expansion energies leaves

$$E_{\delta PPI} = 2K_c^2K_{T0}(X_0/\xi_0)^2|\mathbf{P}|^2 - 3K_{T0}X_0K_c|\mathbf{P}|\delta X/\xi_0 \qquad (71)$$

C.3.7. The Total Energy Inside a Spherical Attached-Aether Region Containing Like-Kind Detached-Aether. The total energy of the analysis-cube is found by summing the tension, quantum and delta energies from Eqs. (67), (69) and (71). $E_{PPI} = E_{TPPI} + E_{QPPI} + E_{\delta PPI} = K_{T0}X_0^2[(1/2) + K_c|\mathbf{P}|/\xi_0 + K_c^2|\mathbf{P}|^2/\xi_0^2 + 3\delta X/X_0 + 3K_c|\mathbf{P}|\delta X/2X_0\xi_0] + (2K_{Q0}/X_0^2)[(1/2) - K_c|\mathbf{P}|/\xi_0 + K_c^2|\mathbf{P}|^2/\xi_0^2 - 3\delta X/X_0 + 3K_c|\mathbf{P}|\delta X/2X_0\xi_0] + 2K_c^2K_{T0}(X_0/\xi_0)^2|\mathbf{P}|^2 - 3K_{T0}K_c(X_0/\xi_0)|\mathbf{P}|\delta X$. Next, use Eq. (22), $2K_{Q0}/X_0^2 = K_{T0}X_0^2$, to get $E_{PPI} = K_{T0}X_0^2 + 4(X_0/\xi_0)^2K_c^2K_{T0}|\mathbf{P}|^2$, or

$$E_P = K_{T0}X_0^2[1 + 4K_c^2P^2/\xi_0^2] \tag{72}$$

(We'll see below that we don't need the second P nor I as subscripts.)

C.3.8. The Force and Energy Fields Outside a Spherical Attached-Aether Region Containing Like-Kind Detached-Aether. Outside of the sphere Eq. (61) informs us that the analysis-cubes will be radially compressed, rather than expanded. A spherical shell originally located a distance r from the center of the sphere with thickness dr is pushed outward by a distance P_R. Since there is no detached-aether outside of the sphere, the density is a constant ρ_0 there and hence the volume of the shell remains constant during its outward displacement. With dr_P the shell thickness after it is displaced by P_R, volume invariance results in $4\pi r^2dr = 4\pi(r+P_R)^2dr_P$, or $dr_P/dr = r^2/(r+P_R)^2 = r^2/[r^2(1+P_R/r)^2] \approx 1 - 2P_R/r$. Now Eq. (61) gives $P_R = P_0R_0^2/r^2$, and therefore $dr_P/dr \approx 1 - 2P_0R_0^2/r^3$, or $dr_P \approx dr - 2drP_0R_0^2/r^3$. This results in the amount of radial compression being $\delta r_P = dr_P - dr \approx 2drP_0R_0^2/r^3$, where dr can be any small radial distance. Assigning dr to the size of an analysis-cube, $dr = X_0$, we obtain the radial compression of the analysis-cubes, $\delta X = 2X_0P_0R_0^2/r^3$. At the edge of the sphere, the analysis-cubes are displaced radially outward by P_0 and the radial compression will gradually lower the displacement of the more distant cubes to the displacement $P_R = P_0R_0^2/r^2$ given by Eq. (61). In this case, all cubes are still displaced in the outward direction however, and so the radial tension-force is increased, it is just that the amount of increase falls off as $1/r^3$.

In the directions perpendicular to **r** the cubes will expand. The area (A) of each spherical shell increases to $4\pi(r+P_R)^2$ from its original $4\pi r^2$. We have $(r+P_R)^2/r^2 = (1+P_R/r)^2 \approx 1+2P_R/r$, and we can see that the relative area expansion is $\delta A/A = 2P_R/r$. Now we form $A = Y^2$, where Y is one of the perpendicular directions, and we see $\delta A/\delta Y = 2Y$, leading to $(\delta A/\delta Y)/A = (2Y)/Y^2$, or $\delta A/A = 2\delta Y/Y$ and hence $\delta Y/Y = P_R/r$. Therefore a cube of size $Y = X_0$ will expand by $\delta Y = X_0P_R/r = X_0P_0R_0^2/r^3$, or

$$\delta Y = \delta X/2 \tag{73}$$

Now consider cube-P, a cube that is displaced by a distance P. Integrating Eq. (32), $W_{TD} = (X_0/\xi_0) K_c\int K_TX_0\, dx$, from 0 to P, the first-order work (against the first-order force $F_{TP} = K_{T0}X_0$) done against the tension on cube-P from the displacement is $W_{TD} = (X_0/\xi_0)K_cK_{T0}X_0P$.

Setting the expansion effects aside to focus on the effect of displacement, the tension-energy of cube-P is $E_T = E_{T0} + W_{TD} = (1/2)K_{T0}X_0^2 + K_cK_{T0}X_0^2P/\xi_0$. With $E_T = (1/2)K_TX_0^2$, we arrive at the expression $K_T = K_{T0}(1 + 2K_cP/\xi_0)$. The tension-force remains $F_T = K_TX_0$, but now K_T includes the next order correction. As detached-aether is slowly injected, the center of the Pth cube will move a distance P. At the beginning of the motion F_{TP} is of course just $K_{T0}X_0$, and it is at the end that $F_{TP} = K_{T0}X_0[1 + 2K_cP/\xi_0]$. With x again defined as the deviation of the cube center from its nominal center (which varies from 0 to P during the detached-aether injection) the full expression for tension-force in the analysis-cube is again given by Eq. (66), $F_{TPP} = K_{T0}X_0[1 + 2K_cx/\xi_0]$.

The second-order effect on the cube tension-energy due to the cube displacement is now included by integrating Eq. (32) from 0 to P, $W_{TD} = (X_0/\xi_0) K_c\int F_{TP}\, dx = (X_0/\xi_0)K_cK_{T0}X_0 \int [1+2K_cx/\xi_0]\, dx$ $= (X_0/\xi_0)K_cK_{T0}X_0x + K_c^2(X_0/\xi_0)^2K_{T0}x^2 = K_cK_{T0}X_0^2P/\xi_0 + K_c^2K_{T0}P^2X_0^2/\xi_0^2$.

The compression energy is now calculated by including a minus sign in Eq. (63) since compression will release energy from the tension, $W_{TC1} = -2 \int K_T X_0 \, dw$. From Eq. (66), $K_T X_0 = K_{T0} X_0[1 + 2K_c x/\xi_0]$. We now use Eq. (62), $x = (P/\delta X)w$, to obtain $K_T X_0 = K_{T0} X_0[1+2K_c(P/\delta X)w/\xi_0]$. As specified in Eq. (63) we integrate from $w = 0$ to $w = \delta X/2$. Hence $W_{TC1} = -2 \int K_T X_0 \, dw = -2K_{T0} X_0 \int [1+2K_c(P/\delta X)w/\xi_0] \, dw = -2K_{T0} X_0 w - 2K_{T0} X_0 K_c(P/\delta X)w^2/\xi_0 = -K_{T0} X_0 \delta X - K_{T0} X_0 K_c P \delta X/2\xi_0$. Recall that Eq. (63) is for one dimension only. Eq. (73) informs that outside the sphere there is an expansion $\delta Y = \delta X/2$ for each of the other two dimensions and for those dimensions we integrate from 0 to $\delta Y/2$, which is equal to 0 to $\delta X/4$. Also, since it is an expansion we do work against the tension and now our signs are positive. For each transverse dimension then we obtain $W_{TC2} = 2K_{T0} X_0 w + 2K_{T0} X_0 K_c(P/\delta X)w^2/\xi_0 = K_{T0} X_0 \delta X/2 + K_{T0} X_0 K_c(P/\delta X)\delta X^2/8\xi_0$, and the total expansion energy will be twice that, since there are two dimensions. Hence, $W_{TC3} = W_{TC1} + 2W_{TC2} = -K_{T0} X_0 \delta X - K_{T0} X_0 K_c P \delta X/2\xi_0 + K_{T0} X_0 \delta X + K_{T0} X_0 K_c P \delta X/4\xi_0 = -K_{T0} X_0 K_c P \delta X/4\xi_0$. The total tension-energy of an arbitrary analysis-cube outside the spherical region is the undisturbed energy plus the displacement energy plus the expansion energy, $E_{TPPO} = (1/2)K_{T0} X_0^2 + K_c K_{T0} X_0^2 P/\xi_0 + K_c^2 K_{T0} P^2 X_0^2/\xi_0^2 - K_{T0} X_0 K_c P \delta X/4\xi_0$, or, $E_{TPPO} = K_{T0} X_0^2[(1/2) + K_c P/\xi_0 + K_c^2(P/\xi_0)^2 - K_c P \delta X/4X_0\xi_0]$. Here we have derived the force for positive P. For negative P (or $\mathbf{P}$ at any angle) the work done is still positive, since such a cube is still expanding outward against the tension. Hence we will replace P by its absolute value, $|\mathbf{P}|$:

$$E_{TPPO} = K_{T0} X_0^2[(1/2) + K_c|\mathbf{P}|/\xi_0 + K_c^2(|\mathbf{P}|/\xi_0)^2 - K_c|\mathbf{P}|\delta X/4X_0\xi_0] \qquad (74)$$

In Eq. (74) and the above paragraph, the subscript TPPO refers to Tension-energy of the Positive-attached-aether due to immersed Positive-detached-aether in the region Outside of the sphere.

Turning to the quantum-field, the first-order energy freed as cube-P is displaced is given by Eq. (33), $W_{QD} = (X_0/\xi_0)K_c\int 2K_Q/X_0^3\, dx$, and the displacement is P. The work done against the quantum-force-field as cube-P is displaced is $W_{QD} = -2PK_cK_{Q0}/X_0^2\xi_0$. Setting the compression effects aside to focus on the effect of displacement, the quantum-energy of cube-P is $E_Q = E_{Q0} + W_{QD} = K_{Q0}/X_0^2 - 2PK_cK_{Q0}/X_0^2\xi_0 = (K_{Q0}/X_0^2)(1 - 2K_cP/\xi_0)$. With $E_Q = K_Q/X_0^2$, we arrive at the expression $K_Q = K_{Q0}(1 - 2K_cP/\xi_0)$. The quantum-force remains $F_Q = 2K_Q/X_0^3$, but now K_Q includes the next order correction. As detached-aether is slowly injected, the center of the Pth cube will move a distance P. At the beginning of the motion F_{QP} is of course just $2K_{Q0}/X_0^3$, and it is at the end that $F_{QP} = (2K_{Q0}/X_0^3)[1 - 2K_cP/\xi_0]$. With x again defined as the deviation of the cube center from its nominal center (which varies from 0 to P during the detached-aether injection) the full expression for the quantum-force in the analysis-cube is again given by Eq. (68), $F_{QPP} = (2K_{Q0}/X_0^3)[1 - 2K_cx/\xi_0]$.

The second order effect on the cube quantum-energy due to the cube displacement is now included with $W_{QD} = -(X_0/\xi_0)K_c \int (2K_{Q0}/X_0^3)[1-2K_cx/\xi_0]\, dx = -(2K_{Q0}K_c/X_0^2\xi_0)x + (2K_{Q0}K_c^2/X_0^2\xi_0^2)x^2 = -(2K_{Q0}K_c/X_0^2\xi_0)P + (2K_{Q0}K_c^2/X_0^2\xi_0^2)P^2$.

The compression energy is calculated using Eq. (64) but with a positive sign, since compression will do work against the quantum-pressure, $W_{QC1} = 4 \int K_Q/X_0^3\, dw$. From Eq. (68), $F_{QPP} = (2K_{Q0}/X_0^3)[1 - 2K_cx/\xi_0]$. We now use Eq. (62), $x = (P/\delta X)w$, to obtain $K_Q/X_0^3 = (K_{Q0}/X_0^3)[1-2K_c(P/\delta X)w/\xi_0]$. As specified in Eq. (64) we integrate from $w = 0$ to $w = \delta X/2$. Hence $W_{QC1} = 4(K_{Q0}/X_0^3) \int [1-2K_c(P/\delta X)w/\xi_0]\, dw = 4(K_{Q0}/X_0^3)w - 4(K_{Q0}/X_0^3)K_c(P/\delta X)w^2/\xi_0 = 2(K_{Q0}/X_0^3)\delta X - (K_{Q0}/X_0^3)K_cP\delta X/\xi_0$.

Recall that Eq. (64) is for one dimension only. Eq. (73) informs that outside the sphere there is an expansion $\delta Y = \delta X/2$ for each of the other two dimensions and for those dimensions we integrate from 0 to $\delta Y/2$, which is equal to 0 to $\delta X/4$. Also, since it is an expansion the quantum-pressure will decrease and our signs are negative. For each transverse dimension then we obtain $W_{QC2} = -4(K_{Q0}/X_0^3)w + 4(K_{Q0}/X_0^3)K_c(P/\delta X)w^2/\xi_0 = -(K_{Q0}/X_0^3)\delta X + (K_{Q0}/X_0^3)K_cP\delta X/4\xi_0$, and the total expansion energy will be twice that, since there are two dimensions. Hence, $W_{QC3} = W_{QC1} + 2W_{QC2} = 2(K_{Q0}/X_0^3)\delta X - (K_{Q0}/X_0^3)K_cP\delta X/\xi_0 - 2(K_{Q0}/X_0^3)\delta X + (K_{Q0}/X_0^3)K_cP\delta X/2\xi_0$. The total quantum-energy of an arbitrary analysis-cube outside the spherical region is the undisturbed energy plus the displacement energy plus the expansion energy, $E_{QPPO} = K_{Q0}/X_0^2 - (2K_{Q0}K_c/X_0^2\xi_0)P + (2K_{Q0}K_c^2/X_0^2\xi_0^2)P^2 - (K_{Q0}/X_0^3)K_cP\delta X/\xi_0 + (K_{Q0}/X_0^3)K_cP\delta X/2\xi_0$, or, $E_{QPPO} = (2K_{Q0}/X_0^2)[(1/2) - (K_c/\xi_0)P + (K_c^2/\xi_0^2)P^2 - K_cP\delta X/4X_0\xi_0]$. Here we have derived the force for positive P. For negative P (or **P** at any angle) the work done is still negative, since such a cube is still expanding outward reducing the quantum-energy. Hence we will replace P by its absolute value, $|\mathbf{P}|$:

$$E_{QPPO} = (2K_{Q0}/X_0^2)[(1/2) - K_c|\mathbf{P}|/\xi_0 + K_c^2(|\mathbf{P}|/\xi_0)^2 - K_c|\mathbf{P}|\delta X/4X_0\xi_0] \qquad (75)$$

In Eq. (75) and the above paragraph, the subscript QPPO refers to Quantum-energy of the Positive-attached-aether due to immersed Positive-detached-aether in the region Outside of the sphere.

For the delta-energy, since Eqs. (66) and (68) are the same both inside and outside of the spherical region, Eq. (70), $\mathbf{F_{\delta PP}} = 4K_cK_{T0}(X_0/\xi_0)x\hat{\mathbf{r}}$, holds both inside and outside as well. The work done on the delta-field due to the cube displacement is calculated from Eq. (32) as $W_{\delta D} = (X_0/\xi_0)\, K_c \int F_{\delta PP}\, dx$, and substituting in Eq. (70) we obtain $W_{\delta D} = (X_0/\xi_0)\, K_c \int 4K_{T0}K_c(X_0/\xi_0)x\, dx = 2K_{T0}K_c^2(X_0/\xi_0)^2x^2 = 2K_{T0}K_c^2(X_0/\xi_0)^2P^2$. (For the displacement, the integral is evaluated between zero and P. For the sign of the energy see section C.3.10.)

The radial compression energy is now calculated by using Eq. (65), $W_{\delta C} = 2\int F\, dw$. We now use Eq. (62), $x = (P/\delta X)w$ and use a positive sign since compression will increase the quantum-energy to obtain $F = 4K_{T0}K_cX_0(P/\delta X)w/\xi_0$. As specified in Eq. (65) we integrate from $w = 0$ to $w = \delta X/2$. Hence $W_{\delta C1} = 2\int 4K_{T0}K_cX_0(P/\delta X)w/\xi_0\, dw = 4K_{T0}X_0K_c(P/\delta X)w^2/\xi_0 = K_{T0}X_0K_cP\delta X/\xi_0$. Recall that Eq. (65) is for one dimension only. Eq. (73) informs that outside the sphere there is an expansion $\delta Y = \delta X/2$ for each of the other two dimensions and for those dimensions we integrate from 0 to $\delta Y/2$, which is equal to 0 to $\delta X/4$. Also, since it is an expansion the delta pressure will decrease and our signs are negative. For each transverse dimension then we obtain $W_{\delta C2} = -4K_{T0}X_0K_c(P/\delta X)w^2/\xi_0 = -K_{T0}X_0K_cP\delta X/4\xi_0$, and the total expansion energy will be twice that, since there are two dimensions. Hence, $W_{\delta C3} = W_{\delta C1} + 2W_{\delta C2} = K_{T0}X_0K_cP\delta X/\xi_0 - K_{T0}X_0K_cP\delta X/2\xi_0 = K_{T0}X_0K_cP\delta X/2\xi_0$. The total delta-energy of an arbitrary analysis-cube outside the spherical region is the displacement energy plus the expansion energy, $E_{\delta PPO} = 2K_{T0}K_c^2(X_0/\xi_0)^2P^2 + K_{T0}X_0K_cP\delta X/2\xi_0$. The calculation will hold for any angle so we can replace P by $|\mathbf{P}|$, and therefore summing the displacement and expansion energies leaves

$$E_{\delta PPO} = 2K_{T0}K_c^2(X_0/\xi_0)^2|\mathbf{P}|^2 + K_{T0}X_0K_c|\mathbf{P}|\delta X/2\xi_0 \tag{76}$$

The total energy of the analysis-cube is found by summing the tension, quantum and delta energies from Eqs. (74), (75) and (76). $E_{PPO} = E_{TPPO} + E_{QPPO} + E_{\delta PPO} = K_{T0}X_0^2[(1/2) + K_c|\mathbf{P}|/\xi_0 + K_c^2|\mathbf{P}|^2/\xi_0^2 - K_c|\mathbf{P}|\delta X/4X_0\xi_0] + (2K_{Q0}/X_0^2)[(1/2) - K_c|\mathbf{P}|/\xi_0 + K_c^2|\mathbf{P}|^2/\xi_0^2 - K_c|\mathbf{P}|\delta X/4X_0\xi_0] + 2K_c^2K_{T0}(X_0/\xi_0)^2|\mathbf{P}|^2 + K_{T0}X_0K_c|\mathbf{P}|\delta X/2\xi_0$. Next, use Eq. (22), $2K_{Q0}/X_0^2 = K_{T0}X_0^2$, to get $E_P = K_{T0}X_0^2[1 + 4K_c^2(P/\xi_0)^2]$, which is Eq. (72). Hence we arrive at Eq. (72) both inside and outside of the sphere, and therefore Eq. (72) doesn't need the I and O subscripts.

C.3.9. The Energy Fields in the General Case. In the more general case, consider first the situation of a spherical positive-attached-aether region containing negative-detached-aether. For that situation, the cubes inside the sphere will compress instead of expanding, while those outside of the sphere will radially expand instead of compressing. Appendix F shows that those changes still result in in Eq. (72), and that is why Eq. (72) doesn't need the second P subscript.

Next, observe that a derivation for the case of a spherical negative-attached-aether region will involve an identical derivation, but with P replaced by N and N replaced by P. This leaves a negative-attached-aether analysis-cube energy of

$$E_N = K_{T0}X_0^2[1 + 4K_c^2N^2/\xi_0^2] \qquad (77)$$

C.3.10. The Physical Nature of the Delta-Force for Like-Kind Immersion. In the sections above we make three assumptions regarding our analysis of the case of positive-detached-aether within positive-attached-aether: 1) that the delta-force is a positive pressure; 2) that displacement of the analysis-cube will not change its size; and 3) that the energies associated with the delta-force are positive for the displacement, negative for expansion, and positive for compression. This section will validate these assumptions while providing a physical understanding of the delta-force.

C.3.10.1. Delta-force Pressure. The delta-force $\mathbf{F}_\delta$ is a force that balances against the sum of $\mathbf{F}_T$ and $\mathbf{F}_Q$. Above we have assigned a sign to $\mathbf{F}_\delta$ so that it adds to the weaker of $\mathbf{F}_T$ and $\mathbf{F}_Q$ as needed to provide the balancing force. If $\mathbf{F}_T$ is less than $\mathbf{F}_Q$, $\mathbf{F}_\delta$ a positive tension providing an additional inward force. If $\mathbf{F}_Q$ is less than $\mathbf{F}_T$, $\mathbf{F}_\delta$ is a positive pressure (a positive quantum-force) providing an additional outward force.

The physical reason that $\mathbf{F}_\delta$ adds to the weaker of the forces, rather than subtracting from the stronger, is because the delta-force originates from the forced expansion and compression of attached-aether as caused by the immersion of detached-aether. For the case of positive-detached-aether, a positive pressure is exerted by the positive-detached-aether onto the positive-attached-aether and a positive tension is exerted onto the negative-attached-aether. As the cubes are then displaced, the work done makes these forces grow, and this force is the delta-force. Therefore the delta-force is a positive pressure (positive tension) for the case of positive-detached-aether immersed into positive-attached-aether (negative-attached-aether).

C.3.10.2. Analysis-Cube Size During Displacement. When cube-P of positive-attached-aether is displaced outward due to injection of positive-detached-aether, cube-P has its tension increased as described by Eq. (66), $F_{TPP} = K_{T0}X_0[1 + 2K_c x/\xi_0]$, and cube-P has its quantum-force decreased as described by Eq. (68), $F_{QPP} = (2K_{Q0}/X_0^3)[1 - 2K_c x/\xi_0]$. If there were no delta-force, cube-P would therefore compress. However, the overlapping cube-N of negative-attached-aether has the opposite behavior (see Appendix F), and with no delta-force it would expand. (Eqs. (F14) and (F15) from Appendix F are $F_{TNP} = K_{T0}X_0[1 - 2K_c x/\xi_0]$ and $F_{QNP} = (2K_{Q0}/X_0^3)[1 + 2K_c x/\xi_0]$, respectively.) In order for the density postulate to hold as they displace, cube-P and cube-N must either both expand, both compress, or both retain their size. Since the sum of $\mathbf{F_T}$ and $\mathbf{F_Q}$ on cube-P, described by Eqs. (66) and (68), is equal and opposite to the sum of $\mathbf{F_T}$ and $\mathbf{F_Q}$ on cube-N, this symmetry informs that the cube sizes will not change during their displacement, which is what is assumed above.

C.3.10.3. The Physics Leading to the Delta-Force Energies. We see that the delta-force arises between the positive-attached-aether and the negative-attached-aether in a way that leads to no displacement-induced size change of the displaced cubes. We also see that outward motion of positive-attached-aether causes $\mathbf{F_{TP}}$ (the positive-attached-aether tension) to grow and $\mathbf{F_{QP}}$ (the positive-attached-aether quantum-force) to recede, and hence $\mathbf{F_{\delta P}}$ (the positive-attached-aether delta-force) must grow to offset this growing difference. Since the displacement of the positive-attached-aether is outward and the displacement of the negative-attached-aether is inward in this case, and since $\mathbf{F_{\delta P}}$ is outward, it is the displacement of the inward-moving negative-attached-aether that does the work against $\mathbf{F_{\delta P}}$ to make it grow, since we must have an inward motion against the force to do work against it to make it grow. (The outward-moving positive-attached-aether would do negative work on $\mathbf{F_{\delta P}}$ and therefore it cannot be the source of growth for $\mathbf{F_{\delta P}}$.) Therefore we see that $\mathbf{F_{\delta P}}$ is a positive pressure that adds to the quantum-pressure within the positive-attached-aether, and yet work is done against it by the motion of the negative-attached-aether. Similarly, the outward motion of the positive-attached-aether increases $\mathbf{F_{\delta N}}$ (the negative-attached-aether delta-force) which is a positive tension within the negative-attached-aether. This is the mechanism by which each type of attached-aether exerts forces on the other in order to maintain equal densities as prescribed by the density postulate.

As just mentioned, it is the moving negative-attached-aether that is doing work against the positive-attached-aether delta force. Inside the spherical region the negative-attached-aether will move radially inward and compress. The inward motion will do work against $\mathbf{F_{\delta P}}$ and hence the first term in the energy expression of Eq. (71) is positive. Outside the spherical region negative-attached-aether will move radially inward and expand. The inward displacement does work against $\mathbf{F_{\delta P}}$ and hence the first term in the energy expression of Eq. (76) is positive.

Appendix F shows that $\mathbf{F_{\delta P}}$ becomes directed inward when negative-detached-aether is immersed, and in that case the force will be a positive tension. In that case it will be the outward moving positive-attached-aether that does positive work against that delta-tension force, and the first term in each of Eqs. (F8) and (F11) are positive.

C.3.10.4. Directional Dependence of the Force Parameter Changes. Here in section C.3 we have assumed that the displacement work increases or decreases the force parameters K_T and K_Q equally in all three directions. However, it is possible that nature only changes the force parameters in the direction that the work is done. If that is the case, the expansion and compression energies calculated above for the cases perpendicular to r will change, but it can be seen that the effect on the total energy will not change, since the delta force contribution will always cancel the combined contribution from the tension and quantum-pressure forces. The displacement work calculation will remain the same as above, since the displacement work is the inner product of the force times the displacement, and the work will change the force parameters in the direction of displacement. Therefore, it will be up to future experimentation to determine whether or not the displacement work changes the force parameters in all three dimensions. Here we have chosen the simpler case (that the force parameters change equally in all dimensions) for analysis.

C.3.10.5. Comments about the Delta Force. A first comment is that Eq. (71), $E_{\delta PPI} = 2K_c^2K_{T0}(X_0/\xi_0)^2|\mathbf{P}|^2 - 3K_{T0}X_0K_c|\mathbf{P}|\delta X/\xi_0$, will result in a negative energy if $[3K_{T0}X_0K_c|\mathbf{P}|\delta X/\xi_0]/[2K_c^2K_{T0}(X_0/\xi_0)^2|\mathbf{P}|^2] = [3\delta X]/[2K_c(X_0/\xi_0)|\mathbf{P}|] > 1$. $|\mathbf{P}|$ will be small for the cube at the center of the sphere and it grows from there, so in some regions the delta-energy will be negative.

The cube expands while it displaces. Even though the displacement itself does not generate forces to expand the cube, immersion of the detached-aether does. We must check to see if this materially affects our derivations above. The energy at any value of x along our expansion is $(1/2)K_TX^2$. At x + dx the energy neglecting the displacement work is $(1/2)K_T[X + (\delta X dx/P)]^2 \approx (1/2)K_TX^2 + K_TX\delta X(dx/P)$. (Over an interval dx, the cube will expand dx/P of its full expansion of δX at P.) The energy of the work done by displacement is $K_TX dx$. And so, the energy change due to expansion divided by the energy change due to displacement work is $K_TX\delta X(dx/P)/K_TX dx = \delta X/P$, which goes to zero as our analysis-cube size shrinks to zero. Hence, this effect does not affect our analysis.

C.3.10.6. The Delta-Force in Other Cases. Here in section C.3.10 we have looked primarily at the case of positive-detached-aether immersion into positive-attached-aether. For discussion of the remaining cases see Appendix F.

C.4 – The Electromagnetic Aetherial Flow Forces

C.4.1. The Electrodynamic Flow Force Law. Empirically we propose:

The Electrodynamic Flow Force Law: In regions where there is flowing detached-aether, both the detached-aether and the attached-aether density disturbances caused by the detached-aether will generate a force upon the attached-aether components that is proportional to the relative flow between the flowing detached-aether and the attached-aether; the force will be aligned with the flow, and only the transverse component of the flow leads to a force. (The longitudinal component of the flow does not lead to any force.)

C.4.2. Transverse Velocities and "Image Charges". It is always possible for any vector field **U** to be decomposed into a transverse component that has zero divergence, $\nabla \cdot \mathbf{U_T}$, and a longitudinal component that has zero curl, $\nabla \times \mathbf{U_L} = 0$. (The Helmholtz decomposition; here $\mathbf{U} = \mathbf{U_T} + \mathbf{U_L}$.) In the electrodynamic flow force law, only the transverse component contributes. Also, "the attached-aether density disturbances caused by the detached-aether" can be thought of as "image charges". Eq. (41) is $\rho_{PA} = \rho_0 - \rho_{PD}/2 + \rho_{ND}/2$, and we see that a moving detached-aether density ρ_{PD} or ρ_{ND} will lead to a change in the attached-aether-density ρ_{PA} (an "image charge") that moves along with the detached-aether, and Eq. (42) has similar characteristics. The electrodynamic flow force law indicates that a flow force will be generated both from the moving detached-aether and from the "image charges".

C.4.3. The Flow Forces due to Detached-Aether Flow. For the case of detached-aether flowing through the positive-attached-aether, the aetherial flow force law is expressed mathematically as:

$$\mathbf{F_{FP1}} = K_{F1}\Delta V\rho_{PD}[\mathbf{U_{PDT}} - \partial\mathbf{P_T}/\partial t] - K_{F1}\Delta V\rho_{ND}[\mathbf{U_{NDT}} - \partial\mathbf{P_T}/\partial t] \qquad (78)$$

And for the flow forces on the negative-attached-aether we have

$$\mathbf{F_{FN1}} = K_{F1}\Delta V\rho_{ND}[\mathbf{U_{NDT}} - \partial\mathbf{N_T}/\partial t] - K_{F1}\Delta V\rho_{PD}[\mathbf{U_{PDT}} - \partial\mathbf{N_T}/\partial t] \qquad (79)$$

In the above, the amount of detached-aether that is flowing is given by its density, ρ, multiplied by its volume ΔV. Here we make the empirical choices that flowing detached-aether exerts a force parallel (anti-parallel) to its motion against the like-kind (unlike-kind) attached-aether, and K_{F1} is the proportionality constant. It is the relative transverse velocity between the detached-aether and the attached-aether that results in the force. The velocity of the positive-detached-aether (negative-detached-aether) is $\mathbf{U}_{\mathbf{PD}}$ ($\mathbf{U}_{\mathbf{ND}}$) and the velocity of the attached positive-aether (negative-aether) is $\partial\mathbf{P}/\partial t$ ($\partial\mathbf{N}/\partial t$). The T subscripts designate the transverse component of the vector quantities.

C.4.4. Flow Forces due to Attached-Aether Density Disturbance Flows. Moving detached-aether leads to the flow forces just described, but the detached-aether also results in the pushing out of like-kind aether and the pulling in of unlike-kind aether as described in Eqs. (41) and (42). These attached-aether density disturbances result in an overall equating of the positive-aether density to the negative-aether density as prescribed by the density postulate. Hence, for every flow of detached-aether there is a corresponding flow of an attached-aetherial density disturbance that has an equal magnitude but of unlike-kind, reversing the sign of the force. In this case we now use a flow force constant K_{F2}, since the underlying physical difference may lead to a different proportionality constant from the detached-aether flow force. The attached-aether density disturbances will therefore lead to force equations similar to those described in section C.4.3:

$$\mathbf{F}_{\mathbf{FP2}} = -K_{F2}\Delta V\rho_{PD}[\mathbf{U}_{\mathbf{PDT}} - \partial\mathbf{P}_{\mathbf{T}}/\partial t] + K_{F2}\Delta V\rho_{ND}[\mathbf{U}_{\mathbf{NDT}} - \partial\mathbf{P}_{\mathbf{T}}/\partial t] \quad (80)$$

$$\mathbf{F}_{\mathbf{FN2}} = -K_{F2}\Delta V\rho_{ND}[\mathbf{U}_{\mathbf{NDT}} - \partial\mathbf{N}_{\mathbf{T}}/\partial t] + K_{F2}\Delta V\rho_{PD}[\mathbf{U}_{\mathbf{PDT}} - \partial\mathbf{N}_{\mathbf{T}}/\partial t] \quad (81)$$

Dimensional Analysis. With ρ_{PD} and ρ_{ND} having dimensions of C divided by m^3, the dimensions of K_{F1} and K_{F2} are force divided by ($C/m^3 \times m^3 \times m/s$), or ($kg\ m/s^2$) x s / (C m), or kg/ (C s).

C.5 – The Tension Force Effect on the Positive-Attached-Aether

Next we will consider the force of tension, $\mathbf{F_T}$, acting upon an analysis-cube of attached-aether. Fig. 2 shows three analysis-cubes of attached-aether as well as two tension forces acting on the central cube. Of course, the real aether analysis-cubes (which are arbitrarily specified for analysis only) will have adjoining faces. Fig. 2 separates the faces only to make the drawing clear. It is known that tension forces in a solid lead to transversely polarized waves, with a familiar example being waves on a string. Since light is also known to be a transversely polarized wave, it has been postulated here that each attached-aether component is a solid under tension. Note that in Fig. 2 the longitudinal tension force will predominantly cancel out, as the longitudinal component of $\mathbf{F_{T2}}$ is almost exactly opposite to the longitudinal component of $\mathbf{F_{T1}}$. This leads to the observation that it is the transverse components of the tension that are responsible for the forces which cause the wave motion within the solid aether.

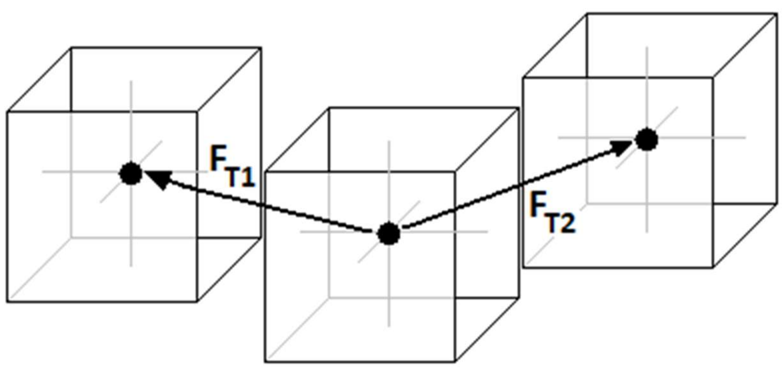

Figure 2. Tension forces on a displaced cube of attached-aether.

Each tension force will be directed from the center of one cube toward the center of the adjacent cube. Recall that the vector **P** is the position of the positive-attached-aether with respect to its undisturbed, equilibrium position. Hence, the cube with an equilibrium position of **r** is in general at **r** + **P**(x, y, z, t) and the cube with an equilibrium position of **r** + **Δx** = **r** + Δx**i** is in general at **r**

$+ \Delta x\mathbf{i} + \mathbf{P}(x + \Delta x, y, z, t)$. (Here Δx is the width of our analysis-cube in the x direction.) The direction of the force is simply the direction of a vector $\mathbf{D}$ between these two cube centers where

$$\mathbf{D} = \mathbf{r} + \Delta x\mathbf{i} + \mathbf{P}(x+\Delta x, y, z, t) - \mathbf{r} - \mathbf{P}(x, y, z, t) = \Delta x\mathbf{i} + \mathbf{P}(x+\Delta x, y, z, t) - \mathbf{P}(x, y, z, t) \qquad (82)$$

Assuming small oscillations, $[\mathbf{P}(x+\Delta x, y, z, t) - \mathbf{P}(x, y, z, t)]/\Delta x = \partial\mathbf{P}/\partial x << 1$. Now form the vector $\mathbf{D}/\Delta x$:

$$\mathbf{D}/\Delta x = \mathbf{i} + \partial\mathbf{P}/\partial x = \mathbf{i} + (\partial P_x/\partial x)\mathbf{i} + (\partial P_y/\partial x)\mathbf{j} + (\partial P_z/\partial x)\mathbf{k} \qquad (83)$$

Keeping terms to first order in small quantities the magnitude of $\mathbf{D}/\Delta x$ is:

$$|\mathbf{D}/\Delta x| = [(1 + \partial P_x/\partial x)^2 + (\partial P_y/\partial x)^2 + (\partial P_z/\partial x)^2]^{1/2} \approx 1 + \partial P_x/\partial x \qquad (84)$$

Next, form a unit vector $\mathbf{d}$ in the direction of $\mathbf{D}/\Delta x$:

$$\mathbf{d} = (\mathbf{D}/\Delta x)/|\mathbf{D}/\Delta x| \approx [(1 + \partial P_x/\partial x)\mathbf{i} + (\partial P_y/\partial x)\mathbf{j} + (\partial P_z/\partial x)\mathbf{k}]/[1 + \partial P_x/\partial x]$$

$$\approx \mathbf{i} + (\partial P_y/\partial x)\mathbf{j} + (\partial P_z/\partial x)\mathbf{k} \qquad (85)$$

With $\mathbf{d}$ now derived, the tension force can be expressed as F_T times $\mathbf{d}$, where F_T is the magnitude of the tension force $\mathbf{F_T}$. The magnitude of $\mathbf{F_T}$ will in general be a function of position and time $F_T(x, y, z, t)$. Hence, $\mathbf{F_T}$ on the left face of the cube is

$$\mathbf{F_T}(x, y, z, t) = -F_T(x, y, z, t)[\mathbf{i} + \{\partial P_y(x, y, z, t)/\partial x\}\mathbf{j} + \{\partial P_z(x, y, z, t)/\partial x\}\mathbf{k}] \qquad (86)$$

(In Eq. (86) the minus sign comes from inspection of Fig. 2.) $\mathbf{F_T}$ on the right face is $\mathbf{F_T}(x+\Delta x, y, z, t) = F_T(x+\Delta x, y, z, t)\ [\mathbf{i} + \{\partial P_y(x+\Delta x, y, z, t)/\partial x\}\mathbf{j} + \{\partial P_z(x+\Delta x, y, z, t)/\partial x\}\mathbf{k}]$. At this point the first order Taylor series expansion can be used: $F_T(x+\Delta x, y, z, t) \approx F_T(x, y, z, t) + (\partial F_T/\partial x)\Delta x$. Here, $(\partial F_T/\partial x)\Delta x << F_T(x, y, z, t)$. This leaves $\mathbf{F_T}$ on the right face as:

$$\mathbf{F_T}(x+\Delta x, y, z, t) =$$

$$[F_T(x, y, z, t) + (\partial F_T/\partial x)\Delta x][\mathbf{i} + \{\partial P_y(x+\Delta x, y, z, t)/\partial x\}\mathbf{j} + \{\partial P_z(x+\Delta x, y, z, t)/\partial x\}\mathbf{k}] \qquad (87)$$

The total force on the cube due to the tension forces on the two YZ faces is the sum of Eqs. (86) and (87), or

$\mathbf{F_{TYZ}} = [F_T(x, y, z, t) + (\partial F_T/\partial x)\Delta x][\mathbf{i} + \{\partial P_y(x+\Delta x, y, z, t)/\partial x\}\mathbf{j} + \{\partial P_z(x+\Delta x, y, z, t)/\partial x\}\mathbf{k}]$

$– F_T(x, y, z, t)[\mathbf{i} + \{\partial P_y(x, y, z, t)/\partial x\}\mathbf{j} + \{\partial P_z(x, y, z, t)/\partial x\}\mathbf{k}]$ (88)

Next, define T_0 as the nominal magnitude of the attached-aetherial-tension per unit area. (T_0 is the tension per unit area in the absence of sources, sinks and waves.) It is assumed that deviations of $F_T(x, y, z, t)$ from $T_0\Delta y\Delta z$ will always be small, or $F_T(x, y, z, t) \approx T_0\Delta y\Delta z$, leaving

$\mathbf{F_{TYZ}} = [T_0\Delta y\Delta z + (\partial F_T/\partial x)\Delta x][\mathbf{i} + \{\partial P_y(x+\Delta x, y, z, t)/\partial x\}\mathbf{j} + \{\partial P_z(x+\Delta x, y, z, t)/\partial x\}\mathbf{k}]$

$– T_0\Delta y\Delta z[\mathbf{i} + \{\partial P_y(x, y, z, t)/\partial x\}\mathbf{j} + \{\partial P_z(x, y, z, t)/\partial x\}\mathbf{k}]$ (89)

It is assumed that $\partial P_y/\partial x$ and $\partial P_z/\partial x$ are small quantities in comparison to 1. Also we note that $\{\partial P_y(x+\Delta x, y, z, t)/\partial x\} – \{\partial P_y(x, y, z, t)/\partial x\} = \Delta x\partial^2 P_y/\partial x^2$ in the limit as $\Delta x \rightarrow 0$ and we have a similar equation for for $\partial^2 P_z/\partial x^2$. Keeping only the terms that are lowest order in small quantities,

$\mathbf{F_{TYZ}} = (\partial F_T/\partial x)\Delta x\mathbf{i} + T_0\Delta x\Delta y\Delta z(\partial^2 P_y/\partial x^2)\mathbf{j} + T_0\Delta x\Delta y\Delta z(\partial^2 P_z/\partial x^2)\mathbf{k}]$ (90)

What remains is to further evaluate the term $(\partial F_T/\partial x)\Delta x$. Recall Eq. (15):

$F_{T0} = K_{T0}X_Q$ (15)

(As discussed above, K_T will vary away from K_{T0}. However, here we are considering the dominant first order term for K_T and we use $K_T = K_{T0}$ here.) Also recall that X_Q is the variable size of the analysis-cube of aether in one dimension and here $X_Q \approx X_0$. Dividing F_{T0} by the area of the cube face perpendicular to x results in $K_{T0}X_0/\Delta y\Delta z = K_{T0}X_0/X_0^2 = K_{T0}/X_0$. (Here we use $\Delta y = \Delta z = X_0$.) Recalling that T_0 equals the magnitude of the tension force per unit area that we just found,

$T_0 = K_{T0}/X_0$ (91)

In the general case, the x displacement of the right side of the cube is $P_x(x + X_0, y, z, t)$ while the displacement at the left side is $P_x(x, y, z, t)$. The wall separation is X_0 plus the difference of these displacements so the tension force per unit area that results from Eq. (15) is

$F_T/\Delta y\Delta z = F_T/X_0^2 = K_{T0}X_Q/X_0^2 = (K_{T0}/X_0^2)[X_0 + P_x(x + X_0, y, z, t) – P_x(x, y, z, t)]$ (92)

Rearranging Eq. (91) $K_{T0} = T_0X_0$, and substituting this into Eq. (92):

$$F_T/X_0^2 = (T_0/X_0)[X_0 + P_x(x + X_0, y, z, t) - P_x(x, y, z, t)] = (T_0/X_0)[X_0 + \Delta x\partial P_x/\partial x] \quad (93)$$

($[P_x(x + X_0, y, z, t) - P_x(x, y, z, t)]/\Delta x = \partial P_x/\partial x$ as $\Delta x = X_0 \rightarrow 0$ for our analysis-cube.) With $\Delta x = X_0$ we have $F_T/X_0^2 = T_0[1 + \partial P_x/\partial x]$. From which $\partial F_T/\partial x = T_0X_0^2[\partial^2P_x/\partial x^2]$ and hence

$$(\partial F_T/\partial x)\Delta x = (\partial F_T/\partial x)X_0 = T_0X_0^3[\partial^2P_x/\partial x^2] = \Delta x\Delta y\Delta zT_0(\partial^2P_x/\partial x^2) \quad (94)$$

Note again that here we have used $\Delta x = \Delta y = \Delta z = X_0$ and we allow our analysis-cube to shrink to zero. Substituting Eq. (94) into Eq. (90) leaves:

$$\mathbf{F_{TYZ}} = \Delta x\Delta y\Delta z[T_0(\partial^2P_x/\partial x^2)\mathbf{i} + T_0(\partial^2P_y/\partial x^2)\mathbf{j} + T_0(\partial^2P_z/\partial x^2)\mathbf{k}] \quad (95)$$

Eq. (95) is the tension force on a small cube of aether that results from the tension forces that are present on the yz faces of the cube. By doing the same derivation for the xy and xz faces analogous expressions for the forces are derived in those directions, $\mathbf{F_{TXZ}} = \Delta x\Delta y\Delta z[T_0(\partial^2P_x/\partial y^2)\mathbf{i} + T_0(\partial^2P_y/\partial y^2)\mathbf{j} + T_0(\partial^2P_z/\partial y^2)\mathbf{k}]$ and $\mathbf{F_{TXY}} = \Delta x\Delta y\Delta z[T_0(\partial^2P_x/\partial z^2)\mathbf{i} + T_0(\partial^2P_y/\partial z^2)\mathbf{j} + T_0(\partial^2P_z/\partial z^2)\mathbf{k}$. The sum of the three force components $\mathbf{F_{TYZ}}$, $\mathbf{F_{TXZ}}$ and $\mathbf{F_{TXY}}$ results in the total force expression, and recalling that there is no net longitudinal tension force, that $\mathbf{P}$ is just the sum of its components, and that the Laplacian is $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2 + \partial^2/\partial z^2$, the equation for the transverse tension force, $\mathbf{F_{TPT}}$, on an arbitrary cube of positive-attached-aether can be expressed as:

$$\mathbf{F_{TPT}} = \Delta x\Delta y\Delta zT_0\nabla^2\mathbf{P} \quad (96)$$

C.6 – Flow Force Effect on the Positive-Attached-Aether

Recall Eqs. (78) and (80):

$$\mathbf{F_{FP1}} = K_{F1}\Delta V\rho_{PD}[\mathbf{U_{PDT}} - \partial\mathbf{P}_T/\partial t] - K_{F1}\Delta V\rho_{ND}[\mathbf{U_{NDT}} - \partial\mathbf{P}_T/\partial t] \quad (78)$$

$$\mathbf{F_{FP2}} = -K_{F2}\Delta V\rho_{PD}[\mathbf{U_{PDT}} - \partial\mathbf{P_T}/\partial t] + K_{F2}\Delta V\rho_{ND}[\mathbf{U_{NDT}} - \partial\mathbf{P_T}/\partial t] \quad (80)$$

The flow force on the positive-attached-aether is given by the sum of Eqs. (78) and (80):

$\mathbf{F_{FP}} = \mathbf{F_{FP1}}+\mathbf{F_{FP2}} = (K_{F1}–K_{F2})\Delta V\rho_{PD}[\mathbf{U_{PDT}}–\partial\mathbf{P}_T/\partial t] – (K_{F1}–K_{F2})\Delta V\rho_{ND}[\mathbf{U_{NDT}}–\partial\mathbf{P_T}/\partial t]$ (97)

We now define K_{F3} and rewrite Eq. (97) in terms of K_{F3}

$K_{F3} = K_{F2} – K_{F1}$ (98)

$\mathbf{F_{FP}} = \mathbf{F_{FP1}}+\mathbf{F_{FP2}} = –K_{F3}\Delta V\rho_{PD}[\mathbf{U_{PDT}} – \partial\mathbf{P}_T/\partial t] + K_{F3}\Delta V\rho_{ND}[\mathbf{U_{NDT}} – \partial\mathbf{P_T}/\partial t]$ (99)

Next, we will drop the terms $\partial\mathbf{P_T}/\partial t$ from Eq. (99) as we will show shortly that $\partial\mathbf{P}/\partial t << \mathbf{U_{PD}}$. Expanding the volume of the analysis-cube ΔV as $\Delta x\Delta y\Delta z$ this leaves us with:

$\mathbf{F_{FP}} = –K_{F3}\rho_{PD}\Delta x\Delta y\Delta z\mathbf{U_{PDT}} + K_{F3}\rho_{ND}\Delta x\Delta y\Delta z\mathbf{U_{NDT}} = K_{F3}\Delta x\Delta y\Delta z[\rho_{ND}\mathbf{U_{NDT}} – \rho_{PD}\mathbf{U_{PDT}}]$ (100)

To see that $\partial\mathbf{P}/\partial t << \mathbf{U_{PD}}$, consider Figure 3. If a sphere of detached-aether moves upward so that the bottom of the sphere occupies the position where its top used to be, the attached-aether of like-kind that was originally at the top of the sphere, and was originally displaced upward (the vector **T** in Fig. 3), will now be at the bottom of the sphere, and it will be displaced downward (the vector **B** in Fig. 3). Therefore, the attached-aether moves downward as a result of the upward flow of the like-kind detached-aether: the flow of attached-aether within the sphere is in the opposite direction of the flow of like-kind detached-aether. Since the unlike-kind attached-aether has the opposite displacement (it is pulled in rather than being pushed out) the flow of attached-aether within the sphere is in the same direction as the flow of unlike-kind detached-aether.

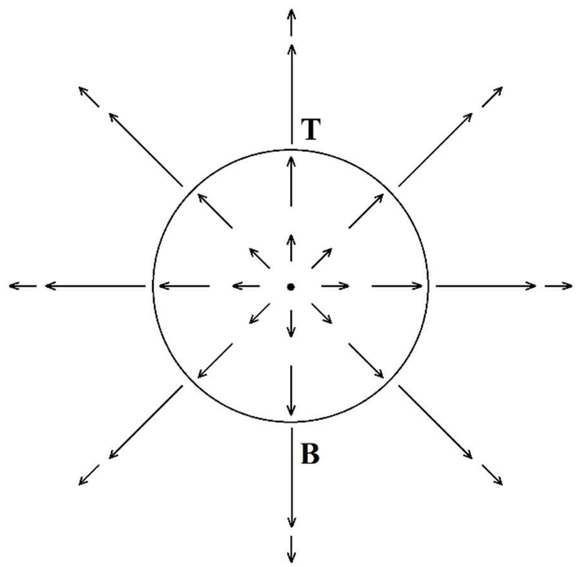

Figure 3. A sphere of detached-aether results in like-kind attached-aether being pushed radially outward according to the density law. At the top pole of the sphere (T) the displacement of the attached-aether is in the upward direction, and at the bottom of the sphere (B) the displacement is in the downward direction.

To evaluate the magnitude of the driven flows, first consider Figure 3 for the case of positive-detached-aether moving with velocity U in the upward (z) direction. Specify t = 0 as the time where the situation is as shown in the figure. Then, at t = 0 the displacement of the positive-attached-aether at the top of the sphere is Pz(x, y, z, t) = Pz(0, 0, R, 0) = dr, where dr is the expansion of the attached-aether due to the presence of the detached-aether within the sphere.

Since Eq. (39) informs that the positive-attached-aether density within the sphere becomes $\rho_0 - \rho_{PD}/2$ due to the detached-aether, the amount $Q/2 = (\rho_{PD}/2)(4/3)\pi R^3$ must be pushed into the volume of a shell surrounding the sphere, and, with no detached-aether in that shell, the density in that shell will be ρ_0. Hence, $\rho_0 4\pi R^2 dr = Q/2$, and we see that $dr/R = (Q/2)/\rho_0 4\pi R^3 = [(\rho_{PD}/2)(4/3)\pi R^3]/\rho_0 4\pi R^3 = \rho_{PD}/6\rho_0$. Next, with U the velocity of the sphere, notice that the sphere will move a distance 2R upward in the time t = 2R/U and at that time Pz(x, y, z, t) = Pz(0, 0, R,

$2R/U) = -dr$, since the point of analysis is now at the bottom of the sphere. Now, define $\partial \mathbf{P_D}/\partial t$ as the driven flow (subscript D) of the attached-aether due to the motion of the detached-aether. We find that $\partial P_D/\partial t = [Pz(0, 0, R, 2R/U) - Pz(0, 0, R, 0)]/[2R/U] = -2dr/[2R/U] = -Udr/R = -U\rho_{PD}/6\rho_0$. The negative-attached-aether will have the opposite displacement, so $\partial N_D/\partial t = +U\rho_{PD}/6\rho_0$. Since throughout this work we have $\rho_{PD} << \rho_0$, for the case presented here we see that $\partial P_D/\partial t << U$ and $\partial N_D/\partial t << U$. We assume these conditions hold generally, and hence can drop the term $\partial \mathbf{P}/\partial t$ with respect to $\mathbf{U_{PD}}$ when arriving at Eq. (100).

C.7 – Longitudinal Tension-Force, Delta-force and Quantum-Force Effects on the Positive-Attached-Aether

In section C.3, Eq. (59), we show that the delta-force ($\mathbf{F_\delta}$), the quantum-force ($\mathbf{F_Q}$) and the longitudinal-tension-force ($\mathbf{F_{TL}}$) sum to zero longitudinally, and both $\mathbf{F_\delta}$ and $\mathbf{F_Q}$ are purely longitudinal forces. (Section C.3 only considered longitudinal forces arising from immersed detached-aether.) Since they sum to zero, $\mathbf{F_\delta}$, $\mathbf{F_Q}$ and $\mathbf{F_{TL}}$ do not appear in the equation of motion of the positive-attached-aether.

C.8 – The Positive-Attached-Aether Equation of Motion

Now that all of the forces acting on the positive-attached-aether have been identified and evaluated, it is possible to determine the equation of motion for the positive-attached-aether. It will be assumed here that the velocity of the attached-aether is small compared to the velocity of light, and hence the equation $F = ma$ will be applied. The mass of an analysis-cube of aether is equal to its mass density times its volume $\Delta x\Delta y\Delta z$ and the acceleration is equal to the second partial derivative of the position of the cube with respect to time, $\partial^2\mathbf{P}/\partial t^2$. We will now define m_0 as the nominal aetherial mass density. Putting this all together, and recalling the forces on the positive-attached-aether for the flow, Eq. (100), $\mathbf{F_{FP}}$, and transverse tension, Eq. (96), $\mathbf{F_{TPT}}$, leaves:

$$\mathbf{F_P} = m\mathbf{a} = m_0\Delta x\Delta y\Delta z(\partial^2\mathbf{P}/\partial t^2) = \mathbf{F_{TPT}} + \mathbf{F_{FP}}$$

$$= \Delta x\Delta y\Delta z T_0\nabla^2\mathbf{P} + K_{F3}\Delta x\Delta y\Delta z[\rho_{ND}\mathbf{U_{NDT}} - \rho_{PD}\mathbf{U_{PDT}}] \qquad (101)$$

Now, eliminate the factor $\Delta x\Delta y\Delta z$ from each term:

$$m_0(\partial^2\mathbf{P}/\partial t^2) = T_0\nabla^2\mathbf{P} + K_{F3}[\rho_{ND}\mathbf{U_{NDT}} - \rho_{PD}\mathbf{U_{PDT}}] \qquad (102)$$

In order to simplify the mathematics, the vector $\mathbf{J}$ will now be defined as the net current density of detached-aether that flows through the attached-aether. $\mathbf{J}$ is the density of the positive-detached-aether times its velocity, minus the density of the negative-detached-aether times its velocity:

$$\mathbf{J} = \rho_{PD}\mathbf{U_{PD}} - \rho_{ND}\mathbf{U_{ND}} \qquad (103)$$

With $\mathbf{J_T}$ the transverse component of $\mathbf{J}$, substituting Eq. (103) into Eq. (102) leaves:

$$m_0(\partial^2\mathbf{P}/\partial t^2) = T_0\nabla^2\mathbf{P} - K_{F3}\mathbf{J_T} \qquad (104)$$

Now divide each side of Eq. (104) by T_0:

$$(m_0/T_0)(\partial^2\mathbf{P}/\partial t^2) = \nabla^2\mathbf{P} - (K_{F3}/T_0)\mathbf{J_T} \qquad (105)$$

And then multiply each side of the equation by minus one and take the Laplacian term over to the left-hand side:

$$\nabla^2\mathbf{P} - (m_0/T_0)(\partial^2\mathbf{P}/\partial t^2) = (K_{F3}/T_0)\mathbf{J_T} \qquad (106)$$

Now set the following relationship:

$$T_0 = m_0c^2 \qquad (107)$$

Rearranging Eq. (107) to $m_0/T_0 = 1/c^2$ Eq. (106) becomes:

$$\nabla^2\mathbf{P} - (1/c^2)(\partial^2\mathbf{P}/\partial t^2) = (K_{F3}/T_0)\mathbf{J_T} \qquad (108)$$

Dimensional analysis. m_0 has dimensions kg/m^3; T_0 has dimensions of force/m^2, or kg m/s^2 m^2, or kg/s^2 m. Hence, m_0/T_0 has dimensions (kg/m^3)/(kg/s^2 m) = (s^2/m^2), the same as $1/c^2$.

C.9 – The Negative-Attached-Aether Equation of Motion

The derivation of the equation of motion for the negative-attached-aether will follow the same steps as does the derivation for the positive-attached-aether, although there are some differences which we can describe term by term. Referring to the derivations above, it is seen that the Laplacian and acceleration terms only involve replacing **P** by **N**, provided we assume that the density ρ_0, mass density m_0, and nominal tension T_0 are equal for both types of aether. The flow force law has the opposite sign. (See Eqs. (78), (79), (80) and (81).) Making these changes leaves:

$$\nabla^2\mathbf{N} - (1/c^2)(\partial^2\mathbf{N}/\partial t^2) = -(K_{F3}/T_0)\mathbf{J_T} \qquad (109)$$

C.10 – Transverse and Longitudinal Separations of the Attached-Aether

Consider now the transverse portions of Eqs. (108) and (109):

$$\nabla^2\mathbf{P_T} - (1/c^2)(\partial^2\mathbf{P_T}/\partial t^2) = (K_{F3}/T_0)\mathbf{J_T} \qquad (110)$$

$$\nabla^2\mathbf{N_T} - (1/c^2)(\partial^2\mathbf{N_T}/\partial t^2) = -(K_{F3}/T_0)\mathbf{J_T} \qquad (111)$$

It can be seen that Eq. (110) for $\mathbf{P_T}$ is nearly identical to Eq. (111) for $\mathbf{N_T}$, as they only differ in the sign of the right-hand side. At this point, recall the physics of the situation: $\mathbf{P_T}$ and $\mathbf{N_T}$ are the transverse portion of the vector displacements of the positive and negative attached-aether, respectively. These vector displacements will be caused by the forces acting upon the attached-aether, which are the tension and flow forces. For the transverse component of the displacements, the flow force on the positive-attached-aether is the equal and opposite of the flow force on the negative-attached-aether. Since the tension results from a restorative force proportional to relative displacements within each individual aether component (see Fig. 2 above), and since both the presence of detached-aether and the flow force act to displace the components in opposite directions, the tension force will also act in opposite directions. Hence, we have:

$$\mathbf{N_T} = -\mathbf{P_T} \tag{112}$$

Recalling Eq. (58), $\mathbf{N_L} = -\mathbf{P_L}$, and summing Eqs. (58) and (112) leaves:

$$\mathbf{N} = -\mathbf{P} \tag{113}$$

C.11 – Maxwell's Equations in Terms of Potentials

Now make the following assignment:

$$\mathbf{P_T} = -\mathbf{N_T} = -K_{F3}\mathbf{A}/\mu_0 T_0 \tag{114}$$

In Eq. (114) μ_0 is the permeability of free space. Substitution of Eq. (114) into Eq. (110) yields the same equation as substitution of Eq. (114) into Eq. (111):

$$\nabla^2\mathbf{A} - (1/c^2)(\partial^2\mathbf{A}/\partial t^2) = -\mu_0\mathbf{J_T} \tag{115}$$

Dimensional analysis. At the end of section C.4 we see the dimensions of K_{F1} and K_{F2} as kg/C s, and hence K_{F3} has dimensions of kg/C s; T_0 has dimensions of kg m s^{-2}/m^2 or kg/(m s^2). μ_0 has units kg m/C^2. Therefore $K_{F3}/\mu_0 T_0$ has dimensions (kg/C s)/[(kg m/C^2)(kg/(m s^2)] = C s/kg, and thus by Eq. (114) **A** has dimensions of kg m/C s.

Now **J** is a vector field which we are free to decompose into its longitudinal and transverse components $\mathbf{J} = \mathbf{J_L} + \mathbf{J_T}$. Also from Eq. (103) we have $\mathbf{J} = \rho_{PD}\mathbf{U_{PD}} - \rho_{ND}\mathbf{U_{ND}}$. We will choose the conventional definition of $\mathbf{J_L}$:

$$\mathbf{J_L} = -(1/4\pi)\,\nabla \int [\,(\nabla' \cdot \mathbf{J})\,/\,|\,\mathbf{x} - \mathbf{x}'\,|\,]\, d^3x' \tag{116}$$

Next note that the solution to Poisson's Equation, Eq. (55), $\nabla^2\phi = -\rho_D/\varepsilon_0$, is:

$$\phi(x,t) = (1/4\pi\varepsilon_0) \int [\rho_D\,(\mathbf{x}\,'\,, t) / |\,\mathbf{x} - \mathbf{x}\,'\,|\,]\, d^3x' \qquad (117)$$

Now take the gradient of $\phi(x,t)$ and then take its partial time derivative:

$$\partial\nabla\phi(x,t)/\partial t = (1/4\pi\varepsilon_0)\, \nabla \int \partial[\rho_D\,(\mathbf{x}\,'\,, t)/\partial t \,/\, |\,\mathbf{x} - \mathbf{x}\,'\,|\,]\, d^3x' \qquad (118)$$

Using the continuity equation, $\partial\rho_D/\partial t = -\nabla \cdot \mathbf{J}$, Eqs. (116) and (118) reveal:

$$\varepsilon_0\partial\nabla\phi(x,t)/\partial t = \mathbf{J_L} \qquad (119)$$

At this point, we can therefore add the quantity $-\mu_0\mathbf{J_L} + \varepsilon_0\mu_0\partial\nabla\phi/\partial t$ to Eq. (115), since this is adding zero. And with $\varepsilon_0\mu_0 = 1/c^2$ we get:

$$\nabla^2\mathbf{A} - (1/c^2)(\partial^2\mathbf{A}/\partial t^2) = -\mu_0\mathbf{J_T} - \mu_0\mathbf{J_L} + (1/c^2)\partial\nabla\phi/\partial t \qquad (120)$$

or,

$$\nabla^2\mathbf{A} - (1/c^2)(\partial^2\mathbf{A}/\partial t^2) = -\mu_0\mathbf{J} + (1/c^2)\partial\nabla\phi/\partial t \qquad (121)$$

Further, since Eq. (114) informs that $\mathbf{A}$ is a transverse vector:

$$\nabla \cdot \mathbf{A} = 0 \qquad (122)$$

It is also timely to recall Eq. (55):

$$\nabla^2\phi = -\rho_D/\varepsilon_0 \qquad (55)$$

Eqs. (55), (121) and (122) are readily recognized as Maxwell's Equations in the Coulomb gauge in terms of potentials.

C.12 – Maxwell's Equations in Terms of Fields

To get the more familiar Maxwell's Equations in terms of fields, start by defining two arbitrary vectors $\mathbf{E}$ and $\mathbf{B}$:

$$\mathbf{E} = -\nabla\phi - \partial\mathbf{A}/\partial t \qquad (123)$$

$$\mathbf{B} = \nabla \times \mathbf{A} \qquad (124)$$

Applying the partial derivative with respect to time to Eq. (123):

$$\partial\mathbf{E}/\partial t = -\partial\nabla\phi/\partial t - \partial^2\mathbf{A}/\partial t^2 \qquad (125)$$

Next, add a term $\nabla(\nabla \cdot \mathbf{A})$ to Eq. (121), which is permissible since by Eq. (122) $\nabla \cdot \mathbf{A} = 0$:

$$\nabla^2\mathbf{A} - (1/c^2)(\partial^2\mathbf{A}/\partial t^2) = -\mu_0\mathbf{J} + (1/c^2)\partial\nabla\phi/\partial t + \nabla(\nabla \cdot \mathbf{A}) \qquad (126)$$

Now rearrange terms (first equality below), and use Eq. (125) for $\partial\mathbf{E}/\partial t$ (second equality below):

$$\nabla(\nabla \cdot \mathbf{A}) - \nabla^2\mathbf{A} = \mu_0\mathbf{J} - (1/c^2)\partial\nabla\phi/\partial t - (1/c^2)(\partial^2\mathbf{A}/\partial t^2) = \mu_0\mathbf{J} + (1/c^2)\partial\mathbf{E}/\partial t \qquad (127)$$

Now use $\nabla \times \mathbf{B} = \nabla \times (\nabla \times \mathbf{A}) = \nabla(\nabla \cdot \mathbf{A}) - \nabla^2\mathbf{A}$ to get:

$$\nabla \times \mathbf{B} = \mu_0\mathbf{J} + (1/c^2)\partial\mathbf{E}/\partial t \qquad (128)$$

Taking the divergence of Eq. (123), $\nabla \cdot \mathbf{E} = -\nabla \cdot \nabla\phi - \partial(\nabla \cdot \mathbf{A})/\partial t$. With $\nabla \cdot \mathbf{A} = 0$, and $\nabla \cdot \nabla\phi = \nabla^2\phi$,

and with Eq. (55) $\nabla^2\phi = -\rho_D/\varepsilon_0$ this leaves:

$$\nabla \cdot \mathbf{E} = \rho_D/\varepsilon_0 \qquad (129)$$

Taking the divergence of Eq. (124), along with the identity $\nabla \cdot (\nabla \times \mathbf{A}) = 0$:

$$\nabla \cdot \mathbf{B} = 0 \qquad (130)$$

Now take the curl of Eq. (123), $\nabla \times \mathbf{E} = -\nabla \times \nabla\phi - \partial(\nabla \times \mathbf{A})/\partial t$ and recalling Eq. (124) $\mathbf{B} = \nabla \times \mathbf{A}$ and using the identity $\nabla \times \nabla\phi = 0$ we get

$\nabla \times \mathbf{E} = -\partial \mathbf{B}/\partial t$ (131)

Eqs. (128) through (131) are Maxwell's Equations in terms of fields.

C.13. Field-Energy Effects on the Detached-Aether

Consider a region where there is an ambient attached-aether longitudinal displacement $\mathbf{P_{AL}} = -\mathbf{N_{AL}}$ not equal to zero. We have seen above in section C.2 that such a displacement is caused by detached-aether within the attached-aether, and Eqs. (72) and (77) of section C.3, show that total analysis-cube energies of $E_P = K_{T0}X_0^2[1+4K_c^2P^2/\xi_0^2]$ and $E_N = K_{T0}X_0^2[1+4K_c^2N^2/\xi_0^2]$ are associated with such displacements.

Now consider a sphere of detached-aether placed within the attached-aether. Without loss of generality we can assume an ambient displacement of $\mathbf{P_{AL}} = -\mathbf{N_{AL}}$ in the x direction, and that our sphere is centered at the origin. For analysis we will divide the sphere into slices of thickness ΔY centered at $y = Y$, and then further divide those slices into strips with thickness ΔZ centered at $z = Z$, as shown in Figures 4 and 5.

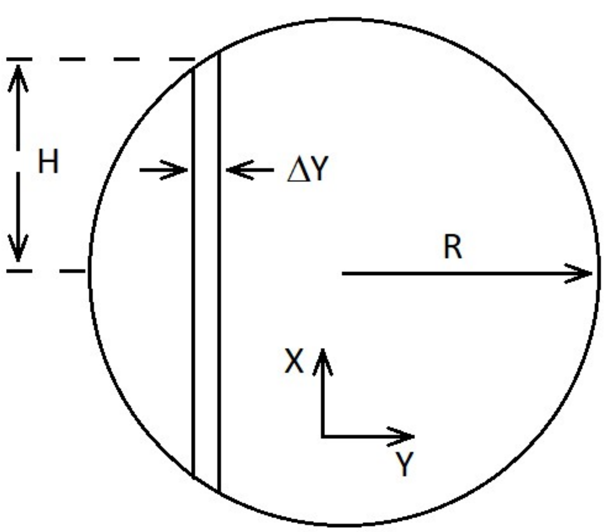

Figure 4. A sphere of detached-aether, and a slice of width ΔY.

Figure 4 shows a view of a sphere of detached-aether with radius R. Examining a slice of thickness ΔY we can see that the half-height of that slice will be $H = (R^2 - Y^2)^{1/2}$.

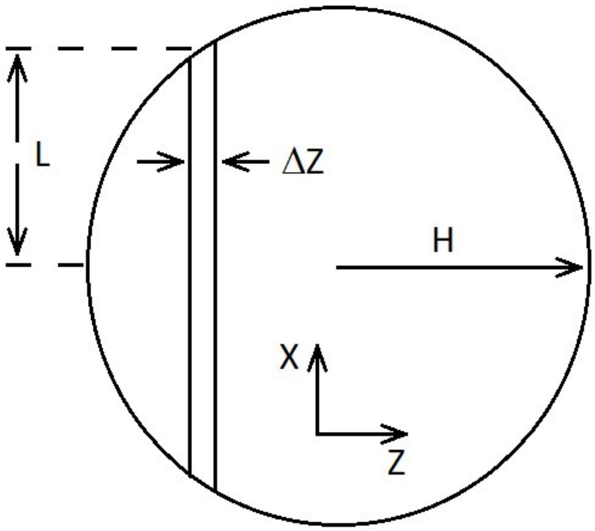

Figure 5. A slice of detached-aether, and a strip of width ΔZ.

Figure 5 shows a view of the slice of detached-aether. The slice has the half-height, H, that we found in Fig. 4. Examining the strip of thickness ΔZ centered at Z we can see that the half-length of that strip will be $L = (H^2 - Z^2)^{1/2}$. The strip shown in Fig. 5 has dimensions of ΔY by ΔZ by L. Our aim is to evaluate the force on our sphere of detached-aether when it is immersed in a region of attached-aether displacement where we have finite $\mathbf{P_L} = -\mathbf{N_L}$. Toward that aim, we first observe that the sphere of detached-aether will cause its own longitudinal attached-aether displacement. Due to the symmetry of the situation, a sphere of positive-detached-aether will push nearby positive-attached-aether outward radially, and it will pull nearby negative-attached-aether in radially. With a detached-aether density inside of the sphere of ρ_{PD}, the positive-attached-aether density inside the sphere will be $\rho_{PIN} = \rho_0 - \rho_{PD}/2$. Without any detached-aether inside of it, a sphere of radius R has $(4/3)\pi R^3\rho_0$ of positive-attached-aether inside. To lower the density to ρ_{PIN} requires that the radius of the sphere expand so that $(4/3)\pi(R+\Delta R)^3\rho_{PIN} = (4/3)\pi R^3\rho_0$, or $\rho_{PIN} =$

$R^3\rho_0/(R+\Delta R)^3 = \rho_0/(1+\Delta R/R)^3 \approx \rho_0(1 - 3\Delta R/R)$. Recalling $\rho_{PIN} = \rho_0 - \rho_{PD}/2$ we obtain $\rho_{PD}/2 \approx 3\rho_0\Delta R/R$, or, $\Delta R/R = \rho_{PD}/6\rho_0$. The displacement of the edge of the sphere is $P_{edge} = R(\rho_{PD}/6\rho_0)$. Now the displacement will linearly grow from the center of the sphere, so that in general, with $\mathbf{P_{sphere}}$ being the displacement caused by the sphere, inside the sphere we have:

$$\mathbf{P_{sphere}} = (\rho_{PD}/6\rho_0)\mathbf{r} = (\rho_{PD}/6\rho_0)(x\mathbf{i} + y\mathbf{j} + z\mathbf{k}). \qquad (132)$$

If there is now an ambient longitudinal attached-aether displacement $\mathbf{P_{AL}}$, which without loss of generality can be considered to be in the X direction, $\mathbf{P_{AL}} = P_A\mathbf{i}$, then the total attached-aether displacement within the sphere becomes $\mathbf{P_{TOT}} = \mathbf{P_{AL}} + \mathbf{P_{sphere}} = [(\rho_{PD}/6\rho_0)x + P_A]\mathbf{i} + (\rho_{PD}/6\rho_0)y\mathbf{j} + (\rho_{PD}/6\rho_0)z\mathbf{k}$. (The displacement of the attached-aether induced by the detached-aether ($\mathbf{P_{sphere}}$) is purely longitudinal, and hence will only add and subtract with respect to the longitudinal ambient displacement, and therefore we look here at $\mathbf{P_{AL}}$, the longitudinal ambient displacement of $\mathbf{P}$.)

Eq. (72) relates that $E_P = K_{T0}X_0^2[1+4K_c^2P^2/\xi_0^2]$. Since the nominal energy (prior to any displacement) is $K_{T0}X_0^2$, we see that the energy due to displacement is $4K_c^2K_{T0}X_0^2P^2/\xi_0^2$. Returning to Figure 5, the strip of positive-attached-aether is centered at z and y. Within the strip, the displacement energy of an analysis-cube centered at x will be:

$$E_{PP}(x,y,z) = 4K_c^2K_{T0}X_0^2P^2/\xi_0^2 = 4K_c^2K_{T0}(X_0^2/\xi_0^2)[P_{TOT}(x,y,z)]^2$$

$$= 4K_c^2K_{T0}(X_0^2/\xi_0^2)[(\rho_{PD}/6\rho_0)^2x^2 + 2(\rho_{PD}/6\rho_0)xP_A + P_A^2 + (\rho_{PD}/6\rho_0)^2y^2 + (\rho_{PD}/6\rho_0)^2z^2] \qquad (133)$$

In Eq. (133) the subscript PP is for the case where positive-detached-aether is immersed in positive-attached-aether. We can now evaluate the force present on the strip of attached-aether depicted in Fig. 5. For a small additional (and virtual) displacement δx, the energy will become:

$$E_{PP}(x+\delta x,y,z) = 4K_c^2K_{T0}(X_0^2/\xi_0^2)[P_{TOT}(x+\delta x,y,z)]^2 \qquad (134)$$

$$= 4K_c^2K_{T0}(X_0^2/\xi_0^2)[(\rho_{PD}/6\rho_0)^2(x+\delta x)^2+2(\rho_{PD}/6\rho_0)(x+\delta x)P_A+P_A^2+(\rho_{PD}/6\rho_0)^2y^2+(\rho_{PD}/6\rho_0)^2z^2]$$

Subtracting Eq. (133) from Eq. (134) leaves the energy change resulting from the additional displacement:

$\delta E_{PP}(x,y,z) = E_{PP}(x+\delta x,y,z) - E_{PP}(x,y,z)$

$= 4K_c^2K_{T0}(X_0^2/\xi_0^2)[2(\rho_{PD}/6\rho_0)^2x\delta x + (\rho_{PD}/6\rho_0)^2\delta x^2 + 2(\rho_{PD}/6\rho_0)\delta xP_A]$ (135)

In the above expression we can drop the term that is second order in the small quantity δx, as we will take the limit as δx → 0. We can now evaluate the force on the strip by considering the sum of all volume elements within the strip. We can drop the term $2(\rho_{PD}/6\rho_0)^2x\delta x$ because for every value of positive x on our strip there is a value of negative x of equal magnitude. (The term $2(\rho_{PD}/6\rho_0)^2x\delta x$ cancels out over the strip because of symmetry.) The surviving term, $8K_c^2K_{T0}(X_0^2/\xi_0^2)(\rho_{PD}/6\rho_0)\delta xP_A$, is independent of x, y or z. Recalling that Eq. (135) refers to the change in energy for an analysis-cube within the strip, we can form the relation for the force on the whole strip by summing over all of the analysis-cubes in the strip (Σ_{strip} is the symbol for that sum). Analyzing a strip of length 2L, width X_0, and height X_0, there will be $2L/X_0$ analysis-cubes in that strip. The force in that strip will be

$F_{stripPP} = \Sigma_{strip}\ \{\delta E_{PP}(x,y,z)/\delta x\} = (2L/X_0)8K_c^2K_{T0}(X_0^2/\xi_0^2)(\rho_{PD}/6\rho_0)\delta xP_A/\delta x$

$= 8LK_c^2K_{T0}X_0\rho_{PD}P_A/3\rho_0\xi_0^2 = 8K_c^2K_{T0}LX_0^2\rho_{PD}P_A/3\rho_0\xi_0^2X_0$ (136)

The force on the strip shown in Fig. 5 is proportional to the volume of the strip ($2LX_0^2$) but independent of x, y and z. The sum of the volume of all of the strips will be the volume of the sphere, V_{sphere}. Hence, we can sum the forces from all such strips to arrive at:

$F_{spherePP} = 4K_c^2K_{T0}V_{sphere}\rho_{PD}P_A/3\rho_0\xi_0^2X_0$ (137)

Immersed positive-detached-aether will also affect the negative-attached-aether. In that case, the negative-attached-aether is pulled into the sphere rather than pushed out. Hence, both the displacement of the negative-attached-aether due to positive-detached-aether ($\mathbf{N_{sphere}}$) and the ambient longitudinal negative-attached-aether displacement ($\mathbf{N_{AL}}$) are equal in magnitude but opposite in sign to the case of the positive-attached-aether. (The analysis simply replaces by $\mathbf{P_{sphere}}$ by $-\mathbf{N_{sphere}}$ and $\mathbf{P_{AL}}$ by $-\mathbf{N_{AL}}$.) $\mathbf{N_{TOT}} = \mathbf{N_{AL}} + \mathbf{N_{sphere}} = -\mathbf{P_{AL}} + -\mathbf{P_{sphere}} = -\mathbf{P_{TOT}}$, and since the energies in Eqs. (134) and (135) are proportional to P_{TOT}^2 for the positive case, replacement of $\mathbf{P_{TOT}}$ by $\mathbf{N_{TOT}} = -\mathbf{P_{TOT}}$ leaves $N_{TOT}^2 = P_{TOT}^2$ in the analysis. This results in the same force on the positive-attached-aether from each component, $F_{spherePN} = F_{spherePP} = 4K_c^2K_{T0}V_{sphere}\rho_{PD}P_A/3\rho_0\xi_0^2X_0$. The total force is thus:

$$F_{sphereP} = F_{spherePP} + F_{spherePN} = 8K_c^2K_{T0}V_{sphere}\rho_{PD}P_A/3\rho_0\xi_0^2X_0 \quad (138)$$

Observe that the force is aligned with the direction of $\mathbf{x}$ in the above analysis, since the energy varies with δx. We have chosen $\mathbf{x}$ to be aligned with $\mathbf{P_{AL}}$, so the force is aligned with $\mathbf{P_{AL}}$. Inside the sphere shown in Fig. 5 the total aetherial displacement $\mathbf{P}$ increases in the direction of $\mathbf{P_{AL}}$ as a result of the detached-aether pushing the attached-aether outward. (On one side of the sphere the attached-aether is pushed in the opposite direction of $\mathbf{P_{AL}}$ while on the other side it is pushed in the same direction as $\mathbf{P_{AL}}$. Within the sphere, $\mathbf{P}$ increases from one side to the other.) Hence, the positive-attached-aether will be forced in the direction opposite of $\mathbf{P_{AL}}$ as lower energy (lower $\mathbf{P}$) is preferred. The force on the positive-detached-aether will be in the opposite direction as the force on the positive-attached-aether. This can be thought of as analogous to pushing a ball downward through a tub of water: as the disturbance (the ball, or the detached-aether) is forced down, the substance it is pushed through (the water, or the attached-aether) is forced in the opposite direction. Hence, the force on the detached-aether is in the direction of $\mathbf{P_{AL}}$.

$$\mathbf{F_{sphereP}} = 8K_c^2K_{T0}V_{sphere}\rho_{PD}\mathbf{P_{AL}}/3\rho_0\xi_0^2X_0 \quad (139)$$

We now recall Eq. (56) $\mathbf{P_L} - \mathbf{N_L} = -\varepsilon_0\nabla\phi/\rho_0$ and using Eq. (113), $\mathbf{N} = -\mathbf{P}$, $\mathbf{P_{AL}} - \mathbf{N_{AL}} = 2\mathbf{P_{AL}} = -\varepsilon_0\nabla\phi/\rho_0$. This allows us to arrive at

$\mathbf{F}_{\mathbf{sphere}} = -4K_c^2K_{T0}V_{sphere}\rho_{PD}\varepsilon_0\nabla\phi/3\xi_0^2X_0\rho_0^2$ (140)

Next, since Eq. (72) relates that $E_P = K_{T0}X_0^2[1 + 4K_c^2P^2/\xi_0^2]$, we see that two overlapped spheres of positive-detached-aether will have an energy in their cubes of $E_P = 2K_{T0}X_0^2[1 + 4K_c^2(2P_0)^2/\xi_0^2] = 2K_{T0}X_0^2[1 + 16K_c^2P_0^2/\xi_0^2]$, while if those same two spheres are at a great distance the energy will be $E_P = 2K_{T0}X_0^2[1 + 4K_c^2P_0^2/\xi_0^2]$. (Here P_0 is the displacement of a cube at an arbitrary position within an isolated sphere; overlapping two spheres will double such displacements.) Since nature favors smaller energy, we see that the force between amounts of positive-detached-aether is repulsive, agreeing with our sign choice leading to Eq. (140).

We now set the value of K_{T0}

$K_{T0} = 3\rho_0^2X_0\xi_0^2/4K_c^2\varepsilon_0$ (141)

Note that Eq. (10) leads to $X_0 = \xi_0/n$, or $n = \xi_0/X_0$. From Eq. (13), $K_{T0} = K_{T\xi 0}/n = K_{T\xi 0}X_0/\xi_0$. Hence K_{T0} varies linearly with X_0 (X_0 is the size of our analysis-cube) and this is consistent with our assignment in Eq. (141). We next observe that the density of the detached-aether multiplied by the volume of the sphere is equal to a quantity called Q, the charge in the sphere, $V_{sphere}\rho_{PD} = Q$. With this, and with Eq. (141), we then arrive at $\mathbf{F}_{\mathbf{LD}}$, the Lorentz force on an amount of positive-detached-aether immersed within a region of ambient attached-aether displacement:

$\mathbf{F}_{\mathbf{LD}} = -Q\nabla\phi$ (142)

While Eq. (142) has only been derived for the case of positive-detached-aether immersion, notice that negative-detached-aether will reverse the sign of the force in the derivation, and therefore Eq. (142) holds for either positive or negative detached-aether immersion.

Dimensional Analysis. $K_{T0} = 3\rho_0^2X_0\xi_0^2/4K_c^2\varepsilon_0$. $1/\varepsilon_0$ has dimensions m^3kg/s^2C^2, ρ_0 has dimensions of C/m^3, X_0 and ξ_0^2 have dimensions of m, and K_c^2 is dimensionless. So, K_{T0} has dimensions of

$(m^3kg/s^2C^2)(C/m^3)^2m^3 = kg/s^2$. We also know that $K_{T0}X_0^2$ is an energy, $kg\text{-}m^2/s^2$, confirming our result that K_{T0} has dimensions of kg/s^2.

C.14. Flow Force Effects on the Detached-Aether

To determine the flow force on the detached-aether, recall Eqs. (78) and (79) from section C.4.3:

$$\mathbf{F_{FP1}} = K_{F1}\Delta V\rho_{PD}[\mathbf{U_{PDT}} - \partial\mathbf{P_T}/\partial t] - K_{F1}\Delta V\rho_{ND}[\mathbf{U_{NDT}} - \partial\mathbf{P_T}/\partial t] \qquad (78)$$

$$\mathbf{F_{FN1}} = K_{F1}\Delta V\rho_{ND}[\mathbf{U_{NDT}} - \partial\mathbf{N_T}/\partial t] - K_{F1}\Delta V\rho_{PD}[\mathbf{U_{PDT}} - \partial\mathbf{N_T}/\partial t] \qquad (79)$$

Eqs. (78) and (79) are the forces from the flow of the detached-aether onto the positive-attached-aether and negative-attached-aether respectively. $\mathbf{F_{LF}}$, the total Lorentz flow force on the detached-aether, will be the reaction force, which is equal to the negative of the sum of the forces given in Eqs. (78) and (79):

$$\mathbf{F_{LF}} = K_{F1}\Delta V\rho_{PD}(\partial\mathbf{P_T}/\partial t - \partial\mathbf{N_T}/\partial t) - K_{F1}\Delta V\rho_{ND}[\partial\mathbf{P_T}/\partial t - \partial\mathbf{N_T}/\partial t]$$

$$= K_{F1}Q(\partial\mathbf{P_T}/\partial t - \partial\mathbf{N_T}/\partial t) \qquad (143)$$

In Eq. (143) Q is defined as $\Delta V(\rho_{PD} - \rho_{ND})$. Now recall Eq. (114), $\mathbf{P_T} = -\mathbf{N_T} = -K_{F3}\mathbf{A}/\mu_0T_0$. From this, and using Eq. (113), $\mathbf{N} = -\mathbf{P}$, $\partial\mathbf{P_T}/\partial t - \partial\mathbf{N_T}/\partial t = -2K_{F3}(\partial\mathbf{A}/\partial t)/\mu_0T_0$ so $\mathbf{F_{LF}} = K_{F1}Q(\partial\mathbf{P_T}/\partial t - \partial\mathbf{N_T}/\partial t) = -(2K_{F3}K_{F1}Q/\mu_0T_0)\partial\mathbf{A}/\partial t$. Now make the assignment

$$\mu_0T_0 = 2K_{F3}K_{F1} \qquad (144)$$

This allows the Lorentz force on any detached-aether due to flow forces to be expressed as:

$$\mathbf{F_{LF}} = -Q\partial\mathbf{A}/\partial t \qquad (145)$$

C.15. Tension Force Effects on the Detached-Aether

Detached-aether will not be directly affected by tension since tension is a force associated with attachment. However, when detached-aether flows through the attached-aether the tension within the attached-aether is affected, and that leads to reaction forces on moving detached-aether.

C.15.1. Tension Forces on the Attached-Aether when Ambient Displacements are Parallel to Detached-Aether Flow. Next we will evaluate the forces present on an analysis-cube of attached-aether due to the presence of flowing detached-aether when there is a parallel, ambient, attached-aether displacement. Without loss of generality the flow and ambient displacement (P_Z) can be assumed to be in the z direction and the gradient of the ambient displacement ($\partial P_{AZ}/\partial x$) can be assumed to be in the x direction. We start with an analysis-cube of positive-attached-aether that has positive-detached-aether flowing upward within it. This flow in the positive z direction leads to a force in the negative z direction, as found in Eq. (100) above.

Figures 6 through 9 show the central cube from Figure 2 under various conditions relevant for our present analysis. Figures 6 and 7 show cutouts of the xz plane through the cube, while Figures 8 and 9 show cutouts of the yz plane. Figures 6 and 8 show the tension forces on the cube without a flow (or flow force) while Figures 7 and 9 add a flow (and flow force) to the situation. From Figures 6 and 8 it is easily seen that the cube of aether has no net tension force upon it, since the tension forces are equal and opposite to each other. However, in Figures 7 and 9 the situation changes. The downward flow force on the cube will result in a downward displacement of the cube until the restoring tension forces have a z component, and the sum of such restoring tension-forces will be equal and opposite to the applied downward flow force. (Were the downward flow force greater, the cube will displace more, which would increase the restoring tension-force. Were it less, the opposite would occur. The equilibrium value is for the forces to balance each other out.)

However, while the total net force in the z direction will cancel out to zero, the situation will result in a force in the x direction, which will now be evaluated.

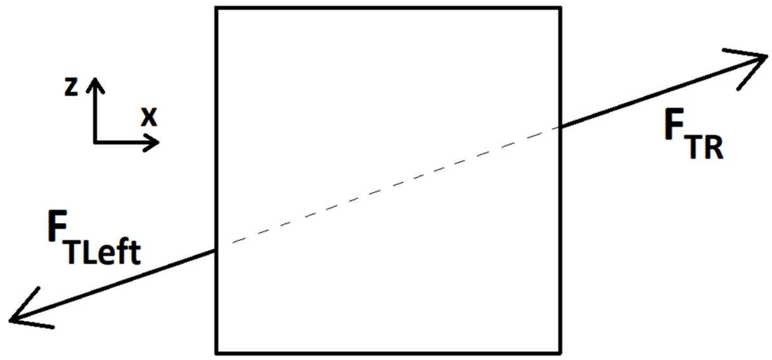

Figure 6. Tension forces on the yz faces of an analysis-cube of positive-attached-aether in the presence of an ambient positive-attached-aether displacement with gradient $\partial P_{AZ}/\partial x$ when there is no flowing detached-aether.

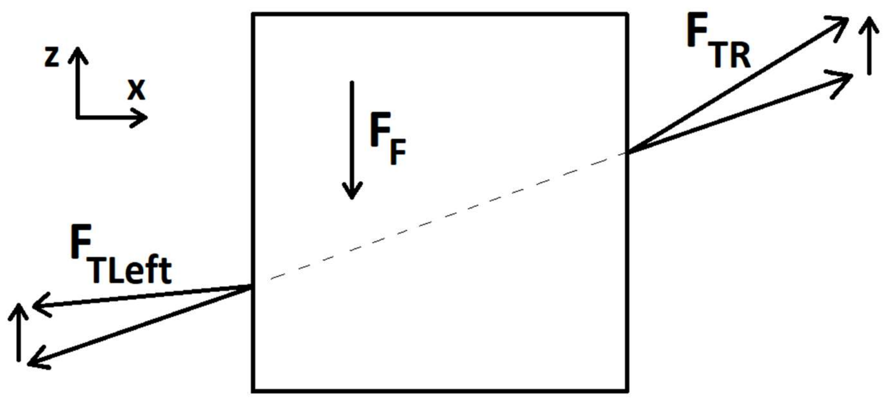

Figure 7. Tension forces on the yz faces of an analysis-cube of positive-attached-aether in the presence of an ambient positive-attached-aether displacement with gradient $\partial P_{AZ}/\partial x$ when there is positive-detached-aether flowing upward within.

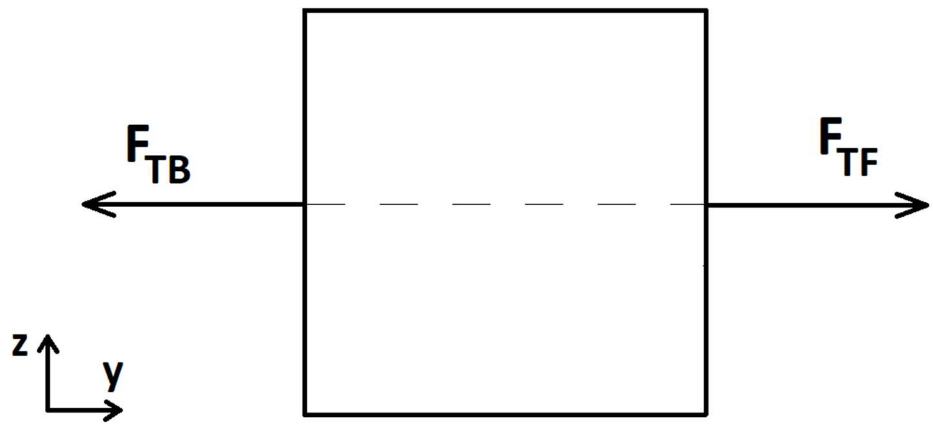

Figure 8. Tension forces on the xz faces of an analysis-cube of positive-attached-aether in the presence of an ambient aetherial gradient $\partial P_{AZ}/\partial x$ when there is no flowing detached-aether.

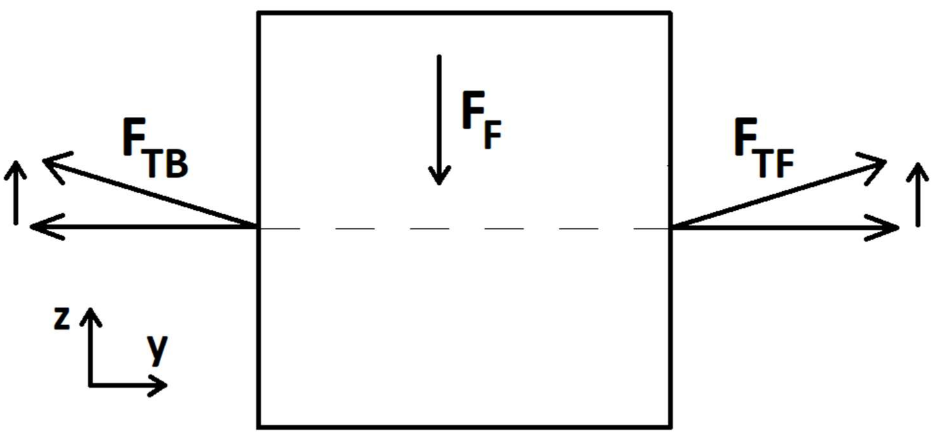

Figure 9. Tension forces on the xz faces of an analysis-cube of positive-attached-aether in the presence of an ambient aetherial gradient $\partial P_{AZ}/\partial x$ when there is upward flowing positive-detached-aether within.

In Figure 6 the tension-force on the left (right) points from the center of the cube shown toward the center of a not-shown cube to the left (right) of the one shown. (See Figure 2 for additional cubes.) The (not shown) cube on the left (right) will be displaced a distance $-dP_{AZ}$ (dP_{AZ}) in the z direction, and the slope of the tension-force is dP_{AZ} divided by the width of the cube $\Delta x = X_0$. (We use the subscript A to denote the ambient aetherial displacement.) Hence, in Figure 6, the tension-force on the left face of the cube is in the direction of ($-\mathbf{i} - [\partial P_{AZ}/\partial x]\mathbf{k}$). On the right side of Figure 6 the tension-force is in the direction of ($\mathbf{i} + [\partial P_{AZ}/\partial x]\mathbf{k}$). In Figure 7 there is an additional z component to the tension-force caused by the displacement of the cube due to the need to offset the flow-force. Here we define the quantity θ_F via:

$$dP_F = \theta_F \Delta x \qquad (146)$$

In Eq. (146) θ_F (for small θ_F) is the change in angle of the tension-force due to the flow force displacing the analysis-cube. On the left face of the cube in Figure 7, this results in a tension-force in the direction ($-\mathbf{i} + [\theta_F - \partial P_{AZ}/\partial x]\mathbf{k}$) and the tension-force on the right side is in the direction of ($\mathbf{i} + [\theta_F + \partial P_{AZ}/\partial x]\mathbf{k}$). The tension-force direction on the front and back sides can be similarly found from Figure 9 as being in the direction of ($\mathbf{j} + \mathbf{k}\theta_F$) and ($-\mathbf{j} + \mathbf{k}\theta_F$), respectively.

The tension-force on each face of the cube is the force per unit area, T_0, multiplied by the area of the face Δx^2. (The area of the yz face could be expressed as $\Delta y \Delta z$, but since this is a cube, $\Delta x = \Delta y = \Delta z = X_0$, so the area is Δx^2, as it is for all six faces.) The force will also involve a unit vector in the direction of the force.

The tension-force on the left side of Figure 7 is therefore $\mathbf{F_{Left}} = T_0 \Delta x^2[-\mathbf{i} + (\theta_F - \partial P_{AZ}/\partial x)\mathbf{k}]/[1 + (\theta_F - \partial P_{AZ}/\partial x)^2]^{1/2}$, while the tension-force on the right side is $\mathbf{F_{Right}} = T_0 \Delta x^2[\mathbf{i} + (\theta_F + \partial P_{AZ}/\partial x)\mathbf{k}]/[1+(\theta_F+\partial P_{AZ}/\partial x)^2]^{1/2}$. The tension-force on the front side is $\mathbf{F_{Front}} = T_0 \Delta x^2[\mathbf{j} + \theta_F\mathbf{k}]/[1+\theta_F^2]^{1/2}$; and the tension-force on the back $\mathbf{F_{Back}} = T_0 \Delta x^2[-\mathbf{j} + \theta_F\mathbf{k}]/[1+\theta_F^2]^{1/2}$. At this point, we use the approximation $(1+\varepsilon)^n \approx 1 + n\varepsilon$ for small ε, note that θ_F and $\partial P_{AZ}/\partial x$ will both be small, and we will keep terms up to second order in small quantities. The four tension-forces are then

$$\mathbf{F_{Left}} \approx T_0\Delta x^2[-\mathbf{i} + (\theta_F - \partial P_{AZ}/\partial x)\mathbf{k} + \mathbf{i}(\theta_F - \partial P_{AZ}/\partial x)^2/2] \quad (147)$$

$$\mathbf{F_{Right}} \approx T_0\Delta x^2[\mathbf{i} + (\theta_F + \partial P_{AZ}/\partial x)\mathbf{k} - \mathbf{i}(\theta_F + \partial P_{AZ}/\partial x)^2/2] \quad (148)$$

$$\mathbf{F_{Front}} \approx T_0\Delta x^2[\mathbf{j} + \theta_F\mathbf{k} - \mathbf{j}\theta_F^2/2] \quad (149)$$

$$\mathbf{F_{Back}} \approx T_0\Delta x^2[-\mathbf{j} + \theta_F\mathbf{k} + \mathbf{j}\theta_F^2/2]. \quad (150)$$

Lastly, there is the force from the flow within the cube. Eq. (100) above, $\mathbf{F_{FP}} = -K_{F3}\rho_{PD}\Delta V\mathbf{U_{PDT}} + K_{F3}\rho_{ND}\Delta V\mathbf{U_{NDT}}$, gives the expression for the total flow-force on the positive-attached-aether due to flows. Since we are only looking at positive-detached-aether flow, this becomes:

$$\mathbf{F_{FlowPP}} = -K_{F3}\rho_{PD}\Delta V\mathbf{U_{PDT}} = -K_{F3}Q_PU\mathbf{k} \quad (151)$$

In the second of Eqs. (151) $Q_P = \rho_{PD}\Delta V$ and $\mathbf{U_{PDT}} = U\mathbf{k}$. Here we are evaluating the force on a uniformly moving Q_P within a small cube. For a uniform sphere of charge moving in the z direction, points within that sphere have a uniform current in the z direction, and that does indeed have zero divergence: the current is transverse. If however we were to consider a moving gaussian distribution, then the current would vary in the z direction, which would have a finite divergence. Dividing such a distribution into spherical shells, we can decompose each shell into a constant current portion and its divergent portion and $\mathbf{U_{PDT}} = U\mathbf{k}$ results from just considering the constant current portion.

Summing the five forces yields the total force on the cube:

$$\mathbf{F_{LT1PP}} = T_0\Delta x^2[4\theta_F\mathbf{k} - 2\theta_F(\partial P_{AZ}/\partial x)\mathbf{i}] - K_{F3}Q_PU\mathbf{k} \quad (152)$$

The subscript reminds us we are evaluating the **L**orentz **T**ension-force of type **1** for the case of **P**ositive-detached-aether flowing through **P**ositive-attached-aether. In this case, equilibrium in the z direction will be obtained when $4T_0\Delta x^2\theta_F = K_{F3}Q_PU$, or $\theta_F = K_{F3}Q_PU/4T_0\Delta x^2$ and hence $\mathbf{F_{LT1PP}}$

$= -T_0\Delta x^2 2\theta_F(\partial P_{AZ}/\partial x)\mathbf{i} = -\mathbf{i}(\partial P_{AZ}/\partial x)K_{F3}Q_PU/2$. Now recall $\mathbf{P_T} = -K_{F3}\mathbf{A}/\mu_0T_0$ from Eq. (114) to obtain:

$$\mathbf{F_{LT1PP}} = \mathbf{i}(\partial A_{AZ}/\partial x)K_{F3}^2Q_PU/2\mu_0T_0. \tag{153}$$

The positive-detached-aether moves through both types of attached-aether. When we evaluate positive-detached-aether flowing through negative-attached-aether the relevant portion of Eq. (81) gives us a flow force of $\mathbf{F_{FlowPN}} = K_{F3}\rho_{PD}\Delta V\mathbf{U_{PDT}}$, which is the negative of the flow force given by Eq. (151). In this case we use $\mathbf{N_T} = K_{F3}\mathbf{A}/\mu_0T_0$ from Eq. (114), which is the negative what was used to get to Eq. (153). Since there are two sign reversals in the derivation we arrive at:

$$\mathbf{F_{LT1PN}} = \mathbf{i}(\partial A_{AZ}/\partial x)K_{F3}^2Q_PU/2\mu_0T_0 \tag{154}$$

C.15.2. Tension-Forces on the Detached-Aether when Ambient Displacements are Parallel to Detached-Aether Flow. Eqs. (153) and (154) are forces on the attached-aether due to flows of positive-detached-aether. In this case, the change in slope of $\partial P_{AZ}/\partial x$ shown in Fig. 7 results in a force on the attached-aether which is primarily in the direction of the tension-force, $\mathbf{F_{TX}}$. (Recall that Fig. 7 greatly exaggerates the z-components of $\mathbf{F_T}$.) The force on the attached-aether can be eliminated if the force from Eqs. (153) and (154) is transferred to the detached-aether. (Empirical observation, see section C.17.) Hence, the total Lorentz Tension force of type 1 on the positive-detached-aether, $\mathbf{F_{LT1P}}$, is the sum of Eqs. (153) and (154):

$$\mathbf{F_{LT1P}} = \mathbf{i}(\partial A_{AZ}/\partial x)K_{F3}^2Q_PU/\mu_0T_0 \tag{155}$$

For the total Lorentz Tension force of type 1 on negative-detached-aether, Eqs. (78), (79), (80) and (81) inform us that there is simply a change in sign in the direction of the flow force. (Recall that we drop the terms $\partial\mathbf{P_T}/\partial t$ and $\partial\mathbf{N_T}/\partial t$ to get to Eq. (100).) Therefore the derivation involves replacing Q_P by $-Q_N$ which leaves:

$\mathbf{F_{LT1N}} = -\mathbf{i}(\partial A_{AZ}/\partial x)K_{F3}^2 Q_N U/\mu_0 T_0$ (156)

Defining $Q = Q_P - Q_N$ and combining Eqs. (155) and (156) leaves:

$\mathbf{F_{LT1}} = \mathbf{i}(\partial A_{AZ}/\partial x)K_{F3}^2 QU/\mu_0 T_0$. (157)

Now, assigning our constant K_{F3} as:

$K_{F3}^2 = \mu_0 T_0$ (158)

allows us to arrive at:

$\mathbf{F_{LT1}} = \mathbf{i}(\partial A_{AZ}/\partial x)QU$ (159)

A similar treatment to the above can be applied if the ambient, parallel, attached-aetherial displacements are instead assumed to vary in the y direction. This leads to:

$\mathbf{F_{LT2}} = \mathbf{j}(\partial A_{AZ}/\partial y)QU$ (160)

C.15.3. Tension-Forces on the Attached-Aether when Ambient Displacements are Perpendicular to Detached-Aether Flow. We now continue the evaluation of the forces present on an analysis-cube of attached-aether due to the simultaneous presence of flowing detached-aether as well as an ambient tension-gradient by considering ambient displacements perpendicular to the velocity of the detached-aether. The situation for the case of positive-attached-aether and flowing positive-detached-aether is shown in Figures 10 and 11.

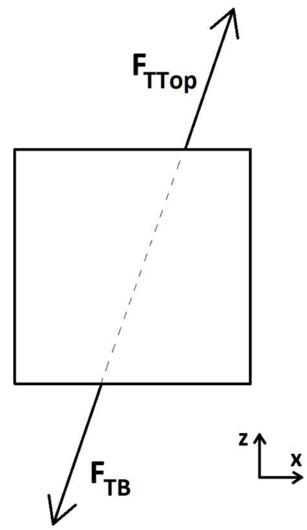

Figure 10. Tension-forces on the xy faces of an analysis-cube of attached-aether in the presence of an ambient aetherial gradient $\partial P_{AX}/\partial z$ for case where there is no flowing detached-aether.

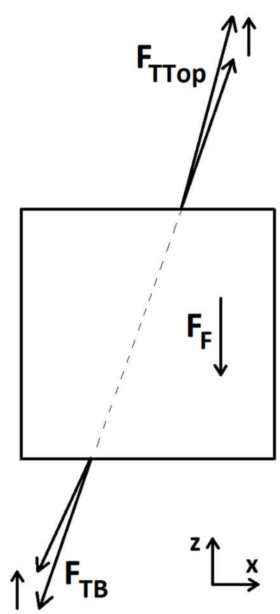

Figure 11. Tension-forces on the xy faces of an analysis-cube of attached-aether in the presence of an ambient aetherial gradient $\partial P_{AX}/\partial z$ for case where positive-detached-aether flows in the positive z direction within. The flow forces the cube downward relative to its neighbors, and the small arrows represent the additional tension forces resulting from that displacement.

Figure 10 shows the tension-forces on the top and bottom faces when there is an ambient gradient in the attached-aether $\partial P_{AX}/\partial z$. When there is no flowing detached-aether within the cube (shown in Figure 10) the tension-forces are equal and opposite so there is no net tension-force on the cube. We define our cube of analysis to be centered at z = 0, x = 0. The ambient gradient $\partial P_{AX}/\partial z$ results from cubes at height z being displaced to $\mathbf{P}(z) = P_X(z) = \mathbf{i}(\partial P_{AX}/\partial z)z$. The tension-force direction in Figure 10 is thus $[\mathbf{k} + (\partial P_{AX}/\partial z)\mathbf{i}]/[1+(\partial P_{AX}/\partial z)^2]^{1/2}$. This direction will point to a cube, cube 2 (not shown). Cube 2 is $\mathbf{k}z$ above the analysis-cube and $\mathbf{i}(\partial P_{AX}/\partial z)z$ to the right of the analysis-cube where $z = \Delta x$. (Cube 2 is centered at $z = \Delta x$, $x = [\partial P_{AX}/\partial z]\Delta x$, where $\Delta x = X_0$.)

Now consider adding flowing positive-detached-aether into our analysis, which will displace the cube of analysis opposite to the direction of the flow; this will change the direction of the tension-force as shown in Figure 11. With such a flow, cube 2 is now at ($z = \Delta x + dP_{FZ}$, $x = [\partial P_{AX}/\partial z]\Delta x$) where dP_{FZ} is the amount of displacement due to the flow. (The analysis-cube moves down a distance dP_{FZ} and we move the origin of our coordinate system down with it, leading to the relative positive displacement of cube 2.) We can now define θ_F as we did in Eq. (146), $dP_{FZ} = \theta_F\Delta x$. Doing so, the tension-force is now directed toward ($z = \Delta x + \theta_F\Delta x$, $x = [\partial P_{AX}/\partial z]\Delta x$), and the direction of the tension-force at the top face of Figure 11 is $[(1+\theta_F)\mathbf{k} + (\partial P_{AX}/\partial z)\mathbf{i}]/[(1+\theta_F)^2+(\partial P_{AX}/\partial z)^2]^{1/2}$. Similarly, the tension-force direction at the bottom face becomes $-[(1-\theta_F)\mathbf{k} + (\partial P_{AX}/\partial z)\mathbf{i}]/[(1-\theta_F)^2+(\partial P_{AX}/\partial z)^2]^{1/2}$.

The tension-force on each face of the cube is the force per unit area, T_0, multiplied by the area of the face Δx^2. We can now include the forces of the other four sides, as well as the flow force, to obtain the total force on the cube of analysis. In this case both the xz and yz faces will have tension-forces normal to the plane prior to any displacement. See Figure 9 and the discussion above and through Eqs. (149) and (150) for the case of two of the tension-forces normal to the plane, here we find $\mathbf{F_{Front}} = T_0\Delta x^2[\mathbf{j} + \theta_F\mathbf{k}]/[1+\theta_F^2]^{1/2}$ and $\mathbf{F_{Back}} = T_0\Delta x^2[-\mathbf{j} + \theta_F\mathbf{k}]/[1+\theta_F^2]^{1/2}$. Similar expressions will pertain to the forces on the right and left cube faces, $\mathbf{F_{Left}} = T_0\Delta x^2[-\mathbf{i} + \theta_F\mathbf{k}]/[1+\theta_F^2]^{1/2}$ and $\mathbf{F_{Right}} = T_0\Delta x^2[\mathbf{i} + \theta_F\mathbf{k}]/[1+\theta_F^2]^{1/2}$. We must also include the force due to the flow, given in Eq. (151) as $\mathbf{F_{FlowPP}} = -K_{F3}Q_PU\mathbf{k}$. Hence, the total force on the analysis-cube is:

$$\mathbf{F_{LT3PP}} = T_0\Delta x^2[(1 + \theta_F)\mathbf{k} + (\partial P_{AX}/\partial z)\mathbf{i}]/[(1 + \theta_F)^2+(\partial P_{AX}/\partial z)^2]^{1/2} \tag{161}$$

$$- T_0\Delta x^2[(1- \theta_F)\mathbf{k} + (\partial P_{AX}/\partial z)\mathbf{i}]/[(1 - \theta_F)^2+(\partial P_{AX}/\partial z)^2]^{1/2} + T_0\Delta x^2[-\mathbf{i} + \theta_F\mathbf{k}]/[1+ \theta_F^2]^{1/2}$$

$$+ T_0\Delta x^2[\mathbf{i} + \theta_F\mathbf{k}]/[1+ \theta_F^2]^{1/2} + T_0\Delta x^2[-\mathbf{j} + \theta_F\mathbf{k}]/[1+ \theta_F^2]^{1/2} + T_0\Delta x^2[\mathbf{j} + \theta_F\mathbf{k}]/[1+ \theta_F^2]^{1/2} - K_{F3}Q_PU\mathbf{k}$$

At this point, we use the approximation $(1+\varepsilon)^n \approx 1 + n\varepsilon$ for small ε, and also note that θ_F and $\partial P_{AX}/\partial z$ will both be small. This leaves us with:

$$\mathbf{F_{LT3PP}} \approx T_0\Delta x^2[(1+ \theta_F)\mathbf{k} + (\partial P_{AX}/\partial z)\mathbf{i}][1 - \theta_F - \theta_F^2/2 - (\partial P_{AX}/\partial z)^2/2] \tag{162}$$

$$- T_0\Delta x^2[(1 - \theta_F)\mathbf{k} + (\partial P_{AX}/\partial z)\mathbf{i}][1+ \theta_F - \theta_F^2/2 - (\partial P_{AX}/\partial z)^2/2] + T_0\Delta x^2[-\mathbf{i} + \theta_F\mathbf{k}][1- \theta_F^2/2]$$

$$+ T_0\Delta x^2[\mathbf{i} + \theta_F\mathbf{k}][1- \theta_F^2/2] + T_0\Delta x^2[-\mathbf{j} + \theta_F\mathbf{k}][1- \theta_F^2/2] + T_0\Delta x^2[\mathbf{j} + \theta_F\mathbf{k}][1- \theta_F^2/2] - K_{F3}Q_PU\mathbf{k}$$

Now, keep terms to second order:

$$\mathbf{F_{LT3PP}} = T_0\Delta x^2\{\mathbf{k}[1 - \theta_F - \theta_F^2/2 - (\partial P_{AX}/\partial z)^2/2] + \mathbf{k}\theta_F - \mathbf{k}\theta_F^2 + (\partial P_{AX}/\partial z)\mathbf{i} - (\partial P_{AX}/\partial z)\theta_F\mathbf{i}\}$$

$$- T_0\Delta x^2\{\mathbf{k}[1+ \theta_F - \theta_F^2/2 - (\partial P_{AX}/\partial z)^2/2] - \mathbf{k}\theta_F - \mathbf{k}\theta_F^2 + (\partial P_{AX}/\partial z)\mathbf{i} + (\partial P_{AX}/\partial z)\theta_F\mathbf{i}\}$$

$$- T_0\Delta x^2\mathbf{i}[1-\theta_F^2/2] + T_0\Delta x^2\theta_F\mathbf{k} + T_0\Delta x^2\mathbf{i}[1-\theta_F^2/2] + T_0\Delta x^2\theta_F\mathbf{k} \tag{163}$$

$$- T_0\Delta x^2\mathbf{j}[1-\theta_F^2/2] + T_0\Delta x^2\theta_F\mathbf{k} + T_0\Delta x^2\mathbf{j}[1-\theta_F^2/2] + T_0\Delta x^2\theta_F\mathbf{k} - K_{F3}Q_PU\mathbf{k}$$

Cancelling terms that are equal and opposite (red and orange), and combining remaining like terms (blue and green) leaves:

$$\mathbf{F_{LT3PP}} = -2T_0\Delta x^2(\partial P_{AX}/\partial z)\theta_F\mathbf{i} + 4T_0\Delta x^2\theta_F\mathbf{k} - K_{F3}Q_PU\mathbf{k} \tag{164}$$

Equilibrium in the z direction will be obtained by $4T_0\Delta x^2\theta_F = K_{F3}Q_PU$, or $\theta_F = K_{F3}Q_PU/4T_0\Delta x^2$ and hence $\mathbf{F_{LT3PP}} = -2T_0\Delta x^2(\partial P_{AX}/\partial z)\theta_F\mathbf{i} = -\mathbf{i}(\partial P_{AX}/\partial z)K_{F3}Q_PU/2$. Now recall $\mathbf{P_T} = -K_{F3}\mathbf{A}/\mu_0T_0$ from Eq. (114) to obtain $\mathbf{F_{LT3PP}} = \mathbf{i}(\partial A_{AX}/\partial z)K_{F3}^2Q_PU/2\mu_0T_0$, and recall Eq. (158), $K_{F3}^2 = \mu_0T_0$, to arrive at

$$\mathbf{F_{LT3PP}} = \mathbf{i}(\partial A_{AX}/\partial z)Q_PU/2 \tag{165}$$

Next, note that if we evaluate positive-detached-aether flowing through negative-attached-aether the relevant portion of Eqs. (79) and (81) gives us a flow force of $\mathbf{F_{FN}} = K_{F3}\rho_{PD}\Delta V\mathbf{U_{PDT}}$, which is the negative of the flow force given by Eq. (151). In this case we use $\mathbf{N_T} = K_{F3}\mathbf{A}/\mu_0T_0$ from Eq. (114), which is the negative what was used to get to Eq. (165). Since there are two sign reversals in the derivation we arrive at:

$$\mathbf{F_{LT3PN}} = \mathbf{i}(\partial A_{AX}/\partial z)Q_PU/2 \tag{166}$$

C.15.4. Tension Forces on the Detached-Aether when Ambient Displacements are Perpendicular to Detached-Aether Flow. Eqs. (165) and (166) are forces on the attached-aether due to flows of positive-detached-aether. In this case, similar to what is described near Eq. (139), the detached-aether will be forced to move in the opposite direction. (Empirical observation, see section C.17.) Hence, the total Lorentz Tension force of type 3 on the positive-detached-aether is:

$$\mathbf{F_{LT3P}} = -\mathbf{i}(\partial A_{AX}/\partial z)Q_PU \tag{167}$$

For the total Lorentz Tension force of type 3 on the negative-detached-aether, Eqs. (78), (79), (80) and (81) inform us that there is simply a change in sign in the direction of the flow force. (Recall that we drop the terms $\partial\mathbf{P_T}/\partial t$ and $\partial\mathbf{N_T}/\partial t$ to get to Eq. (100).) Therefore the derivation involves replacing Q_P by $-Q_N$ which leaves:

$$\mathbf{F_{LT3N}} = \mathbf{i}(\partial A_{AX}/\partial z)Q_NU \tag{168}$$

Again using the definition $Q = Q_P - Q_N$ leaves:

$$\mathbf{F}_{\mathbf{LT3}} = -\mathbf{i}(\partial A_{AX}/\partial z)QU \qquad (169)$$

It is also possible that $\partial A_{AY}/\partial z$ is non-zero, with a similar derivation to above leading to:

$$\mathbf{F}_{\mathbf{LT4}} = -\mathbf{j}(\partial A_{AY}/\partial z)QU \qquad (170)$$

C.15.5. The General Expression for the Tension-Force on the Detached-Aether due to Flowing-Detached-Aether. Above, the forces $\mathbf{F}_{\mathbf{LT1}}$, from Eq. (159), $\mathbf{F}_{\mathbf{LT2}}$, from Eq. (160), $\mathbf{F}_{\mathbf{LT3}}$, from Eq. (169), and $\mathbf{F}_{\mathbf{LT4}}$, from Eq. (170) are derived under the assumption that the velocity U is in the z direction, so we replace U by U_Z and the total force is the sum of these four forces, or:

$$\mathbf{F}_{\mathbf{LT5}} = \mathbf{i}(\partial A_{AZ}/\partial x)QU_Z + \mathbf{j}(\partial A_{AZ}/\partial y)QU_Z - \mathbf{i}(\partial A_{AX}/\partial z)QU_Z - \mathbf{j}(\partial A_{AY}/\partial z)QU_Z \qquad (171)$$

However, the velocity may not be in the z direction. With a velocity in the x direction, $U = U_X$, we just make the substitutions z to x, x to y, y to z, **i** to **j**, **j** to **k**, and **k** to **i**, in the analysis to obtain the expression for $U = U_X$. Then do the same again for the expression for $U = U_Y$.

$$\mathbf{F}_{\mathbf{LT6}} = \mathbf{j}(\partial A_{AX}/\partial y)QU_X + \mathbf{k}(\partial A_{AX}/\partial z)QU_X - \mathbf{j}(\partial A_{AY}/\partial x)QU_X - \mathbf{k}(\partial A_{AZ}/\partial x)QU_X \qquad (172)$$

$$\mathbf{F}_{\mathbf{LT7}} = \mathbf{k}(\partial A_{AY}/\partial z)QU_Y + \mathbf{i}(\partial A_{AY}/\partial x)QU_Y - \mathbf{k}(\partial A_{AZ}/\partial y)QU_Y - \mathbf{i}(\partial A_{AX}/\partial y)QU_Y \qquad (173)$$

The total Lorentz force on moving detached-aether due to the tension is:

$$\mathbf{F}_{\mathbf{LT}} = \mathbf{F}_{\mathbf{LT5}} + \mathbf{F}_{\mathbf{LT6}} + \mathbf{F}_{\mathbf{LT7}} = \qquad (174)$$

$$+\,\mathbf{i}(\partial A_{AZ}/\partial x)QU_Z + \mathbf{j}(\partial A_{AZ}/\partial y)QU_Z - \mathbf{i}(\partial A_{AX}/\partial z)QU_Z - \mathbf{j}(\partial A_{AY}/\partial z)QU_Z$$

$$+\,\mathbf{j}(\partial A_{AX}/\partial y)QU_X + \mathbf{k}(\partial A_{AX}/\partial z)QU_X - \mathbf{j}(\partial A_{AY}/\partial x)QU_X - \mathbf{k}(\partial A_{AZ}/\partial x)QU_X$$

$$+\,\mathbf{k}(\partial A_{AY}/\partial z)QU_Y + \mathbf{i}(\partial A_{AY}/\partial x)QU_Y - \mathbf{k}(\partial A_{AZ}/\partial y)QU_Y - \mathbf{i}(\partial A_{AX}/\partial y)QU_Y$$

C.16. The Lorentz Force Equation

The total force on a volume of detached-aether, Q, is the force given by Eq. (142) from the energy effects due to displacement, plus the flow-force given by Eq. (145) plus the force due to tension effects given by Eq. (174):

$$\mathbf{F_L} = \mathbf{F_{LD}} + \mathbf{F_{LF}} + \mathbf{F_{LT}} = -Q\nabla\phi - Q\partial\mathbf{A}/\partial t \tag{175}$$

$$+ \mathbf{i}(\partial A_{AZ}/\partial x)QU_Z + \mathbf{j}(\partial A_{AZ}/\partial y)QU_Z - \mathbf{i}(\partial A_{AX}/\partial z)QU_Z - \mathbf{j}(\partial A_{AY}/\partial z)QU_Z$$

$$+ \mathbf{j}(\partial A_{AX}/\partial y)QU_X + \mathbf{k}(\partial A_{AX}/\partial z)QU_X - \mathbf{j}(\partial A_{AY}/\partial x)QU_X - \mathbf{k}(\partial A_{AZ}/\partial x)QU_X$$

$$+ \mathbf{k}(\partial A_{AY}/\partial z)QU_Y + \mathbf{i}(\partial A_{AY}/\partial x)QU_Y - \mathbf{k}(\partial A_{AZ}/\partial y)QU_Y - \mathbf{i}(\partial A_{AX}/\partial y)QU_Y$$

Now form $\nabla \times \mathbf{A} = [\mathbf{i}\partial/\partial x + \mathbf{j}\partial/\partial y + \mathbf{k}\partial/\partial z] \times [\mathbf{i}A_{AX} + \mathbf{j}A_{AY} + \mathbf{k}A_{AZ}] = \mathbf{k}\partial A_{AY}/\partial x - \mathbf{j}\partial A_{AZ}/\partial x - \mathbf{k}\partial A_{AX}/\partial y + \mathbf{i}\partial A_{AZ}/\partial y + \mathbf{j}\partial A_{AX}/\partial z - \mathbf{i}\partial A_{AY}/\partial z$, and then form $\mathbf{U} \times \nabla \times \mathbf{A} = U_X(-\mathbf{j}\partial A_{AY}/\partial x - \mathbf{k}\partial A_{AZ}/\partial x + \mathbf{j}\partial A_{AX}/\partial y + \mathbf{k}\partial A_{AX}/\partial z) + U_Y(\mathbf{i}\partial A_{AY}/\partial x - \mathbf{i}\partial A_{AX}/\partial y - \mathbf{k}\partial A_{AZ}/\partial y + \mathbf{k}\partial A_{AY}/\partial z) + U_Z(\mathbf{i}\partial A_{AZ}/\partial x + \mathbf{j}\partial A_{AZ}/\partial y - \mathbf{i}\partial A_{AX}/\partial z - \mathbf{j}\partial A_{AY}/\partial z)$, which allows Eq. (175) to become:

$$\mathbf{F_L} = \mathbf{F_{LD}} + \mathbf{F_{LF}} + \mathbf{F_{LT}} = -Q\nabla\phi - Q\partial\mathbf{A}/\partial t + Q\mathbf{U} \times \nabla \times \mathbf{A} \tag{176}$$

Substituting in Eq. (123), $\mathbf{E} = -\nabla\phi - \partial\mathbf{A}/\partial t$, and Eq. (124), $\mathbf{B} = \nabla \times \mathbf{A}$, this leaves:

$$\mathbf{F_L} = Q(\mathbf{E} + \mathbf{U}\times\mathbf{B}) \tag{177}$$

Eq. (177) is recognized as the Lorentz Force Equation.

C.17. Comments on Electromagnetism

C.17.1. Issues for Further Study. In section C.15.2, leading to Eq. (155), we assumed that the force on the detached-aether will be in the same direction as the force on the attached-aether, while in section C.15.4, leading to Eq. (167), we assumed the force on the detached-aether will be in the opposite direction as the force on the attached-aether. For the Coulomb force, described in section C.13, we saw that energy considerations led to the force on the detached-aether being in the opposite direction as the force on the attached-aether. For the analysis in sections C.15.2 and C.15.4 there is no energy consideration to aid nature in making the choice, and so either choice is possible, but it is curious why the choice made in section C.15.2 differs from that made in sections C.15.4 and C.13. This curiosity is a matter for future consideration and research; there should be a physical cause for the difference.

In section C.14 we assumed that only the reaction force from the direct forces of Eqs. (78) and (79) act on the detached-aether, and that there will be no reaction force from the indirect forces of Eqs. (80) and (81) upon the detached-aether. However in sections C.6 and C.15 both the direct and indirect forces are seen to act upon the attached-aether. This assumed behavior is an additional curiosity and another matter for future consideration and research.

Despite their curious nature, we have proceeded with these empirical choices, as it is with those choices that we have arrived at Maxwell's Equations and the Lorentz force equation. These choices are a topic for further study, and we will not speculate further on them in this work.

C.17.2. Summary Comments. Together, Maxwell's Equations and the Lorentz Force Equation are a set of equations with considerable complexity. In this section C, we have seen how that complexity arises from several underlying simple physical causes. The quantum-pressure, internal tension, delta field and a flow law are all quite simple on their own. And yet, when properly analyzed, we see how those simple attributes of the aether lead quite straight forwardly to the complexities of Maxwell's Equations and the Lorentz Force Equation.

We have also discovered the fundamental nature of the longitudinal and transverse separations inherent in aetherial interactions. We have seen that the physics of the aether results in a derivation of Maxwell's equations in terms of potentials in the Coulomb, or instantaneous, gauge. Hence, the physics is that of longitudinal solutions described by Poisson's Equation, which is instantly transmitted as the sources move, plus transverse solutions which are described by the results of a straightforward analysis based on F = ma. (Of course, the longitudinal solutions may not be fully instantaneous and there may be a finite velocity of transmission. At the time of this writing however the equations of electrodynamics are consistent with instantaneity. This matter of instantaneous solutions described by Poisson's Equation is another matter for future research.)

Part D. Gravitation.

In this section the aether will be analyzed in the absence of electromagnetic effects. (Electromagnetic effects are analyzed in section C above.)

D.1 – Attached-Aether Density Change Due to Mass

We will define ρ_E, the extrinsic-energy-density for massive bodies, as

$$\rho_E = \gamma M c^2/V \tag{178}$$

In Eq. (178) M is the mass in a small volume V and $\gamma = (1 - v^2/c^2)^{-1/2}$, where v is the velocity of the mass and c the speed of light, and all quantities are measured with respect to the rest frame of the aether. We now propose:

The Extrinsic-Energy Force-Reduction Law. The presence of extrinsic-energy decreases the positive (negative) attached-aether tension and the negative (positive) attached-aether quantum-force by an amount proportional to the amount of extrinsic-energy present with a constant of proportionality K_{G1} (K_{G2}).

With X_Q the size of the analysis-cube, the Extrinsic-Energy Force-Reduction Law can be expressed mathematically for the nominal analysis-cube as

$$F_{TP} = K_{TP}X_Q = K_{T0}(1- K_{G1}\rho_E)X_Q \quad (179)$$

$$F_{TN} = K_{TN}X_Q = K_{T0}(1- K_{G2}\rho_E)X_Q \quad (180)$$

$$F_{QP} = 2K_{QP}/X_Q^3 = 2K_{Q0}(1- K_{G2}\rho_E)/X_Q^3 \quad (181)$$

$$F_{QN} = 2K_{QN}/X_Q^3 = 2K_{Q0}(1- K_{G1}\rho_E)/X_Q^3 \quad (182)$$

From Eqs. (179) to (182) the force parameters are:

$$K_{TP} = K_{T0}(1- K_{G1}\rho_E) \quad (183)$$

$$K_{TN} = K_{T0}(1- K_{G2}\rho_E) \quad (184)$$

$$K_{QP} = K_{Q0}(1- K_{G2}\rho_E) \quad (185)$$

$$K_{QN} = K_{Q0}(1- K_{G1}\rho_E) \quad (186)$$

The energy formulas, Eqs. (27) and 29), become

$$E_{TP} = (1/2)K_{T0}(1 - K_{G1}\rho_E)X_Q^2 \quad (187)$$

$$E_{TN} = (1/2)K_{T0}(1 - K_{G2}\rho_E)X_Q^2 \quad (188)$$

$$E_{QP} = K_{Q0}(1 - K_{G2}\rho_E)/X_Q^2 \quad (189)$$

$$E_{QN} = K_{Q0}(1 - K_{G1}\rho_E)/X_Q^2 \quad (190)$$

Consider first the undisturbed, positive analysis-cube. The total cube energy is $E_P = E_{TP} + E_{QP} = (1/2)K_{T0}(1 - K_{G1}\rho_E)X_Q^2 + K_{Q0}(1 - K_{G2}\rho_E)/X_Q^2$, the extremum of which can be found by setting

$$(dE_P/dX_Q)_1 = K_{T0}(1 - K_{G1}\rho_E)X_Q - 2K_{Q0}(1 - K_{G2}\rho_E)/X_Q^3 = 0 \quad (191)$$

We see that $K_{T0}(1 - K_{G1}\rho_E)X_Q = 2K_{Q0}(1 - K_{G2}\rho_E)/X_Q^3$ at the extremum. Defining X_1 as the value obtained at the extremum, $X_1^4 = 2K_{Q0}(1 - K_{G2}\rho_E)/K_{T0}(1 - K_{G1}\rho_E)$. With $K_{G1}\rho_E << 1$ and $K_{G2}\rho_E << 1$, this becomes $X_1^4 \approx (2K_{Q0}/K_{T0})(1 - K_{G2}\rho_E + K_{G1}\rho_E)$. Recalling Eq. (23), $X_0 = (2K_{Q0}/K_{T0})^{1/4}$,

$$X_1 \approx X_0(1 + [K_{G1}–K_{G2}]\rho_E/4) \quad (192)$$

To see whether the extremum is a minimum or a maximum, the second derivative of E_P is evaluated: $(d^2E_P/dX_Q^2)_{X1} = [K_{T0}(1 – K_{G1}\rho_E) + 6K_{Q0}(1 – K_{G2}\rho_E)/X_Q^4]_{X1} = K_{T0}(1 – K_{G1}\rho_E) + 6K_{Q0}(1 – K_{G2}\rho_E)/[2K_{Q0}(1 – K_{G2}\rho_E)/K_{T0}(1– K_{G1}\rho_E)] = K_{T0}(1 – K_{G1}\rho_E) + 3K_{T0}(1 – K_{G1}\rho_E) = 4K_{T0}(1 – K_{G1}\rho_E)$ which is manifestly positive, indicating an energy minimum, showing that X_1 will be the equilibrium analysis-cube size. The density is proportional to the inverse of the volume, or $\rho_{PA}/\rho_0 = (X_0/X_1)^3$, or $\rho_{PA}/\rho_0 = (1 + [K_{G1}–K_{G2}]\rho_E/4)^{–3} \approx (1 – 3[K_{G1}–K_{G2}]\rho_E/4)$. We now define

$$\rho_G = 3[K_{G1}–K_{G2}]\rho_E\rho_0/2 \quad (193)$$

leaving

$$\rho_{PA} = \rho_0 – \rho_G/2 \quad (194)$$

Next consider the negative analysis-cube. The total energy is the sum $E_N = E_{TN} + E_{QN} = (1/2)K_{T0}(1 – K_{G2}\rho_E)X_Q^2 + K_{Q0}(1 – K_{G1}\rho_E)/X_Q^2$, the extremum of which can be found by setting

$$(dE_N/dX_Q)_2 = K_{T0}(1 – K_{G2}\rho_E)X_Q – 2K_{Q0}(1 – K_{G1}\rho_E)/X_Q^3 = 0 \quad (195)$$

We see that $K_{T0}(1 – K_{G2}\rho_E)X_Q = 2K_{Q0}(1 – K_{G1}\rho_E)/X_Q^3$ at the extremum. Defining X_2 as the value obtained at the extremum, $X_2^4 = 2K_{Q0}(1 – K_{G1}\rho_E)/K_{T0}(1 – K_{G2}\rho_E)$. With $K_{G1}\rho_E << 1$ and $K_{G2}\rho_E << 1$, this becomes $X_2^4 = (2K_{Q0}/K_{T0})(1 – K_{G1}\rho_E + K_{G2}\rho_E)$. Recalling Eq. (23), $X_0 = (2K_{Q0}/K_{T0})^{1/4}$,

$$X_2 \approx X_0(1 + [K_{G2}–K_{G1}]\rho_E/4) \quad (196)$$

To see whether the extremum is a minimum or a maximum, the second derivative of E_N is evaluated: $(d^2E_N/dX_Q^2)_{X2} = [K_{T0}(1 – K_{G2}\rho_E) + 6K_{Q0}(1 – K_{G1}\rho_E)/X_Q^4]_{X2} = K_{T0}(1 – K_{G2}\rho_E) + 6K_{Q0}(1 – K_{G1}\rho_E)/[2K_{Q0}(1 – K_{G1}\rho_E)/K_{T0}(1 – K_{G2}\rho_E)] = K_{T0}(1 – K_{G2}\rho_E) + 3K_{T0}(1 – K_{G2}\rho_E) = 4K_{T0}(1 – K_{G2}\rho_E)$ which is manifestly positive, indicating an energy minimum, showing that X_2 will be the equilibrium analysis-cube size. The density is proportional to the inverse of the volume, or ρ_{NA}/ρ_0

$= (X_0/X_2)^3$, which is $\rho_{NA}/\rho_0 = (1 + [K_{G2}–K_{G1}]\rho_E/4)^{-3} \approx (1 - 3[K_{G2}–K_{G1}]\rho_E/4)$. Using Eq. (193), $\rho_G = 3[K_{G1}–K_{G2}]\rho_E\rho_0/2$, leaves

$$\rho_{NA} = \rho_0 + \rho_G/2 \qquad (197)$$

Recall the density postulate from section A.4.3:

The Density Postulate. In any volume, the density of the positive-aether equals the density of the negative-aether minus an amount proportional to the extrinsic-energy within the volume.

Since here we have no detached-aether, $\rho_N = \rho_{NA}$ and $\rho_P = \rho_{PA}$ and we obtain $\rho_P - \rho_N = -\rho_G$, or $\rho_P = \rho_N - \rho_G$, and since Eq. (193) shows that ρ_G is proportional to ρ_E, we see immediately that Eqs. (194) and (197) are a mathematical representation of the density postulate for the case where no detached-aether is present.

D.2 – Attached-Aether Displacements Due to Mass

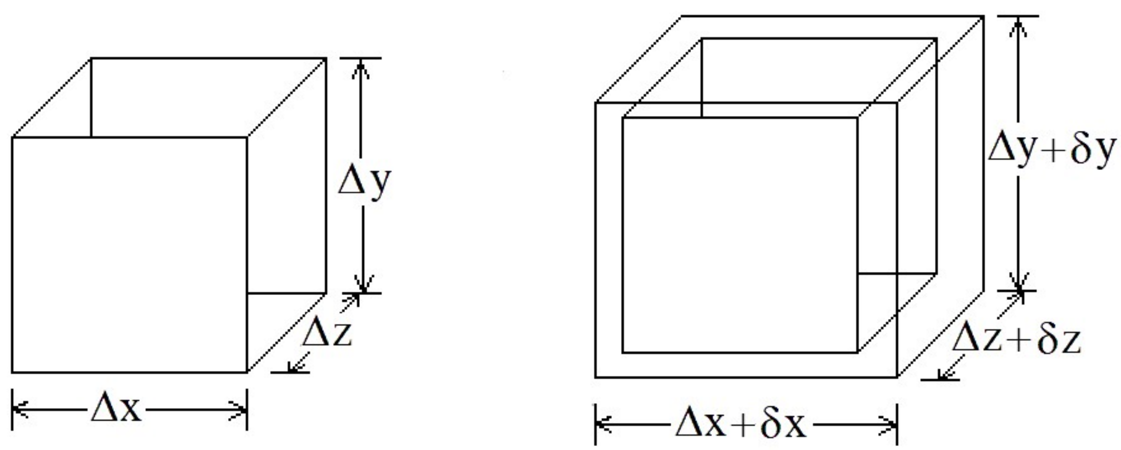

Figure 12. Analysis-cube of undistorted aether (left). Analysis-cube of expanded aether showing the original cube within (right).

Figure 12 shows two analysis-cubes of positive-aether. On the left is an undistorted cube of positive-attached-aether, which is Δx wide by Δy high by Δz deep. In the undistorted state, the density of the positive-attached-aether, ρ_{PA}, is equal to the nominal density, ρ_0. The density of anything is the amount of it divided by the volume it occupies. Hence, on the cube on the left, the amount of positive-attached-aether within the volume is $\rho_0\Delta x\Delta y\Delta z$.

If an amount of extrinsic-energy is added to the cube on the left (often the extrinsic-energy is mass) the density of the attached-positive-aether will decrease as given by Eq. (194), $\rho_{PA} = \rho_0 - \rho_G/2$, where $\rho_G = 3[K_{G1}-K_{G2}]\rho_E\rho_0/2$ is given in Eq. (193). In order for the density of any substance to decrease, the volume containing the substance must of course increase. The required expanded volume is shown on the right side of Fig. 12. In that second cube, the width is increased by an amount δx, while the height and depth are changed by δy and δz, respectively. The amount of positive-attached-aether remains $\rho_0(\Delta x\Delta y\Delta z)$, but with the larger volume the positive-attached-aether density is now:

$$\rho_{PA} = \rho_0(\Delta x\Delta y\Delta z)/[(\Delta x + \delta x)(\Delta y + \delta y)(\Delta z + \delta z)]$$

$$\approx \rho_0(\Delta x\Delta y\Delta z)/(\Delta x\Delta y\Delta z + \delta x\Delta y\Delta z + \delta y\Delta x\Delta z + \delta z\Delta x\Delta y)$$

$$= \rho_0/(1 + \delta x/\Delta x + \delta y/\Delta y + \delta z/\Delta z) \approx \rho_0(1 - \delta x/\Delta x - \delta y/\Delta y - \delta z/\Delta z) \tag{198}$$

It is assumed that the nominal density of the positive-attached-aether is far greater than the density change ρ_G, and so the δ quantities are very much smaller than the Δ quantities in Eq. (198). This allows the approximations made in Eq. (198).

To further refine Eq. (198), recall that the vector $\mathbf{P_G}$ is the displacement of the positive-attached-aether with respect to its equilibrium position. Hence, the x component of $\mathbf{P_G}$ at x is the displacement of the center of the cube shown in Fig. 12, and the x component of $\mathbf{P_G}$ at $x+\Delta x/2$ is the displacement of the right yz face of that cube. Therefore $\delta x/2 = P_{GX}(x + \Delta x/2, y, z, t) - P_{GX}(x, y, z, t)$. Dividing each side by $\Delta x/2$ leaves $\delta x/\Delta x = [P_{GX}(x + \Delta x/2, y, z, t) - P_{GX}(x, y, z, t)]/\Delta x/2 \approx \partial P_{GX}/\partial x$. Repeating the derivation for y and z will lead to similar expressions. Hence, Eq. (198)

can be re-expressed as $\rho_{PA} \approx \rho_0(1 - \delta x/\Delta x - \delta y/\Delta y - \delta z/\Delta z) \approx \rho_0(1 - \partial P_{GX}/\partial x - \partial P_{GY}/\partial y - \partial P_{GZ}/\partial z)$, which results in

$$\rho_{PA} \approx \rho_0(1 - \nabla \cdot \mathbf{P_G}) \tag{199}$$

Eq. (199) can be manipulated by bringing $\rho_0\nabla \cdot \mathbf{P_G}$ to the left-hand side, bringing $-\rho_{AP}$ to the right-hand side, dividing through by ρ_0, and then utilizing Eq. (194) from above:

$$\nabla \cdot \mathbf{P_G} \approx (\rho_0 - \rho_{PA})/\rho_0 = \rho_G/2\rho_0 \tag{200}$$

We can decompose $\mathbf{P_G}$ into its longitudinal ($\mathbf{P_{GL}}$) and transverse ($\mathbf{P_{GT}}$) components, where $\mathbf{P_{GL}}$ is defined as having zero curl and $\mathbf{P_{GT}}$ is defined as having zero divergence. ($\nabla \times \mathbf{P_{GL}} = 0$ and $\nabla \cdot \mathbf{P_{GT}} = 0$.) Since the longitudinal component of the vector field has zero curl it can be expressed as the gradient operator applied to a scalar field:

$$\mathbf{P_{GL}} = \nabla\Psi_{GP} \tag{201}$$

Ψ_{GP} is just a function that has a single scalar value at every point in space and time.

Since the divergence of the transverse component of $\mathbf{P_G}$ is zero, the divergence of $\mathbf{P_G}$ is equal to the divergence of its longitudinal portion alone, $\nabla \cdot \mathbf{P_G} = \nabla \cdot \mathbf{P_{GL}}$. And now, with $\mathbf{P_{GL}} = \nabla\Psi_{GP}$:

$$\nabla \cdot \mathbf{P_G} = \nabla \cdot \mathbf{P_{GL}} = \nabla \cdot \nabla\Psi_{GP} = \nabla^2\Psi_{GP} \tag{202}$$

Next, combine Eqs. (200) and (202) to yield:

$$\nabla^2\Psi_{GP} = \nabla \cdot \mathbf{P_{GL}} = \nabla \cdot \mathbf{P_G} = \rho_G/2\rho_0 \tag{203}$$

A similar derivation can be applied to the negative-attached-aether. Simply replace P_G by N_G and, since Eq. (197) shows that ρ_G enters with the opposite sign for that case, the derivation arrives at:

$$\mathbf{N_{GL}} = \nabla\Psi_{GN} \tag{204}$$

and

$$\nabla^2\Psi_{GN} = \nabla \cdot \mathbf{N}_{GL} = \nabla \cdot \mathbf{N}_G = -\rho_G/2\rho_0 \tag{205}$$

Subtract Eq. (204) from Eq. (201):

$$\mathbf{P}_{GL} - \mathbf{N}_{GL} = \nabla\Psi_{GP} - \nabla\Psi_{GN} \tag{206}$$

Subtract Eq. (205) from Eq. (203):

$$\nabla^2(\Psi_{GP} - \Psi_{GN}) = \rho_G/\rho_0 \tag{207}$$

Now define ϕ_G by $\phi_G = -(\Psi_{GP} - \Psi_{GN})\rho_0/\varepsilon_0$, where ε_0 is the permittivity of free space. This results in Eq. (207) becoming Poisson's Equation:

$$\nabla^2\phi_G = -\rho_G/\varepsilon_0 \tag{208}$$

With this definition for ϕ_G we also use Eq. (206) to get:

$$\mathbf{P}_{GL} - \mathbf{N}_{GL} = \nabla(\Psi_{GP} - \Psi_{GN}) = -\varepsilon_0\nabla\phi_G/\rho_0 \tag{209}$$

Dimensional analysis. From Eq. (209) $\varepsilon_0\nabla\phi_G/\rho_0$ is a length. ε_0 is in $m^{-3}kg^{-1}s^4A^2$, ρ_0 is in A s m^{-3} and ∇ is in m^{-1}. Hence ϕ_G is in m^2 kg s^{-3} A^{-1} = kg m^2/s^2 C = J/C, or Volts.

It is useful to observe that by comparing Eqs. (203) and (205):

$$\nabla \cdot \mathbf{N}_{GL} = -\nabla \cdot \mathbf{P}_{GL} \tag{210}$$

Eq. (210) shows that the divergence of $\mathbf{N}_{GL}$ is equal to the negative of the divergence of $\mathbf{P}_{GL}$. At this point it is relevant to recall the physics of the situation. Referring back to Fig. 12 and the analysis that is used to derive the equations above, we see that the presence of extrinsic-energy within a cube of positive-attached-aether pushes the boundary of the cube outward. Since the situation is symmetric, the effect will be the same in each dimension. That same extrinsic-energy within a cube of negative-attached-aether also pulls the boundary of the negative-attached-aether cube inward. Since extrinsic-energy is the only physical cause for $\mathbf{P}_{GL}$ and $\mathbf{N}_{GL}$:

$$\mathbf{N}_{GL} = -\mathbf{P}_{GL} \tag{211}$$

D.3 – The Extrinsic-Energy-Immersion Force (The Gamma-Force)

D.3.1. A Spherical Positive-Attached-Aether Region Containing Extrinsic-Energy. Consider now the injection of extrinsic-energy into a spherical region of the positive-attached-aether. With Eq. (203), $\nabla \cdot \mathbf{P_G} = \rho_G/2\rho_0$, we arrive at the solutions

$$\mathbf{P_{GIN}} = (\rho_G/6\rho_0)(x\mathbf{i} + y\mathbf{j} + z\mathbf{k}) = (\rho_G/6\rho_0)\mathbf{r} \qquad (r < R_0) \qquad (212)$$

$$\mathbf{P_{GOUT}} = P_0R_0^2\hat{\mathbf{r}}/r^2 = (\rho_G/6\rho_0)R_0^3\hat{\mathbf{r}}/r^2 \qquad (r > R_0) \qquad (213)$$

In Eqs. (212) and (213) R_0 is the radius of our spherical region and $P_0 = (\rho_G/6\rho_0)R_0$ is the magnitude of P_G at R_0. Verifying Eqs. (212) and (213) are solutions to Eq. (203): $\nabla \cdot \mathbf{P_{GIN}} = (1/r^2)\partial(r^2[(\rho_G/6\rho_0)r])/\partial r = (1/r^2)\partial(\rho_G/6\rho_0)r^3)/\partial r = \rho_G/2\rho_0$ and $\nabla \cdot \mathbf{P_{GOUT}} = (1/r^2)\partial(r^2[P_0R_0^2/r^2])/\partial r = (1/r^2)(\partial P_0R_0^2/\partial r) = 0$.

D.3.2. The Tension-Force Inside a Spherical Positive-Attached-Aether Region Containing Extrinsic-Energy. Eq. (212) informs us that adding extrinsic-energy into a spherical region will cause the positive-attached-aether cubes within that region to expand equally in each direction, which will cause cubes not at the center to also displace. As we slowly add extrinsic-energy into our spherical region, the displacement of any cube within that region will do work against the tension, with the work given by Eq. (32), $W_{TD} = (X_0/\xi_0)\, K_c \int K_T X_0\, dx$, where the work will be positive as the motion is against the force.

We now define ρ_e as the instantaneous energy density. As we add extrinsic-energy to the sphere, ρ_e will grow from 0 to its final value ρ_E, with the final value obtained once all extrinsic-energy is added. We see from Eq. (212) that the expansion of the cube, and therefore any displacement of the cube, is linearly proportional to ρ_G, and we see from Eq. (193) that ρ_G is linearly proportional to ρ_E, and hence the expansion and any displacement will be linearly proportional to ρ_E.

Consider now a cube-J, where prior to injection of extrinsic-energy, the center of cube-J is separated from the center of the spherical region by $(J–1)X_0$. The energy to displace cube-J is given by Eq. (32), and the displacement is $(J–1)\delta X$. (A displacement of $\delta X/2$ comes from each of cube-1 and cube-J and an additional δX comes from each of the cubes between cube-1 and cube-J.) As a first order approximation $F_T = K_{T0}X_0$. Integrating Eq. (32), $W_{TD} = (X_0/\xi_0)\, K_c \int K_T X_0\, dx$, from 0 to $(J–1)\delta X$, the work done against the tension on cube-J from the displacement is $W_{TD} = K_c K_{T0} X_0^2 (J–1)\delta X/\xi_0$.

Setting the expansion effects aside, the tension-energy of cube-J is $E_T = E_{T0} + W_{TD} = (1/2)K_{T0}X_0^2 + K_c K_{T0} X_0^2 (J–1)\delta X/\xi_0$. With $E_T = (1/2)K_T X_0^2$, we arrive at the expression $K_T = K_{T0}(1 + 2K_c(J–1)\delta X/\xi_0)$. The tension-force remains $F_T = K_T X_0$, but now K_T includes the next order correction.

At the beginning of the extrinsic-energy injection (when no immersion has yet occurred) F_T is of course just $K_{T0}X_0$, and it is at the end of immersion that $F_{T\_END} = K_{T0}X_0[1 + 2K_c(J–1)\delta X/\xi_0]$. Defining x as the distance the cube moves due to extrinsic-energy immersion, we again arrive at Eq. (66), $F_{TPP} = K_{T0}X_0[1 + 2K_c x/\xi_0]$. To verify Eq. (66), when there is no injection $x = 0$ and $F_T = K_{T0}X_0$; when the injection is complete, $x = (J–1)\delta X$ and $F_T = F_{T\_END} = K_{T0}X_0[1 + 2K_c(J–1)\delta X/\xi_0]$. The linearity in x follows because δX (and hence $(J–1)\delta X$) is linear with the amount of extrinsic-energy injected.

With the second order expression for the tension just derived for the displacement case, we can now include the second order effect in the cube tension-energy. The work done on the field due to the cube displacement again makes use of Eq. (32), $W_{TD} = (X_0/\xi_0)\, K_c \int K_T X_0\, dx$, $W_{TDJ} =$

$(X_0/\xi_0)K_cK_{T0}X_0 \int [1 + 2K_cx/\xi_0]\, dx = K_cK_{T0}X_0{}^2x/\xi_0 + (X_0/\xi_0)^2K_c{}^2K_{T0}x^2 = K_cK_{T0}X_0{}^2(J-1)\delta X/\xi_0 + (X_0/\xi_0)^2K_c{}^2K_{T0}(J-1)^2\delta X^2 = K_cK_{T0}X_0{}^2P_G/\xi_0 + (X_0/\xi_0)^2K_c{}^2K_{T0}P_G{}^2$.

For the displacement, the integral is evaluated between zero and its final displacement $P_G = (J-1)\delta X$. The work done has a positive sign because the tension increases in this case.

The expansion energy is now calculated using Eq. (63), $W_{TC} = 2 \int K_TX_0\, dw$. From Eq. (66), $K_TX_0 = K_{T0}X_0[1 + 2K_cx/\xi_0]$. We now use Eq. (62) with P_G for P, $x = (P_G/\delta X)w$, and we obtain $K_TX_0 = K_{T0}X_0[1+2K_c(P_G/\delta X)w/\xi_0]$. As specified in Eq. (63) we integrate from $w = 0$ to $w = \delta X/2$. Hence $W_{TC1} = 2 \int K_TX_0\, dw = 2K_{T0}X_0 \int [1+2K_c(P_G/\delta X)w/\xi_0]\, dw = 2K_{T0}X_0w + 2K_{T0}X_0K_c(P_G/\delta X)w^2/\xi_0 = K_{T0}X_0\delta X + K_{T0}X_0K_cP_G\delta X/2\xi_0$. Recall that Eq. (63) is for one dimension only, and that inside the sphere the expansion will be the same in all three dimensions. Hence, $W_{TC3} = 3K_{T0}X_0\delta X + 3K_{T0}X_0K_cP_G\delta X/2\xi_0$. The total tension-energy of an arbitrary analysis-cube inside the spherical region is the undisturbed energy plus the displacement energy plus the expansion energy, $E_{TPGI} = (1/2)K_{T0}X_0{}^2 + K_cK_{T0}X_0(X_0/\xi_0)P_G + K_c{}^2K_{T0}(X_0/\xi_0)^2P_G{}^2 + 3K_{T0}X_0\delta X + 3K_{T0}X_0K_cP_G\delta X/2\xi_0$, or, $E_{TPGI} = K_{T0}X_0{}^2[(1/2) + K_cP_G/\xi_0 + K_c{}^2(P_G/\xi_0)^2 + 3\delta X/X_0 + 3K_cP_G\delta X/2X_0\xi_0]$. Here we have derived the force for positive P_G. For $\mathbf{P_G}$ of any angle the work done is the same, due to radial symmetry. Hence we will replace P_G by its absolute value, $|\mathbf{P_G}|$:

$$E_{TPGI} = K_{T0}X_0{}^2[(1/2) + K_c|\mathbf{P_G}|/\xi_0 + K_c{}^2(|\mathbf{P_G}|/\xi_0)^2 + 3\delta X/X_0 + 3K_c|\mathbf{P_G}|\delta X/2X_0\xi_0] \qquad (214)$$

In Eq. (214) and the above paragraph, the subscript TPGI refers to the Tension-energy of the Positive-attached-aether including the Gravitational effects of extrinsic-energy in the region Inside the sphere.

D.3.3. The Quantum-Force Inside a Spherical Positive-Attached-Aether Region Containing Extrinsic-Energy. Eq. (33) relates that the displacement of any cube within the sphere does work against the quantum-force-field of $W_{QD} = (X_0/\xi_0)K_c \int 2K_Q/X_0^3\,dx$. Before adding any extrinsic-energy to our sphere, $F_{QP} = 2K_{Q0}/X_0^3$ is the quantum-force within the analysis-cubes in their nominal state.

Consider again cube-J of the previous section, where J is an integer with the center of cube-J separated from the center of the extrinsic-energy-sphere by $(J–1)X_0$. The energy to displace cube-J is given by Eq. (33), and the displacement is $(J–1)\delta X$. (A displacement of $\delta X/2$ comes from each of cube-1 and cube-J and an additional δX comes from each of the cubes between cube-1 and cube-J.) To first order, F_{QP} remains $2K_{Q0}/X_0^3$. Integrating Eq. (33) from 0 to $(J–1)\delta X$, the quantum-force displacement-work is $W_{QD} = –[2K_cK_{Q0}/X_0^2\xi_0](J–1)\delta X$. Here, the negative sign is applied to the work because we are expanding the sphere, and the displacement leads to a work that reduces the quantum-energy.

Setting the expansion effects aside, the quantum-energy of cube-J is $E_Q = E_{Q0} + W_{DQ} = K_{Q0}/X_0^2 – [2K_cK_{Q0}/X_0^2\xi_0](J–1)\delta X = K_{Q0}/X_0^2[1 – 2K_c(J–1)\delta X/\xi_0]$. With $E_Q = K_Q/X_0^2$, we arrive at the expression $K_Q = K_{Q0}[1 – 2K_c(J–1)\delta X/\xi_0]$. The quantum-force remains $F_Q = 2K_Q/X_0^3$, but now K_Q includes the next order correction.

At the beginning of the extrinsic-energy motion (when no immersion has yet occurred) F_Q is of course just $2K_{Q0}/X_0^3$, and it is at the end of immersion that $F_{QP\_END} = (2K_{Q0}/X_0^3)[1 – 2K_c(J–1)\delta X/\xi_0]$. With x again a variable that goes from 0 at no injection to $(J–1)\delta X$ at full injection, we see that Eq. (68) is again obtained, $F_{QPP} = (2K_{Q0}/X_0^3)[1 – 2K_cx/\xi_0]$, and $K_Q = K_{Q0}[1 – 2K_cx/\xi_0]$. To verify Eq. (68), when there is no injection, $F_{QP} = (2K_{Q0}/X_0^3)$. When the injection is complete,

$x = (J–1)\delta X$ and $F_{QP} = F_{QP\_END}$. The linearity in x follows because δX (and hence $(J–1)\delta X$) is linear with the amount of extrinsic-energy injected.

With the second order expression for the quantum-force just derived, we can now include the second order effect in the cube quantum-energy. The work done on the field due to the cube displacement is from Eq. (33), $W_{QD} = (X_0/\xi_0)\, K_c \int 2K_Q/X_0^3\, dx = -\,(2K_cK_{Q0}/X_0^2\xi_0) \int [1 - 2K_cx/\xi_0]\, dx = -(2K_cK_{Q0}/X_0^2\xi_0)x + (2K_c^2K_{Q0}/X_0^2\xi_0^2)x^2 = -(2K_cK_{Q0}/X_0^2\xi_0)(J–1)\delta X + (2K_c^2K_{Q0}/X_0^2\xi_0^2)(J–1)^2\delta X^2 = -(2K_cK_{Q0}/X_0^2\xi_0)P_G + (2K_c^2K_{Q0}/X_0^2\xi_0^2)P_G^2$. For the displacement, the integral is evaluated between zero and its final displacement $P_G = (J–1)\delta X$. The leading negative sign is because the work decreases the quantum-force in this case.

The expansion energy is now calculated using Eq. (64), $W_{QC} = -4 \int K_Q/X_0^3\, dw$, where a leading minus sign is included since quantum-energy is reduced by the expansion. From Eq. (68), $F_{QPP} = (2K_{Q0}/X_0^3)[1 - 2K_cx/\xi_0]$. We again use Eq. (62) with P_G for P, $x = (P_G/\delta X)w$, and we obtain $2K_Q/X_0^3 = (2K_{Q0}/X_0^3)[1–2K_c(P_G/\delta X)w/\xi_0]$. As specified in Eq. (64) we integrate from $w = 0$ to $w = \delta X/2$. Hence $W_{QX1} = -4 \int K_Q/X_0^3\, dw = -4K_{Q0}/X_0^3 \int [1–2K_c(P_G/\delta X)w/\xi_0]\, dw = -(4K_{Q0}/X_0^3)w + (4K_{Q0}/X_0^3)K_c(P_G/\delta X)w^2/\xi_0 = -(2K_{Q0}/X_0^3)\delta X + (K_{Q0}/X_0^3)K_cP_G\delta X/\xi_0$. Recall that Eq. (64) is for one dimension only, and that inside the sphere the expansion will be the same in all three dimensions. Hence, $W_{TC3} = -(6K_{Q0}/X_0^3)\delta X + (3K_{Q0}/X_0^3)K_cP_G\delta X/\xi_0$. The total quantum-energy of an arbitrary analysis-cube inside the spherical region is the undisturbed energy plus the displacement energy plus the expansion energy, $E_{QPGI} = K_{Q0}/X_0^2 - (2K_cK_{Q0}/X_0^2\xi_0)P_G + (2K_{Q0}K_c^2/X_0^2\xi_0^2)P_G^2 - (6K_{Q0}/X_0^3)\delta X + (3K_{Q0}/X_0^3)K_cP_G\delta X/\xi_0$, or, $E_{QPGI} = (2K_{Q0}/X_0^2)[(1/2) - K_cP_G/\xi_0 + K_c^2P_G^2/\xi_0^2 - 3\delta X/X_0 + 3K_cP_G\delta X/2X_0\xi_0]$. Here we have derived the force for positive P_G. For $\mathbf{P_G}$ at any angle the work is the same. Hence we will replace P_G by its absolute value, $|\mathbf{P_G}|$:

$E_{QPGI} = (2K_{Q0}/X_0^2)[(1/2) - K_c|\mathbf{P_G}|/\xi_0 + K_c^2|\mathbf{P_G}|^2/\xi_0^2 - 3\delta X/X_0 + 3K_c|\mathbf{P_G}|\delta X/2X_0\xi_0]$ (215)

In Eq. (215) and the above paragraph, the subscript QPGI refers to the Quantum-energy of the Positive-attached-aether due to the Gravitational effects of extrinsic-energy in the region Inside of the sphere.

D.3.4. The Gamma-Force and Gamma-Energy Fields Inside a Spherical Positive-Attached-Aether Region Containing Extrinsic-Energy. Injection of extrinsic-energy causes positive-attached-aether to expand leading to the forces described in Eqs. (66) and (68), $F_{TPP} = K_{T0}X_0[1 + 2K_cx/\xi_0]$ and $F_{QPP} = (2K_{Q0}/X_0^3)[1 - 2K_cx/\xi_0]$, respectively. In order to achieve a force balance within the attached-aether, we propose a balancing force called the gamma-force:

$\mathbf{F_\gamma} = -\mathbf{F_{TL}} - \mathbf{F_Q}$ (216)

At this point recall that the tension $\mathbf{F_{TL}}$ is directed inward (toward the center of the sphere) while the quantum-force $\mathbf{F_Q}$ is directed outward, and with $\mathbf{F_\gamma}$ directed oppositely we arrive at $\mathbf{F_{\gamma PI}} = K_{T0}X_0[1 + 2K_cx/\xi_0]\hat{\mathbf{r}} - (2K_{Q0}/X_0^3)[1 - 2K_cx/\xi_0]\hat{\mathbf{r}}$. And now recall Eq. (21), $K_{T0}X_0 = 2K_{Q0}/X_0^3$, leaving $\mathbf{F_{P\gamma I}} = K_{T0}X_0[1 + 2K_cx/\xi_0 - 1 + 2K_cx/\xi_0]\hat{\mathbf{r}}$, or

$\mathbf{F_{\gamma PI}} = 4K_cK_{T0}(X_0/\xi_0)x\hat{\mathbf{r}}$ (217)

Eq. (217) informs us of the total gamma-force needed to balance the difference between the tension and quantum forces, but it does not tell us how that balancing force arises. There are three possibilities for the gamma-force: 1) it could be a tension; 2) it could be a quantum-force; or 3) it could have components of both a tension and a quantum-force. Since the tension exceeds the quantum-force for the positive-attached-aether case (the sphere is expanded) the gamma-force could be a negative tension, a positive quantum-force or some combination of gamma-tension and gamma-quantum-force summing to the net force field given in Eq. (217). We propose the following choice:

$\mathbf{F}_{\gamma PT} = 4K_c(1+ K_{GC})K_{T0}(X_0/\xi_0)x\hat{\mathbf{r}}$ (218)

$\mathbf{F}_{\gamma PQ} = -4K_cK_{GC}K_{T0}(X_0/\xi_0)x\hat{\mathbf{r}}$ (219)

It can be seen that Eq. (218) is a negative tension-force, as it is directed outward, while Eq. (219) is a negative quantum-force, as it is directed inward. In Eqs. (218) and (219), K_{GC} is a coupling constant and the equations reveal a coupling between the gamma-force components. The work done on the gamma-fields due to the cube displacement is calculated as

$W_{\gamma PDI} = (X_0/\xi_0)\, K_c \int F_\gamma\, dx$ (220)

Notice that the factors (X_0/ξ_0) and K_c in Eq. (220) are motivated by the same physics as that found in Eqs. (32) and (33), as they result from the depth of the analysis-cube and a reduction factor. (See section B.5.) Substituting Eqs. (218) and (219) into Eq. (220) and integrating (and assuming both components lead to negative energy as described below in section D.3.5) we obtain $W_{\gamma PDT} = -2K_c^2(1 + K_{GC})K_{T0}(X_0/\xi_0)^2(J-1)^2\delta X^2 = -2K_c^2(1 + K_{GC})K_{T0}(X_0/\xi_0)^2P_G^2$ and $W_{\gamma PDQ} = -2K_c^2K_{GC}K_{T0}(X_0/\xi_0)^2(J-1)^2\delta X^2 = -2K_c^2K_{GC}K_{T0}(X_0/\xi_0)^2P_G^2$. Summing, $W_{\gamma PD} = W_{\gamma PDT} + W_{\gamma PDQ}$ or,

$W_{\gamma PD} = -2K_c^2(1 + 2K_{GC})K_{T0}(X_0/\xi_0)^2P_G^2$ (221)

(For the displacement, the integrals are evaluated between zero and $P_G = (J-1)\delta X$.)

The expansion energy is now calculated using Eq. (63), $W_{TC} = 2 \int F_T\, dw$. (Eqs. (218) and (219) each have the functional form of a tension, $F=kx$.) From Eq. (218), $F_{\gamma PT} = 4K_c(1+ K_{GC})K_{T0}(X_0/\xi_0)x$. We now use Eq. (62) with P_G for P, $x = (P_G/\delta X)w$, and for the tension component of the gamma-force we have $F_{P\gamma T} = 4K_c(1+ K_{GC})K_{T0}(X_0/\xi_0)(P_G/\delta X)w$. As specified in Eq. (63) we integrate from $w = 0$ to $w = \delta X/2$. Hence $W_{\gamma PCT1} = 2 \int 4K_c(1+K_{GC})K_{T0}(X_0/\xi_0)(P_G/\delta X)w\, dw = 4K_c(1+K_{GC})K_{T0}(X_0/\xi_0)(P_G/\delta X)w^2 = K_c(1+K_{GC})K_{T0}(X_0/\xi_0)P_G\delta X$. Recall that Eq. (63) is for one dimension only, and that inside the sphere the expansion will be the same in all three dimensions:

$$W_{\gamma PCT3} = 3K_c(1+K_{GC})K_{T0}(X_0/\xi_0)P_G\delta X \qquad (222)$$

(Here the subscripts γPCT3 are for the Compression energy from the Tension component of the γ force for the Positive aether inside the sphere for the full 3-dimensional analysis. Other subscripts in this section are similarly defined.)

For the quantum-pressure component of the gamma-force Eq. (219) gives us $F_{\gamma PQ} = -4K_cK_{GC}K_{T0}(X_0/\xi_0)x$, and for the expansion energy we again use $x = (P_G/\delta X)w$ to get $F_{\gamma PQ} = -4K_cK_{GC}K_{T0}(X_0/\xi_0)(P_G/\delta X)w$. As specified in Eq. (63) we integrate from $w = 0$ to $w = \delta X/2$. Hence $W_{\gamma PCQ1} = -2 \int 4K_cK_{GC}K_{T0}(X_0/\xi_0)(P_G/\delta X)w \, dw = -4K_cK_{GC}K_{T0}(X_0/\xi_0)(P_G/\delta X)w^2 = -K_cK_{GC}K_{T0}(X_0/\xi_0)P_G\delta X$. Recall that Eq. (63) is for one dimension only, and that inside the sphere the expansion will be the same in all three dimensions:

$$W_{\gamma PCQ3} = -3K_cK_{GC}K_{T0}(X_0/\xi_0)P_G\delta X \qquad (223)$$

The total gamma-energy of an arbitrary analysis-cube inside the spherical region is the sum of the displacement and expansion energies found in Eqs. (221), (222) and (223), $E_{\gamma PGI} = -2K_c^2(1 + 2K_{GC})K_{T0}(X_0/\xi_0)^2P_G^2 + 3K_c(1+K_{GC})K_{T0}(X_0/\xi_0)P_G\delta X - 3K_cK_{GC}K_{T0}(X_0/\xi_0)P_G\delta X = -2K_c^2K_{T0}(X_0/\xi_0)^2P_G^2 - 4K_c^2K_{GC}K_{T0}(X_0/\xi_0)^2P_G^2 + 3K_cK_{T0}(X_0/\xi_0)P_G\delta X$. Here we have derived the force for positive P_G. For $\mathbf{P_G}$ at any angle the work done is the same. Hence we will replace P_G by its absolute value, $|\mathbf{P_G}|$:

$$E_{\gamma PGI} = K_{T0}X_0^2[-2K_c^2(|\mathbf{P_G}|/\xi_0)^2 - 4K_c^2K_{GC}(|\mathbf{P_G}|/\xi_0)^2 + 3K_c|\mathbf{P_G}|\delta X/X_0\xi_0] \qquad (224)$$

In Eq. (224) and the above paragraph, the subscript γPGI refers to γ-energy of the Positive-attached-aether due to the Gravitational effects of extrinsic-energy in the region Inside of the sphere.

The total energy of an analysis-cube is found by summing the tension, quantum and gamma energies found in Eqs. (214), (215) and (224). $E_{PG} = E_{TPGI} + E_{QPGI} + E_{\gamma PGI} = K_{T0}X_0^2[(1/2) + K_c|\mathbf{P_G}|/\xi_0 + K_c^2|\mathbf{P_G}|^2/\xi_0^2 + 3\delta X/X_0 + 3K_c|P_G|\delta X/2X_0\xi_0] + (2K_{Q0}/X_0^2)[(1/2) - K_c|\mathbf{P_G}|/\xi_0 + K_c^2|\mathbf{P_G}|^2/\xi_0^2 - 3\delta X/X_0 + 3K_c|P_G|\delta X/2X_0\xi_0] + K_{T0}X_0^2[-2K_c^2(|P_G|/\xi_0)^2 - 4K_c^2K_{GC}(|\mathbf{P_G}|/\xi_0)^2 + 3K_c|P_G|\delta X/X_0\xi_0]$, or, using Eq. (22), $K_{T0}X_0^2 = 2K_{Q0}/X_0^2$,

$$E_{PG} = K_{T0}X_0^2 - 4K_c^2K_{GC}K_{T0}(X_0/\xi_0)^2|\mathbf{P_G}|^2 = K_{T0}X_0^2(1 - 4K_c^2K_{GC}|\mathbf{P_G}|^2/\xi_0^2) \tag{225}$$

A similar expression, Eq. (G9), is derived in Appendix G for the negative-attached-aether:

$$E_{NG} = K_{T0}X_0^2 - 4K_c^2K_{GC}K_{T0}(X_0/\xi_0)^2|\mathbf{N_G}|^2 = K_{T0}X_0^2(1 - 4K_c^2K_{GC}|\mathbf{N_G}|^2/\xi_0^2) \tag{226}$$

While Eqs. (225) and (226) have been derived inside a spherical region containing extrinsic-energy, Appendix H shows that Eqs. (225) and (226) hold outside of the spherical region as well. Lastly, note that Eqs. (225) and (226) arise from the displacements $\mathbf{P_G}$ and $\mathbf{N_G}$ in the presence of the tension, quantum and gamma force fields. In general, displacements may be the result of superposition of individual displacements from several sources, but the work done will still be calculated from the displacements and the force fields. Therefore Eqs. (225) and (226) hold generally.

D.3.5. The Physical Nature of the Gamma-Force Inside a Sphere of Positive-Attached-Aether Containing Extrinsic-energy. In the sections above we make four assumptions in our analysis regarding the case of extrinsic-energy within positive-attached-aether: 1) the gamma-force has a negative tension component; 2) the gamma-force has a negative pressure component; 3) displacement of the analysis-cube will not change its size; and 4) the energies associated with the gamma-force are negative for the displacement. This section will validate these assumptions while providing a physical understanding of the gamma-force.

D.3.5.1. The Negative Gamma-force Tension and Pressure Components. The gamma-force $\mathbf{F}_\gamma$ is a force that balances against the sum of $\mathbf{F}_T$ and $\mathbf{F}_Q$. Above in Eqs. (218) and (219) we have assigned two components to $\mathbf{F}_\gamma$ and arranged the sum of these components such that the total $\mathbf{F}_\gamma$ will subtract from the stronger of $\mathbf{F}_T$ and $\mathbf{F}_Q$ as needed to provide the balancing force. Since $\mathbf{F}_Q$ is less than $\mathbf{F}_T$ for the case discussed, the total $\mathbf{F}_\gamma$ is a negative tension providing an additional outward force.

The reason $\mathbf{F}_\gamma$ has two components, and the reason the total $\mathbf{F}_\gamma$ subtracts from the stronger of the forces, rather than adding to the weaker, is because the gamma-force originates from the force reductions caused by immersion of extrinsic-energy as specified in the Extrinsic-Energy Force-Reduction Law. As the cubes are then displaced, the work done makes these force reductions grow, and these negative forces are the two components of the gamma-force. These forces then balance against the sum of $\mathbf{F}_T$ and $\mathbf{F}_Q$ as described above in Eqs. (218) and (219).

D.3.5.2. Analysis-Cube Size During Displacement. As described above, when cube-P of positive-attached-aether is moved outward due to injection of extrinsic-energy, cube-P has its tension increased as described by Eq. (66), $F_{TPP} = K_{T0}X_0[1 + 2K_cx/\xi_0]$, and cube-P has its quantum-force decreased as described by Eq. (68), $F_{QPP} = (2K_{Q0}/X_0^3)[1 - 2K_cx/\xi_0]$. If there were no gamma-force, cube-P would therefore compress. However, the overlapping cube-N of negative-attached-aether has the opposite behavior (see Appendix G, section G.4.2), and with no gamma-force it would expand. (Eqs. (G1) and (G3) from Appendix G are $F_{TN} = K_{T0}X_0[1 - 2K_cx/\xi_0]$ and $F_{QN} = (2K_{Q0}/X_0^3)[1 + 2K_cx/\xi_0]$, respectively.) In order for the density postulate to hold as they displace, cube-P and cube-N must either both expand, both compress, or both retain their size. Since the sum of $\mathbf{F}_T$ and $\mathbf{F}_Q$ on cube-P, described by Eqs. (66) and (68), is equal and opposite to the sum of

$\mathbf{F_T}$ and $\mathbf{F_Q}$ on cube-N, this symmetry informs that the cube sizes will not change during their displacement, which is what is assumed above.

D.3.5.3. The Physics Leading to the Gamma-Force Energies. We see that the gamma-force arises between the positive-attached-aether and the negative-attached-aether in a way that leads to no displacement-induced size change of the displaced cubes. We also see that motion of positive-attached-aether causes $\mathbf{F_{TP}}$ (the positive-attached-aether tension) to grow and $\mathbf{F_{QP}}$ (the positive-attached-aether quantum-force) to recede, and hence $\mathbf{F_\gamma}$ (the total gamma-force) must grow in magnitude to offset this growing difference. In this case, $\mathbf{F_\gamma} = \mathbf{F_{\gamma PT}} + \mathbf{F_{\gamma PQ}}$, and inspection of Eqs. (218) and (219) reveals that $F_{\gamma PT} > F_{\gamma PQ}$.

First consider the gamma-force component $\mathbf{F_{\gamma PT}}$. Since the displacement of the positive-attached-aether is outward and the displacement of the negative-attached-aether is inward as extrinsic-energy is injected, and since $\mathbf{F_{\gamma PT}}$ is outward, it is the displacement of the inward-moving negative-attached-aether that does the work against $\mathbf{F_{\gamma PT}}$ to make it grow, since we must have inward motion against the force to do work against it to make it grow. (The outward-moving positive-attached-aether would do negative work on $\mathbf{F_{\gamma PT}}$ since it is moving in the direction of $\mathbf{F_{\gamma PT}}$ and therefore it cannot be the source of growth for $\mathbf{F_{\gamma PT}}$.) Therefore we see that $\mathbf{F_{\gamma PT}}$ is a negative tension that subtracts from the nominal tension within the positive-attached-aether, and yet work is done against it by the motion of the negative-attached-aether. Similarly, Appendix G discusses how the outward motion of the positive-attached-aether increases $\mathbf{F_{\gamma NQ}}$ (the quantum-pressure component of the negative-attached-aether gamma-force), which is a negative pressure.

Now let us look at the gamma-force component $\mathbf{F_{\gamma PQ}}$. Since the displacement of the positive-attached-aether is outward and the displacement of the negative-attached-aether is inward as extrinsic-energy is injected, and since $\mathbf{F_{\gamma PQ}}$ is inward, it is the displacement of the outward-moving positive-attached-aether that does the work against $\mathbf{F}_{\gamma PQ}$ to make it grow, since we must have outward motion against that force to do work against it to make it grow. (The inward-moving negative-attached-aether would do negative work on $\mathbf{F}_{\gamma PQ}$ since it is in the direction of $\mathbf{F}_{\gamma PQ}$ and therefore it cannot be the source of growth for $\mathbf{F}_{\gamma PQ}$.) Therefore we see that $\mathbf{F}_{\gamma PQ}$ is a negative pressure that subtracts from the nominal quantum-pressure within the positive-attached-aether, and work is done against it by the motion of the positive-attached-aether. Similarly, Appendix G discusses how the inward motion of the negative-attached-aether increases $\mathbf{F}_{\gamma NT}$ (the tension component of the negative-attached-aether gamma-force), which is a negative tension.

Since $\mathbf{F}_{\gamma PT}$ and $\mathbf{F}_{\gamma PQ}$ are forces of a negative tension and negative pressure, respectively, they both contribute negative energies to (relax positive energies of) the aetherial cubes. $\mathbf{F}_{\gamma PT}$ and $\mathbf{F}_{\gamma PQ}$ both result from work increasing the effects of the Extrinsic-Energy Force-Reduction Law, which are negative energies, which is why the displacement energies are negative for the gamma-force.

The leading $\mathbf{F}_{\gamma PT}$ term in Eq. (218), $4K_cK_{T0}(X_0/\xi_0)x\hat{\mathbf{r}}$, and the leading $\mathbf{F}_{\gamma NQ}$ term in Eq. (G6) from Appendix G, $-4K_cK_{T0}(X_0/\xi_0)x\hat{\mathbf{r}}$, are what provide the mechanism by which each type of attached-aether exerts forces on the other to maintain the equal-density postulate. The leading $\mathbf{F}_{\gamma PT}$ term in Eq. (218) cancels the sum of the tension-force $\mathbf{F}_{PT}$ and quantum-force $\mathbf{F}_{PQ}$, and when integrated over the displacement it also contributes a negative work to offset the net work done by $\mathbf{F}_{PT}$ and $\mathbf{F}_{PQ}$. The trailing $\mathbf{F}_{\gamma PT}$ term in Eq. (218), $4K_cK_{GC}K_{T0}(X_0/\xi_0)x\hat{\mathbf{r}}$, and $\mathbf{F}_{\gamma PQ} = -4K_cK_{GC}K_{T0}(X_0/\xi_0)x\hat{\mathbf{r}}$ in Eq. (219) are forces which balance each other while contributing more negative work to reduce the total energy in the positive-attached-aether. It is this reduction in total energy that leads to Newtonian gravity, as discussed below.

D.3.5.4. The Gamma-Force in Other Cases. Here in section D.3.5 we have looked at the case of extrinsic-energy immersion into positive-attached-aether. For discussion of the remaining cases see Appendices G and H.

D.4 – Tension, Quantum and Gamma Field Effects on Mass: Force on a Spherical Mass in a Gravitational Field

Consider a region where there is an ambient attached-aether displacement $\mathbf{P}_{AG} = -\mathbf{N}_{AG}$ (a gravitational field) not equal to zero. We have seen above in section D.2 that such displacement is caused by extrinsic-energy somewhere within the aether. To see the effect that $\mathbf{P}_{AG}$ has on masses we will begin by gradually add a sphere of mass (extrinsic-energy) with final density ρ_E into the positive-attached-aether region that has the ambient $\mathbf{P}_{AG}$. Without loss of generality we can assume that the ambient displacement $\mathbf{P}_{AG}$ is in the x direction, and that our sphere is centered at the origin. The energy of cubes within the sphere is derived above to result in Eq. (225):

$$E_{PG} = K_{T0}X_0^2(1 - 4K_c^2K_{GC}|\mathbf{P}_G|^2/\xi_0^2) \qquad (225)$$

For analysis we will divide the sphere into slices of thickness ΔY centered at y = Y, and then further divide those slices into strips with thickness ΔZ centered at z = Z, as shown in Figures 13 and 14.

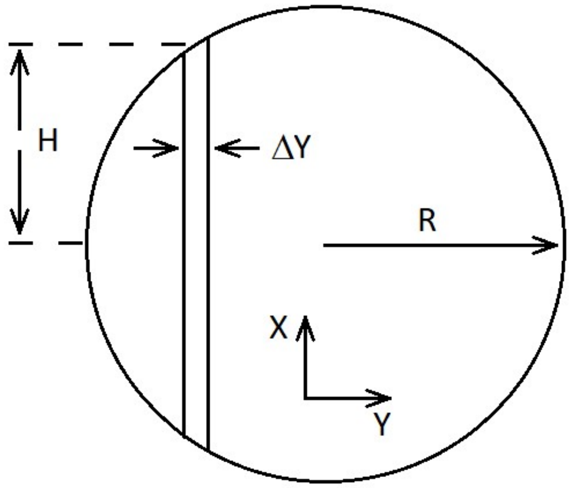

Figure 13. A sphere of attached-aether containing extrinsic-energy (ρ_E) and a slice of width ΔY.

Figure 13 shows a view of a sphere of attached-aether containing ρ_E with radius R. Examining a slice of thickness ΔY we can see that the half-height of that slice will be $H = (R^2 - Y^2)^{1/2}$.

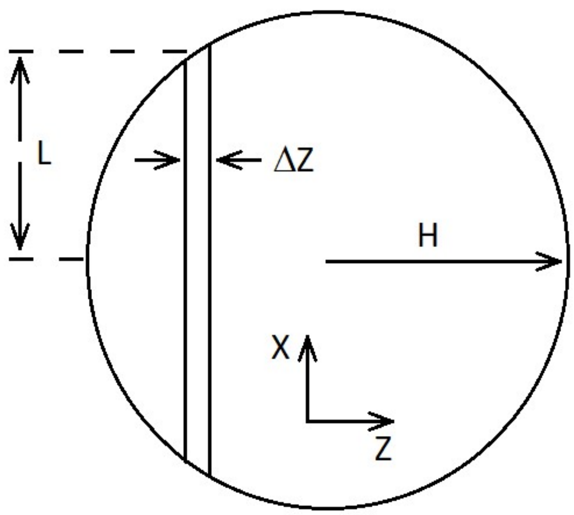

Figure 14. A slice of attached-aether containing extrinsic-energy (ρ_E) and a strip of width ΔZ.

Figure 14 shows a view of the slice of attached-aether containing ρ_E. The slice has the half-height, H, that we found in Fig. 13. Examining the strip of thickness ΔZ centered at Z we can see that the half-length of that strip will be $L = (H^2 - Z^2)^{1/2}$. The strip shown in Fig. 14 has dimensions of ΔY by ΔZ by L.

Our aim is to evaluate the force on our sphere of mass when it is immersed in a region of attached-aether where we have finite displacement $\mathbf{P_{AG}}$. Toward that aim, we first observe that the sphere of mass will cause its own longitudinal attached-aether displacement. Due to the symmetry of the situation, a sphere of mass will push nearby positive-attached-aether outward radially. With ρ_E causing a decrease in aether density inside of the sphere it occupies, the positive-attached-aether density inside that sphere, ρ_{in}, is shown above to lead to Eq. (194), which here becomes $\rho_{in} = \rho_0 - \rho_G/2$. With no ρ_E inside of it, a sphere of radius R has $(4/3)\pi R^3\rho_0$ of attached-aether inside. To decrease the density to ρ_{in} requires that the radius of the sphere expand so that $(4/3)\pi(R+\Delta R)^3\rho_{in} = (4/3)\pi R^3\rho_0$, or $\rho_{in} = R^3\rho_0/(R+\Delta R)^3 = \rho_0/(1+\Delta R/R)^3 \approx \rho_0(1 - 3\Delta R/R)$. With $\rho_{in} = \rho_0 - \rho_G/2$ we obtain $\rho_G/2 \approx 3\rho_0\Delta R/R$, or, $\Delta R/R = \rho_G/6\rho_0$. The displacement of the edge of the sphere is $\Delta R = R(\rho_G/6\rho_0)$. Now the magnitude of the displacement will grow linearly from the center of the sphere, so that in general, with $\mathbf{P_{GS}}$ being the displacement caused by the sphere, inside the sphere we have:

$$\mathbf{P_{GS}} = (\rho_G/6\rho_0)\mathbf{r} = (\rho_G/6\rho_0)(x\mathbf{i} + y\mathbf{j} + z\mathbf{k}) \qquad (227)$$

Since we now have an ambient attached-aether displacement $\mathbf{P_{AG}}$, which without loss of generality is considered to be in the x direction, $\mathbf{P_{AG}} = P_{AG}\mathbf{i}$, and the total attached-aether displacement within the sphere is $\mathbf{P_G} = \mathbf{P_{AG}} + \mathbf{P_{GS}} = [P_{AG} + (\rho_G/6\rho_0)x]\mathbf{i} + (\rho_G/6\rho_0)y\mathbf{j} + (\rho_G/6\rho_0)z\mathbf{k}$.

Returning to Figure 14, within the strip, and using Eqs. (225) and (227), the energy of a small positive-attached-aether analysis-cube centered at x, y, z is:

$$E_{PG}(x,y,z) = K_{T0}X_0^2(1 - 4K_c^2K_{GC}|\mathbf{P_{AG}}+\mathbf{P_{GS}}|^2/\xi_0^2)$$

$$= K_{T0}X_0^2\{1 - 4K_c^2K_{GC}|[P_{AG} + (\rho_G/6\rho_0)x]\mathbf{i} + (\rho_G/6\rho_0)y\mathbf{j} + (\rho_G/6\rho_0)z\mathbf{k}|^2/\xi_0^2\}$$

$$= K_{T0}X_0^2\{1 - 4K_c^2K_{GC}[P_{AG}^2+2P_{AG}(\rho_G/6\rho_0)x+(\rho_G/6\rho_0)^2x^2+(\rho_G/6\rho_0)^2y^2+(\rho_G/6\rho_0)^2z^2]/\xi_0^2\} \qquad (228)$$

We can now evaluate the force present on the strip depicted in Fig. 14. For a small additional (and virtual) displacement δx, E_{PG} will become:

$$E_{PG}(x+\delta x,y,z) = \tag{229}$$

$$K_{T0}X_0^2\{1-4K_c^2K_{GC}[P_{AG}^2+2P_{AG}(\rho_G/6\rho_0)(x+\delta x) +(\rho_G/6\rho_0)^2(x+\delta x)^2+(\rho_G/6\rho_0)^2y^2+(\rho_G/6\rho_0)^2z^2]/\xi_0^2\}$$

Subtracting Eq. (228) from Eq. (229) leaves the energy change during the virtual displacement:

$$\delta E_{PG}(x,y,z) = E_{PG}(x+\delta x,y,z) - E_{PG}(x,y,z) =$$

$$-4K_{T0}X_0^2K_c^2K_{GC}[2P_{AG}(\rho_G/6\rho_0)\delta x + (\rho_G/6\rho_0)^2(2x\delta x+\delta x^2)]/\xi_0^2 \tag{230}$$

In the above expression we can drop the term that is second order in the small quantity δx, as we will take the limit as $\delta x \rightarrow 0$. We can now evaluate the force on the strip by considering the sum of all volume elements within the strip. The term $-4K_{T0}X_0^2K_c^2K_{GC}(\rho_G/6\rho_0)^2(2x\delta x)/\xi_0^2$ can be dropped because for every value of positive x in our strip there is a value of negative x of equal magnitude. The surviving term is $-4K_{T0}X_0^2K_c^2K_{GC}2P_{AG}(\rho_G/6\rho_0)\delta x/\xi_0^2$ and this term is independent of x, y or z. Recalling that Eq. (230) refers to the change in energy for a single analysis-cube within the strip, we can form the relation for the force on the whole strip by summing over all of the analysis-cubes within the strip (Σ_{strip} is the symbol for that sum). The volume of the strip is $2L\Delta y\Delta z$, and therefore the number of analysis-cubes within the strip is $2L\Delta y\Delta z/X_0^3$, and the magnitude of the force on the strip is

$$F_{stripPG} = \Sigma_{strip} \{\delta\Delta E_{PG}(x,y,z)/\delta x\} = [4K_{T0}X_0^2K_c^2K_{GC}2P_{AG}(\rho_G/6\rho_0)/\xi_0^2][2L\Delta y\Delta z/X_0^3]$$

$$= 8L\Delta y\Delta z K_{T0}K_c^2K_{GC}P_{AG}\rho_G/3\rho_0X_0\xi_0^2 \tag{231}$$

The force on the strip shown in Fig. 14 is proportional to the volume of the strip ($2L\Delta y\Delta z$) but independent of x, y and z. The sum of the volume of all of the strips will be the volume of the sphere, V_{sphere}. Hence, we can sum the forces from all such strips to arrive at:

$$F_{spherePG} = 4V_{sphere}K_{T0}K_c^2K_{GC}P_{AG}\rho_G/3\rho_0X_0\xi_0^2 \tag{232}$$

Immersed extrinsic-energy will also affect the negative-attached-aether. In that case, the negative-attached-aether is pulled into the sphere rather than pushed out. Hence, both the effect on the negative-attached-aether from the extrinsic-energy ($\mathbf{N_{sphere}}$) and the ambient negative-attached-aether displacement ($\mathbf{N_{AG}}$) are equal in magnitude but opposite in sign to the case of the positive-attached-aether. Since the energies in Eqs. (228) and (229) are proportional to $|\mathbf{P_{AG}}+\mathbf{P_{GS}}|^2$ for the positive case, replacement of $\mathbf{P_{GS}}$ by $-\mathbf{N_{GS}}$ and $\mathbf{P_{AG}}$ by $-\mathbf{N_{AG}}$ results in the same force magnitude for the negative case, $F_{sphereNG} = F_{spherePG} = 4V_{sphere}K_{T0}K_c^2K_{GC}P_{AG}\rho_G/3\rho_0X_0\xi_0^2$. The total force magnitude is thus:

$$F_{sphereG} = F_{sphereNG} + F_{spherePG} = 8V_{sphere}K_{T0}K_c^2K_{GC}P_{AG}\rho_G/3\rho_0X_0\xi_0^2 \tag{233}$$

Observe that the force is aligned with the direction of $\mathbf{x}$ in the above analysis, since the energy varies with δx. We have chosen $\mathbf{x}$ to be aligned with $\mathbf{P_{AG}}$, so the force is aligned with $\mathbf{P_{AG}}$. Inside the sphere shown in Fig. 14 the total aetherial displacement $\mathbf{P_G}$ increases in the direction of $\mathbf{P_{AG}}$ as a result of the extrinsic-energy pushing the attached-aether outward. (On one side of the sphere the attached-aether is pushed in the opposite direction of $\mathbf{P_{AG}}$ while on the other side it is pushed in the same direction as $\mathbf{P_{AG}}$. Within the sphere, $\mathbf{P_G}$ increases from one side to the other.) Hence, the positive-attached-aether will be forced in the direction of $\mathbf{P_{AG}}$ as lower energy is preferred. (Eq. (225), $E_{PG} = K_{T0}X_0^2(1 - 4K_{GC}K_c^2|\mathbf{P_G}|^2/\xi_0^2)$ shows that larger values of $\mathbf{P_G}$ lead to lower energy.) The force on the extrinsic-energy will be in the opposite direction as the force on the positive-attached-aether. As mentioned in section C.13, this can be thought of as analogous to pushing a ball downward through a tub of water: as the disturbance (the ball, or the extrinsic-energy) is forced down, the substance it is pushed through (the water, or the attached-aether) is forced in the opposite direction. Hence, the force on the extrinsic-energy (the ball) is in the direction opposite to that on $\mathbf{P_{AG}}$ (the water).

$\mathbf{F}_{\mathbf{sphereG}} = -8V_{\text{sphere}}K_{T0}K_c^2K_{GC}\mathbf{P}_{\mathbf{AG}}\rho_G/3\rho_0X_0\xi_0^2$ (234)

We now recall Eq. (209), $\mathbf{P}_{\mathbf{GL}} - \mathbf{N}_{\mathbf{GL}} = -\varepsilon_0\nabla\phi_G/\rho_0$, and recall that before the injection of the extrinsic-energy $\mathbf{P}_{\mathbf{AG}} = \mathbf{P}_{\mathbf{GL}}$ and now with Eq. (211) $\mathbf{N}_{\mathbf{GL}} = -\mathbf{P}_{\mathbf{GL}}$ we have $\mathbf{P}_{\mathbf{GL}} - \mathbf{N}_{\mathbf{GL}} = 2\mathbf{P}_{\mathbf{GL}} = 2\mathbf{P}_{\mathbf{AG}} = -\varepsilon_0\nabla\phi_G/\rho_0$, or $\mathbf{P}_{\mathbf{AG}} = -\varepsilon_0\nabla\phi_G/2\rho_0$. Thus we arrive at

$\mathbf{F}_{\mathbf{sphereG}} = 4V_{\text{sphere}}K_{T0}K_c^2K_{GC}\varepsilon_0\nabla\phi_G\rho_G/3\rho_0^2X_0\xi_0^2$ (235)

To verify our sign, note that Eq. (225) relates that $E_{PG} = K_{T0}X_0^2(1 - 4K_c^2K_{GC}|\mathbf{P}_{\mathbf{G}}|^2/\xi_0^2)$ and we see that two overlapped spheres of extrinsic-energy will have an energy in their cubes of $E_{PG1} = 2K_{T0}X_0^2[1 - 4K_c^2K_{GC}(2P_G)^2/\xi_0^2] = 2K_{T0}X_0^2[1 - 16K_c^2K_{GC}P_G^2/\xi_0^2]$, while if those same two spheres are at a great distance the energy will be $E_{PG2} = 2K_{T0}X_0^2[1 - 4K_c^2K_{GC}P_G^2/\xi_0^2]$. Since nature favors smaller energy, we see that the force between amounts of extrinsic-energy is attractive, agreeing with our sign choice leading to Eq. (235).

We now recall $K_{T0} = 3\rho_0^2X_0\xi_0^2/4K_c^2\varepsilon_0$ from Eq. (141) and we also set ρ_G multiplied by the volume of the sphere multiplied by K_{GC} to a quantity called Q_G

$Q_G = K_{GC}V_{\text{sphere}}\rho_G$ (236)

We then arrive at the force on an amount of extrinsic-energy immersed within a region of ambient attached-aether displacement:

$\mathbf{F}_{\mathbf{G}} = Q_G\nabla\phi_G$ (237)

D.5 – The Gravitational Potential of a Uniform Sphere of Mass

We will now evaluate the gravitational potential of a uniform sphere of mass. In that case we have $(4/3)\pi R_S^3\rho_{ES} = \gamma M_Sc^2$, or $\rho_{ES} = 3\gamma M_Sc^2/4\pi R_S^3$, where R_S is the radius of the sphere, M_S is the mass of the sphere, and ρ_{ES} is the extrinsic-energy density inside of the sphere. Recalling Eq. (193), $\rho_G = 3[K_{G1}–K_{G2}]\rho_E\rho_0/2$, we can define ρ_{GS} to be related to ρ_{ES} as

$$\rho_{GS} = 3[K_{G1}–K_{G2}]\rho_{ES}\rho_0/2 = 9[K_{G1}–K_{G2}]\gamma M_S c^2\rho_0/8\pi R_S^3 \qquad (r < R_S) \qquad (238)$$

Recall Eq. (208), $\nabla^2\phi_G = –\rho_G/\varepsilon_0$. For the case of spherical symmetry, the derivatives with respect to the angular variables vanish leaving Eq. (208) as:

$$\partial^2\phi_G/\partial r^2 + (2/r)\partial\phi_G/\partial r = –\rho_G/\varepsilon_0 \qquad (239)$$

The analysis will be divided into two regions, one inside of R_S where $\phi_G = \phi_{SI}$ and the other outside of R_S where $\phi_G = \phi_{SO}$. Inside of R_S the solution to Eq. (239) is

$$\phi_{SI} = –\rho_{GS}r^2/6\varepsilon_0 \qquad (240)$$

It is easy to show that the potential given in Eq. (240) is a solution to Eq. (239). $\partial\phi_{SI}/\partial r = –\rho_{GS}r/3\varepsilon_0$, $(2/r)\partial\phi_{SI}/\partial r = –2\rho_{GS}/3\varepsilon_0$, and $\partial^2\phi_{SI}/\partial r^2 = –\rho_{GS}/3\varepsilon_0$ so $\partial^2\phi_{SI}/\partial r^2 + (2/r)\partial\phi_{SI}/\partial r = –\rho_{GS}/\varepsilon_0$.

Now form $\nabla\phi_{SI}$:

$$\nabla\phi_{SI} = –\rho_{GS}\mathbf{r}/3\varepsilon_0 \qquad (241)$$

We again recall Eq. (209), $\mathbf{P_{GL}} – \mathbf{N_{GL}} = –\varepsilon_0\nabla\phi_G/\rho_0$ and Eq. (211), $\mathbf{N_{GL}} = –\mathbf{P_{GL}}$, and hence $\mathbf{P_{GL}} – \mathbf{N_{GL}} = 2\mathbf{P_{GL}} = –\varepsilon_0\nabla\phi_G/\rho_0$, or

$$\mathbf{P_{GL}} = –\varepsilon_0\nabla\phi_G/2\rho_0 \qquad (242)$$

Since ϕ_{SI} is a specific form of ϕ_G

$$\mathbf{P_{GLIN}} = –\varepsilon_0\nabla\phi_{SI}/2\rho_0 = \rho_{GS}\mathbf{r}/6\rho_0 = –\mathbf{N_{GLIN}} \qquad (243)$$

And now use Eq. (238) for ρ_{GS},

$$\mathbf{P_{GLIN}} = –\mathbf{N_{GLIN}} = ([K_{G1}–K_{G2}]\rho_{ES}/4)\mathbf{r} = (3[K_{G1}–K_{G2}]\gamma M_S c^2/16\pi R_S^3)\mathbf{r} \qquad (244)$$

Outside of R_S, ρ_M and ρ_G are zero, and the solution to Eq. (239) is

$$\phi_{SO} = C_2/r – C_3 \qquad (245)$$

It is easy to show that the potential ϕ_{SO} given in Eq. (245) is a solution to Eq. (239). $\partial\phi_{SO}/\partial r = -C_2/r^2$, $(2/r)\partial\phi_{SO}/\partial r = -2C_2/r^3$, and $\partial^2\phi_{SO}/\partial r^2 = 2C_2/r^3$ so $\partial^2\phi_{SO}/\partial r^2 + (2/r)\partial\phi_{SO}/\partial r = 0$. The constants of integration, C_2 and C_3, are set so that $\phi_{SO}(R_S) = \phi_{SI}(R_S) = -\rho_{GS}R_S^2/6\varepsilon_0$:

$$\phi_{SO} = \rho_{GS}R_S^3/3r\varepsilon_0 - \rho_{GS}R_S^2/2\varepsilon_0 \qquad (246)$$

(Be reminded that Eq. (246) involves the density ρ_{GS} inside the sphere because it sets the boundary condition. The density ρ_G outside the sphere is zero.) Eq. (246) leads to

$$\nabla\phi_{SO} = -(\rho_{GS}R_S^3/3r^2\varepsilon_0)\hat{\mathbf{r}} \qquad (247)$$

Now use Eq. (242), $\mathbf{P_{GL}} = -\varepsilon_0\nabla\phi_G/2\rho_0$, where now ϕ_{SO} is a specific form of ϕ_G

$$\mathbf{P_{GLOUT}} = -\varepsilon_0\nabla\phi_{SO}/2\rho_0 = (\rho_{GS}R_S^3/6r^2\rho_0)\hat{\mathbf{r}} = -\mathbf{N_{GLOUT}} \qquad (248)$$

Using Eq. (238) for ρ_{GS} leaves

$$\mathbf{P_{GLOUT}} = -\mathbf{N_{GLOUT}} = (3[K_{G1}-K_{G2}]\gamma M_S c^2/16\pi r^2)\hat{\mathbf{r}} \qquad (249)$$

Note that Eqs. (244) and (249) obtain equal values at $r = R_S$, as they must.

D.6 – Newton's Law of Universal Gravitation

It is possible to further manipulate Eq. (237) into a more familiar form for the case of two interacting masses. Consider two homogenous spheres S_1 and S_2, with masses M_1 and M_2, respectively, and radii R_1 and R_2, respectively. Without loss of generality we can consider S_2 to be centered at the origin. With S_1 centered at $r > R_1 + R_2$, Eq. (247) informs that $\nabla\phi_{SO2}$ from M_2 is $\nabla\phi_{SO2} = -(\rho_{GS2}R_2^3/3r^2\varepsilon_0)\hat{\mathbf{r}}$ and therefore Eq. (237) for the force between the two masses becomes $\mathbf{F_{GM1M2}} = Q_{G1}\nabla\phi_{SO2} = -(Q_{G1}\rho_{GS2}R_2^3/3r^2\varepsilon_0)\hat{\mathbf{r}}$. Recall Eq. (238), $\rho_{GS} = 3[K_{G1}-K_{G2}]\rho_{ES}\rho_0/2$, to get $\mathbf{F_{GM1M2}} = Q_{G1}\nabla\phi_{SO2} = -(Q_{G1}[K_{G1}-K_{G2}]\rho_{E2}R_2^3\rho_0/2r^2\varepsilon_0)\hat{\mathbf{r}}$. And with $\rho_{E2} = \gamma_2M_2c^2/[(4/3)\pi R_2^3]$ this becomes $\mathbf{F_{GM1M2}} = -(3\gamma_2M_2c^2Q_{G1}[K_{G1}-K_{G2}]\rho_0/8\pi r^2\varepsilon_0)\hat{\mathbf{r}}$. Next, recall from above Eq. (236) that Q_G

is $K_{GC}V_{sphere}\rho_G$, or, in this case, using Eq. (193), $Q_{G1} = K_{GC}(4/3)\pi R_1^3\rho_{G1} = K_{GC}2\pi R_1^3[K_{G1}–K_{G2}]\rho_{E1}\rho_0$. With $\rho_{E1} = \gamma_1 M_1 c^2/[(4/3)\pi R_1^3]$, $Q_{G1} = 3K_{GC}[K_{G1}–K_{G2}]\rho_0\gamma_1 M_1 c^2/2$, leaving

$\mathbf{F}_{\mathbf{ME1ME2}} = \mathbf{F}_{\mathbf{GNEWTON}} =$

$$–(9\gamma_2 M_2 c^2 K_{GC}[K_{G1}–K_{G2}]^2\rho_0^2\gamma_1 M_1 c^2/16\pi r^2\varepsilon_0)\hat{\mathbf{r}} = –(G_N\gamma_1 M_1\gamma_2 M_2/r^2)\hat{\mathbf{r}} \tag{250}$$

In the low velocity limit, $\gamma_1 = \gamma_2 = 1$, and Eq. (250) is recognized as Newton's Law of Universal Gravitation where G_N is the combination of constants

$$G_N = 9K_{GC}[K_{G1}–K_{G2}]^2\rho_0^2 c^4/16\pi\varepsilon_0 = 6.6743\text{x}10^{–11}\ \text{N m}^2/\text{kg}^2 \tag{251}$$

Dimensional analysis. From Eqs. (183) to (186) we see that $K_{G1}\rho_E$ and $K_{G2}\rho_E$ are dimensionless, K_{GC} is also dimensionless and so $[K_{G1}–K_{G2}]^2\rho_0^2$ has the dimensions of ρ_0^2/ρ_E^2, and hence G_N has the dimensions of $\rho_0^2 c^4/\rho_E^2\varepsilon_0$. From Eq. (178) ρ_E is in $kg(m/s)^2m^{-3}$ = kg $s^{-2}m^{-1}$, ρ_0 is in A s m^{-3}, and ε_0 is in $m^{-3}kg^{-1}$ s^4A^2, so the dimensions of G_N are $(A\ s\ m^{-3})^2(m/s)^4/(kg\ s^{-2}\ m^{-1})^2(m^{-3}\ kg^{-1}\ s^4\ A^2)$ $= (A^2\ s^{-2}\ m^{-2})/(kg\ m^{-5}\ A^2) = m^3/(kg\ s^2) = N\ m^2/kg^2$ as expected.

D.7 – Gravitational Redshift

Eq. (250), $\mathbf{F}_{\mathbf{GNEWTON}} = –(G_N\gamma_1 M_1\gamma_2 M_2/r^2)\hat{\mathbf{r}}$, can be used to evaluate the gravitational redshift for photon emission from stars by assigning $\gamma_1 M_1 = M_{STAR}$ for a star of mass M_{STAR}, and $\gamma_2 M_2 = hf/c^2$ as the equivalent mass of the photon, where h is Planck's constant and f is the frequency of the light. Here, we will evaluate a star comoving with the aetherial rest frame, and hence $\gamma_1 = 1$. By integrating the force over the distance traveled, we can find the energy change as the photon moves radially outward from the surface (at R_0) of the star:

$$E_{PHOTON}(r) – E_{PHOTON}(R_0) = G_N\gamma_1 M_1\gamma_2 M_2/r – G_N\gamma_1 M_1\gamma_2 M_2/R_0$$

$$= (G_N M_{STAR} hf/c^2)(1/r – 1/R_0) \tag{252}$$

Evaluating the above expression for infinite r, and assigning E_{PHOTON} at infinity to hf, results in what is commonly called the Newtonian Limit for the gravitational redshift, $E_{PHOTON}(R_0) = hf + G_N M_{STAR} hf/R_0 c^2 = hf(1 + G_N M_{STAR}/R_0 c^2)$.

D.8 – Masses of the Tension, Quantum and Gamma Fields

Section D.3 derives the tension, quantum and gamma energies of an analysis-cube inside a positive-attached-aether sphere containing extrinsic-energy as Eqs. (214), (215) and (224), respectively:

$$E_{TPGI} = K_{T0}X_0^2[(1/2) + K_c|\mathbf{P_G}|/\xi_0 + K_c^2(|\mathbf{P_G}|/\xi_0)^2 + 3\delta X/X_0 + 3K_c|\mathbf{P_G}|\delta X/2X_0\xi_0] \quad (214)$$

$$E_{QPGI} = (2K_{Q0}/X_0^2)[(1/2) - K_c|\mathbf{P_G}|/\xi_0 + K_c^2|\mathbf{P_G}|^2/\xi_0^2 - 3\delta X/X_0 + 3K_c|\mathbf{P_G}|\delta X/2X_0\xi_0] \quad (215)$$

$$E_{\gamma PGI} = K_{T0}X_0^2[-2K_c^2(|\mathbf{P_G}|/\xi_0)^2 - 4K_c^2K_{GC}(|\mathbf{P_G}|/\xi_0)^2 + 3K_c|\mathbf{P_G}|\delta X/X_0\xi_0] \quad (224)$$

Appendix H shows that outside the extrinsic-energy sphere these equations become:

$$E_{TPGO} = K_{T0}X_0^2[(1/2) + K_c|\mathbf{P_G}|/\xi_0 + K_c^2(|\mathbf{P_G}|/\xi_0)^2 - K_c|\mathbf{P_G}|\delta X/4X_0\xi_0] \quad (H2)$$

$$E_{QPGO} = (2K_{Q0}/X_0^2)[(1/2) - K_c|\mathbf{P_G}|/\xi_0 + K_c^2(|\mathbf{P_G}|/\xi_0)^2 - K_c|\mathbf{P_G}|\delta X/4X_0\xi_0] \quad (H4)$$

$$E_{\gamma PGO} = K_{T0}X_0^2[-2K_c^2(|\mathbf{P_G}|/\xi_0)^2 - 4K_c^2K_{GC}(|\mathbf{P_G}|/\xi_0)^2 + K_c|\mathbf{P_G}|\delta X/2X_0\xi_0] \quad (H6)$$

The red terms in the six equations above will be dropped since they involve $\delta X/X_0$ leaving:

$$E_{TPG} = (K_{T0}X_0^2)[(1/2) + K_c|\mathbf{P_G}|/\xi_0 + K_c^2|\mathbf{P_G}|^2/\xi_0^2] \quad (253)$$

$$E_{QPG} = (2K_{Q0}/X_0^2)[(1/2) - K_c|\mathbf{P_G}|/\xi_0 + K_c^2|\mathbf{P_G}|^2/\xi_0^2] \quad (254)$$

$$E_{\gamma PG} = -2K_{T0}K_c^2(X_0/\xi_0)^2|\mathbf{P_G}|^2 - 4K_c^2K_{GC}K_{T0}(X_0/\xi_0)^2|\mathbf{P_G}|^2 \quad (255)$$

Since these fields have energy, they will also have an equivalent gravitational-mass. Here we will make the assignments:

The Gravitational-Mass Assignment. The tension-energy E_{TPG} (E_{TNG}) leads to a positive (negative) gravitational-mass while the quantum-energy E_{QPG} (E_{QNG}) leads to a negative (positive) gravitational-mass.

For the positive-aether, the gamma-force has a negative tension, which leads to a negative gravitational-mass, and it also has a negative quantum-force, which leads to a positive gravitational-mass. For the energy, both the negative gamma tension and the negative gamma quantum-force lead to negative contributions, since they are reductions in the overall positive force quantities. In the energy term $-4K_c^2K_{GC}K_{T0}(X_0/\xi_0)^2|\mathbf{P_G}|^2$ from Eq. (255), half comes from the negative tension and half comes from the quantum-force. Since the tension and the quantum-force lead to opposite signs of mass, this term will not contribute to the mass. The remaining gamma-energy term, $-2K_{T0}K_c^2(X_0/\xi_0)^2|\mathbf{P_G}|^2$, is the dominant tension term and it will contribute a negative mass. We now apply the usual formula $E = mc^2$ along with the gravitational-mass assignments to Eqs. (253), (254) and (255). With use subscript of PTQγ for the positive-aether tension, quantum and gamma fields, we obtain $m_{PTQ\gamma} = \{(K_{T0}X_0^2)[(1/2) + K_c|\mathbf{P_G}|/\xi_0 + K_c^2|\mathbf{P_G}|^2/\xi_0^2] - (2K_{Q0}/X_0^2)[(1/2) - K_c|\mathbf{P_G}|/\xi_0 + K_c^2|\mathbf{P_G}|^2/\xi_0^2] - 2K_{T0}K_c^2(X_0/\xi_0)^2|\mathbf{P_G}|^2\}/c^2$. Using Eq. (22), $K_{T0}X_0^2 = 2K_{Q0}/X_0^2$ we get

$$m_{PTQ\gamma} = 2(K_{T0}X_0^2/c^2)[K_c|\mathbf{P_G}|/\xi_0 - K_c^2|\mathbf{P_G}|^2/\xi_0)^2] \qquad (256)$$

Appendix G derives the tension, quantum and gamma energies of an analysis-cube within a negative-attached-aether sphere containing extrinsic-energy as Eqs. (G2), (G4) and (G8), respectively. Appendix H derives what those equations become outside of the extrinsic-energy sphere as Eqs. (H8), (H10) and (H12). Eliminating the $\delta X/X_0$ terms as we did for the positive case we obtain

$$E_{TNG} = (K_{T0}X_0^2)[(1/2) - K_c|\mathbf{N_G}|/\xi_0 + K_c^2|\mathbf{N_G}|^2/\xi_0^2] \qquad (257)$$

$E_{QNG} = (2K_{Q0}/X_0^2)[(1/2) + K_c|\mathbf{N_G}|/\xi_0 + K_c^2|\mathbf{N_G}|^2/\xi_0^2]$ (258)

$E_{\gamma NG} = -2K_c^2K_{T0}(X_0/\xi_0)^2|\mathbf{N_G}|^2 - 4K_c^2K_{GC}K_{T0}(X_0/\xi_0)^2|\mathbf{N_G}|^2$ (259)

For the negative-aether, the gamma-force has a negative quantum-force, which leads to a negative gravitational-mass, and it also has a negative tension, which leads to a positive gravitational-mass. For the energy, both the negative gamma tension and the negative gamma quantum-force lead to negative contributions, since they are reductions in the overall positive force quantities. In the energy term $-4K_c^2K_{GC}K_{T0}(X_0/\xi_0)^2|\mathbf{N_G}|^2$ from Eq. (259), half comes from the negative tension and half comes from the quantum-force. Therefore this term will not contribute to the mass. The remaining gamma-energy term, $-2K_{T0}K_c^2(X_0/\xi_0)^2|\mathbf{N_G}|^2$, is the dominant quantum-energy term and it will contribute a negative mass. We again use the usual formula $E = mc^2$. Hence we obtain $m_{NTQ\gamma} = \{-(K_{T0}X_0^2)[(1/2) - K_c|\mathbf{N_G}|/\xi_0 + K_c^2|\mathbf{N_G}|^2/\xi_0^2] + (2K_{Q0}/X_0^2)[(1/2) + K_c|\mathbf{N_G}|/\xi_0 + K_c^2|\mathbf{N_G}|^2/\xi_0^2] - 2K_c^2K_{T0}(X_0/\xi_0)^2|\mathbf{N_G}|^2\}/c^2$. Using Eq. (22), $K_{T0}X_0^2 = 2K_{Q0}/X_0^2$ we get

$m_{NTQ\gamma} = 2(K_{T0}X_0^2/c^2)[K_c|\mathbf{N_G}|/\xi_0 - K_c^2|\mathbf{N_G}|^2/\xi_0^2]$ (260)

Now make use of Eq. (221), $\mathbf{N_{GL}} = -\mathbf{P_{GL}}$, which leads to $|\mathbf{N_{GL}}| = |\mathbf{P_{GL}}|$ to arrive at the total gravitational-mass of the fields:

$m_{TQ\gamma} = m_{PTQ\gamma} + m_{NTQ\gamma} = 4(K_{T0}X_0^2/c^2)[K_c|\mathbf{P_G}|/\xi_0 - K_c^2|\mathbf{P_G}|^2/\xi_0^2]$ (261)

It is useful to divide Eq. (261) by the volume of the analysis-cube X_0^3 and then form the two components of the field-mass density

$\delta\rho_{M1} = 4K_{T0}K_c|\mathbf{P_G}|/\xi_0X_0c^2 = K_{G3}P_G$ (*See note after Eq. (263).) (262)

$\delta\rho_{M2} = -4K_c^2K_{T0}|\mathbf{P_G}|^2/X_0\xi_0^2c^2 = -K_{G4}P_G^2$ (263)

(*Note that we must neglect any first-field-mass contribution to $\mathbf{P_G}$ when using Eq. (262) to find the first-field-mass density. See section D.12 below.)

From the above, the constants K_{G3} and K_{G4} are:

$K_{G3} = 4K_{T0}K_c/\xi_0X_0c^2$ (264)

$K_{G4} = 4K_c^2K_{T0}/X_0\xi_0^2c^2$ (265)

Dimensional Analysis. K_c is dimensionless, $K_{T0}X_0^2$ is an energy, kg m^2/s^2, and X_0 is a length, so K_{T0} is in kg/s^2. ξ_0 is a length. Hence K_{G3} is in (kg/s^2)/[m^2(m/s)^2] = kg/m^4, and $\delta\rho_{M1} = K_{G3}P_G$ is in kg/m^3 as expected. K_{G4} is in (kg/s^2)/[m^3(m/s)^2] = kg/m^5, and $\delta\rho_{M2} = -K_{G4}P_G^2$ is in kg/m^3 also as expected.

D.9 – Tension, Quantum and Gamma Field-Mass Effects on Two Interacting Spherically Symmetric Masses

Section D.8 has shown that field-mass effects have two terms, one with positive gravitational-mass, from Eq. (262), and one with negative gravitational-mass from Eq. (263). This section will investigate those field-mass effects in the two-body problem involving a first body of mass m and a second body of mass M. The second body will be assumed to have a uniform mass-density within a sphere of radius R.

D.9.1. The First-Field-Mass Effect. Eq. (262) gives $\delta\rho_{M1} = K_{G3}P_G$ as a positive field-mass-density. For r < R, Eq. (244) informs that $\mathbf{P_{GLIN}} = (3[K_{G1}–K_{G2}]\gamma M_Sc^2/16\pi R_S^3)\mathbf{r}$, and therefore, recalling that P_{GLIN} is the magnitude of $\mathbf{P_{GLIN}}$, $\delta\rho_{M1} = K_{G3}P_{GLIN} = K_{G3}(3[K_{G1}–K_{G2}]\gamma_M Mc^2/16\pi R^3)r$ for r < R. The total first-field-mass ΔM_{1IN} within a sphere of radius r can be found by integrating $\delta\rho_{M1}$ within that sphere, $\Delta M_{1IN} = 4\pi\int \delta\rho_{M1}\, r^2\, dr = 4\pi\int K_{G3}(3[K_{G1}–K_{G2}]\gamma_M Mc^2/16\pi R^3)r^3 dr$, leaving:

$\Delta M_{1IN} = K_{G3}(3[K_{G1}–K_{G2}]\gamma_M Mc^2/16R^3)r^4 = K_{G5}(\gamma_M M/R^3)r^4$ (for r < R) (266)

In Eq. (266) K_{G5} is a combination of other constants:

$K_{G5} = K_{G3}(3[K_{G1}–K_{G2}]c^2/16)$ (267)

For r > R, Eq. (249) informs that $\mathbf{P_{GLOUT}} = (3[K_{G1}–K_{G2}]\gamma M_S c^2/16\pi r^2)\hat{\mathbf{r}}$, and therefore, $\delta\rho_{M1} = K_{G3}P_{GLOUT} = 3K_{G3}[K_{G1}–K_{G2}]\gamma_M Mc^2/16\pi r^2$ for r > R. The total first-field-mass ΔM_{1SHELL} within a spherical shell of outer radius r and inner radius R can be found by integrating $\delta\rho_{M1}$ within that shell, $\Delta M_{1SHELL} = 4\pi\int \delta\rho_{M1}\ r^2 dr = 4\pi\int(3K_{G3}[K_{G1}–K_{G2}]\gamma_M Mc^2/16\pi r^2)r^2 dr = (3K_{G3}[K_{G1}–K_{G2}]\gamma_M Mc^2/4)r = 4K_{G5}\gamma_M Mr$ evaluated between R and r, or $\Delta M_{1OUT} = 4K_{G5}\gamma_M M(r – R)$. To get the full mass inside of r, but outside of R, we must also add the mass inside of R by evaluating Eq. (266) at R to get:

$$\Delta M_{1OUT} = 4K_{G5}\gamma_M M(r – R) + K_{G5}\gamma_M MR = 4K_{G5}\gamma_M Mr – 3K_{G5}\gamma_M MR \quad \text{(for } r > R) \qquad (268)$$

D.9.2. The Second-Field-Mass Effect. Eq. (263) gives $\delta\rho_{M2} = –K_{G4}P_G^2$ as a negative field-mass-density. For r < R, Eq. (244) informs that $\mathbf{P_{GLIN}} = (3[K_{G1}–K_{G2}]\gamma M_S c^2/16\pi R_S^3)\mathbf{r}$, and therefore, recalling that P_{GLIN} is the magnitude of $\mathbf{P_{GLIN}}$, $\delta\rho_{M2} = –K_{G4}P_{GLIN}^2 = –K_{G4}(3[K_{G1}–K_{G2}]\gamma_M Mc^2/16\pi R^3)^2 r^2$ for r < R. The total second-field-mass within a sphere of radius r, ΔM_{2IN}, is found by integrating $\delta\rho_{M2}$ within the sphere, $\Delta M_{2IN} = 4\pi\int \delta\rho_{M2}\ r^2\ dr = 4\pi\int –K_{G4}(3[K_{G1}–K_{G2}]\gamma_M Mc^2/16\pi R^3)^2 r^4 dr = –4\pi K_{G4}(3[K_{G1}–K_{G2}]\gamma_M Mc^2/16\pi R^3)^2 r^5/5$, or:

$$\Delta M_{2IN} = –(4\pi K_{G4}/5)(3[K_{G1}–K_{G2}]\gamma_M Mc^2/16\pi R^3)^2 r^5 = –K_{G6}(\gamma_M M/R^3)^2 r^5 \quad \text{(for } r < R) \qquad (269)$$

In Eq. (269) K_{G6} is a combination of other constants:

$$K_{G6} = (4\pi K_{G4}/5)(3[K_{G1}–K_{G2}]c^2/16\pi)^2 = 9K_{G4}[K_{G1}–K_{G2}]^2 c^4/320\pi \qquad (270)$$

For r > R, Eq. (249) informs that $\mathbf{P_{GLOUT}} = (3[K_{G1}–K_{G2}]\gamma_M Mc^2/16\pi r^2)\hat{\mathbf{r}}$, and therefore, $\delta\rho_{M2} = –K_{G4}P_G^2 = –K_{G4}P_{GLOUT}^2 = –K_{G4}(3[K_{G1}–K_{G2}]\gamma_M Mc^2/16\pi r^2)^2$ for r > R. The total second-field-mass ΔM_{2OUT} within a spherical shell of outer radius r and inner radius R can be found by integrating $\delta\rho_{M2}$ within that shell, $\Delta M_{2SHELL} = 4\pi\int \delta\rho_{M2}\ r^2 dr = –4\pi\int K_{G4}(3[K_{G1}–K_{G2}]\gamma_M Mc^2/16\pi r^2)^2 r^2 dr =$

$4\pi K_{G4}(3[K_{G1}–K_{G2}]\gamma_M Mc^2/16\pi)^2/r = 5K_{G6}(\gamma_M M)^2/r$ evaluated between R and r, and we must also add the mass from Eq. (269) evaluated at R leaving $\Delta M_{2OUT} = 5K_{G6}(\gamma_M M)^2/r – 5K_{G6}(\gamma_M M)^2/R – K_{G6}(\gamma_M M)^2/R$, or:

$$\Delta M_{2OUT} = 5K_{G6}(\gamma_M M)^2/r – 6K_{G6}(\gamma_M M)^2/R \qquad \text{(for } r > R) \qquad (271)$$

Dimensional Analysis. From Eqs. (183) to (186) we see that $K_{G1}\rho_E$ and $K_{G2}\rho_E$ are dimensionless and so $[K_{G1}–K_{G2}]$ has the dimensions of $1/\rho_E$. From Eq. (178) ρ_E is in kg $(m/s)^2$ m^{-3} = kg s^{-2} m^{-1}, and so $[K_{G1}–K_{G2}]$ is in kg^{-1} s^2 m. In section D.8 we found K_{G3} is in kg/m^4. With Eq. (267) $K_{G5} = K_{G3}(3[K_{G1}–K_{G2}]c^2/16)$ and so K_{G5} is in $(kg/m^4)(kg^{-1}\ s^2\ m)(m/s)^2 = m^{-1}$. Hence ΔM_{1IN} from Eq. (266) and ΔM_{1OUT} from Eq. (268) are in kg, as expected. In section D.8 we found K_{G4} is in kg/m^5. With Eq. (270) $K_{G6} = 9K_{G4}[K_{G1}–K_{G2}]^2c^4/320\pi$, K_{G6} is in $(kg/m^5)(kg^{-1}\ s^2\ m)^2(m/s)^4 = kg^{-1}m$. Hence ΔM_{2IN} from Eq. (269) and ΔM_{2OUT} from Eq. (271) are in kg, as expected.

D.9.3. Gravitational Force on a Mass m due to a Spherically Symmetric Mass M including the Field-Mass Effects. The field-masses determined in Eqs. (266), (268), (269) and (271) are all spherically symmetric about the center of the large spherically symmetric mass M. Recall that without the field-masses, the force on m from M is Newton's Law of Universal Gravitation, Eq. (250). We will now include the field-masses. For spherically symmetric mass distributions, the force on a small mass m at a distance r from the center of M will be the same as if all of the mass within the sphere of radius r was instead centered at the center of M. (This result is shown in elementary physics courses.)

To find the force on m including the field-mass effects for r < R we use Eq. (250), $\mathbf{F}_{\mathbf{GNEWTON}} = –(G_N\gamma_1 M_1\gamma_2 M_2/r^2)\hat{\mathbf{r}}$, substitute in $\gamma_m m$ for $\gamma_1 M_1$ and for $\gamma_2 M_2$ we substitute in $\gamma_M M(r/R)^3$ plus the field-masses of Eqs. (266) and (269). Note also that no additional factors of γ need be multiplied to the field-masses, as the field-masses result from aetherial distortion and aetherial distortions of course do not move respect to the aether (any associated γ is unity). This leaves $\mathbf{F_{GIN}} = -G_N\gamma_m m[\gamma_M M(r/R)^3 + K_{G5}(\gamma_M M/R^3)r^4 - K_{G6}(\gamma_M M/R^3)^2 r^5]\hat{\mathbf{r}}/r^2$, or:

$$\mathbf{F_{GIN}} = -(G_N\gamma_m m\gamma_M M/R^3)[r + K_{G5}r^2 - K_{G6}(\gamma_M M/R^3)r^3]\hat{\mathbf{r}} \qquad (r < R) \qquad (272)$$

For r > R, we again use Eq. (250), and again substitute in $\gamma_m m$ for M_1 but this time substitute in $\gamma_M M$ plus the field-masses of Eqs. (268) and (271). This leaves $\mathbf{F_{GOUT}} = -G_N\gamma_M m(\gamma_M M + 4K_{G5}\gamma_M Mr - 3K_{G5}\gamma_M MR + 5K_{G6}\gamma_M{}^2M^2/r - 6K_{G6}\gamma_M{}^2M^2/R)\,\hat{\mathbf{r}}/r^2$, or:

$\mathbf{F_{GOUT}}$ (r > R)

$$= -G_N\gamma_m m(\gamma_M M/r^2 + 4K_{G5}\gamma_M M/r - 3K_{G5}\gamma_M MR/r^2 + 5K_{G6}\gamma_M{}^2M^2/r^3 - 6K_{G6}\gamma_M{}^2M^2/r^2R)\hat{\mathbf{r}} \qquad (273)$$

We now introduce an effective-mass M_{EFF},

$$M_{EFF} = \gamma_M M - 3K_{G5}\gamma_M MR - 6K_{G6}\gamma_M{}^2M^2/R \qquad (274)$$

With Eq. (274), Eq. (273) becomes:

$$\mathbf{F_{GOUT}} = -G_N\gamma_m m(M_{EFF}/r^2 + 4K_{G5}\gamma_M M/r + 5K_{G6}\gamma_M{}^2M^2/r^3)\hat{\mathbf{r}} \qquad (m << M\ ;\ r > R) \qquad (275)$$

In deriving Eq. (275) we have not included any field-mass effects originating from m. That is because to arrive at Eq. (275) we assume m << M. Notice that we have now derived a new gravitation formula for situations where the condition m << M applies. The new equation includes Newtonian universal gravitation via the term $G_N\gamma_m mM_{EFF}/r^2$ and we have also discovered two additional terms that are associated with the field-masses.

D.10 – Galactic Gravitational Phenomena; the First-Field-Mass Identified as Dark-Matter; a Rough Determination of K_{G5}

Observations indicate that many stars are attracted to the center of galaxies by a force stronger than the $G_N mM/r^2$ given by Newton's law, and the presence of some unseen "dark-matter" would explain these results. In Eqs. (272) and (275) we see that the K_{G5} terms, which are associated with the first-field-mass, are a strong candidate for such dark-matter.

D.10.1. Gravitational Phenomena in Spiral Galaxies. As a first example of a system involving a large spherical mass distribution interacting with a small mass, we consider the case of spiral galaxies. In such galaxies there is typically a dense core, which we will treat as a large spherical mass M, as well as distant individual stars with masses m much smaller than that of the core, allowing the analysis of section D.9 to be used. It is known that the orbital velocities of stars far from the center of spiral galaxies are approximately constant independent of distance from the core.

Eqs. (272) and (275) fit the observations of spiral galaxies well. In the far reaches of the galaxy, where the distance to the star (r) becomes much greater than the radius of the galactic core (R), we can see that the K_{G5} term of Eq. (275), $-G_N\gamma_m m4K_{G5}\gamma_M M/r$, eventually dominates, as the other terms fall off with faster powers of r. Hence at large values of r the centripetal acceleration can be set equal to the dominant force term, $\gamma_m mv^2/r = G_N\gamma_m m4K_{G5}\gamma_M M/r$, and we see that for large r the velocity is independent of r, consistent with observations. Yet inside the core, where r < R, we must use Eq. (272) where the K_{G5} term is $-(G_N\gamma_m m\gamma_M MK_{G5}r^2/R^3)$, which drops quadratically to zero with r, indicating that relatively little first-field-mass will be found in the center of the galactic core. Identifying the first-field-mass as dark matter is therefore in good agreement with observations.

D.10.2. Gravitational Phenomena in Ultra Diffuse Galaxies. It is observed that in some ultra-diffuse-galaxies (UDGs) dark-matter dominates while in other UDGs dark-matter has much less of an effect. Such observations are consistent with the analysis of section D.9 if it is assumed that the cores of UDGs that show large dark-matter effects contain a central massive core, while UDGs where dark-matter effects are much less do not contain such a core. For the case where a central massive core exists, the mass M and radius R in the analysis of section D.9 will correspond to the mass M_C and radius R_C of the core, respectively. For that case, the first-field-mass (dark-matter) effects will be observed through the term $-G_N\gamma_m m4K_{G5}\gamma_M M_C/r$ of Eq. (275) outside of the core ($r > R_C$). However for the second case, where there is no dense core, a spherically symmetric, diffuse stellar density will lead to M and R of section D.9 corresponding to the mass M_{UDG} and radius R_{UDG} of the whole UDG, respectively. For that case, the first-field-mass (dark-matter) effects will be observed through the term $-(G_N\gamma_m m\gamma_M M_U K_{G5}r^2/R_U^3)$ of Eq. (272). Hence the ratio between these cases is $(G_N\gamma_m m\gamma_M M_U K_{G5}r^2/R_U^3)/(G_N\gamma_m m4K_{G5}\gamma_M M_C/r) = M_U r^3/4M_C R_U^3$. For stars located at $r = R_U/2$, and assuming $M_C = M_U/2$, the ratio is 1/16, showing that UDGs with a dense core will show a much greater indication of dark-matter than those without a dense core.

D.10.3. Using Milky Way Observations to Determine an Approximate Value of K_{G5}. A 2010 paper [8] gives a mass estimate of the Milky Way as $6.9\text{x}10^{11}$ M_{SUN} within 80 kpc ($2.469\text{x}10^{21}$ m). With the mass of the sun (M_{SUN}) equal to about $1.988\text{x}10^{30}$ kg, this leaves the mass of the Milky Way as $M_{MWT} = 1.372\text{x}10^{42}$ kg within 80 kpc, where the subscript T denotes the total mass, including a postulated "dark matter" component. A 2011 paper [9] gives a stellar mass within the virial radius of the Milky Way as $6.43\text{x}10^{10}$ M_{SUN}, or $M_{MWS} = 1.279\text{x}10^{41}$ kg while giving a value of $M_{MWT} = 1.26\text{x}10^{12}$ M_{SUN} for the total mass within the virial radius. A 2020 reference [10] gives $M_{MWDM} = 8.3\text{x}10^{11}$ M_{SUN} for the Milky Way dark mass and $M_{MWT} = 8.9\text{x}10^{11}$ M_{SUN} for the total mass within the virial radius. Reference 10 considers several possible model parameters to arrive at its numbers, with the virial radius of the models ranging between 186 and 202 kpc.

Presently, there remains considerable uncertainty in the values of the mass of our galaxy. Our purpose here is to use an admittedly very simplistic modeling of the Milky Way to 1) verify the essential agreement of our gravitational equations, Eqs. (272) and (275), with experimental data and 2) arrive at an estimate for the value of K_{G5} from those data.

Reference 8 gives a most likely value of circular star velocities at 80 kpc as 193 km/s. The central massive region of the Milky Way has a radius R << 80 kpc, and for our purposes we will assume a model wherein the vast majority of stellar mass lies within 80 kpc of the galactic center and hence Eq. (275) applies at 80 kpc. As shown below in section D.12.2, the third term of Eq. (275) will be negligible at 80 kpc. We will set $\gamma_M = \gamma_m = 1$ for our estimate. We can now equate the force from the first two terms of Eq. (275) to the centripetal acceleration of the distant stars, $G_N m(M_{EFF}/r^2+4K_{G5}M/r) = mv^2/r$, or, with $M_{EFF} \approx M$ ($M_{EFF} \approx M$ is shown below), $G_N M/r^2 + 4G_N K_{G5} M/r = v^2/r$ or:

$$K_{G5} = (v^2 - G_N M/r)/4G_N M = 9.900\text{x}10^{-22}\ \text{m}^{-1} \approx 10^{-21}\ \text{m}^{-1} \qquad (276)$$

The second equation in Eq. (276) results from substituting in the values $v = 1.93\text{x}10^5$ m/s [8], $r = 80$ kpc $= 2.469\text{x}10^{21}$ m [8], $M = M_{MWS} = 1.279\text{x}10^{41}$ kg [9], and $G_N = 6.674\text{x}10^{-11}$ m^3/kgs^{-2}. With this value of K_{G5} the dark matter induced force per kg at 80 kpc is $4G_N K_{G5} M/r = 4(6.67\text{x}10^{-11}$ m^3/kgs^{-2})(9.9x10^{-22} m^{-1})(1.279x10^{41} kg)/(2.469x10^{21} m) = 1.383x10^{-11} m/s^2. Meanwhile, the Newtonian gravitational term at 80 kpc is $G_N M/r^2 = (6.67\text{x}10^{-11}$ m^3/kg-s^2)(1.279x10^{41} kg)/(2.469x10^{21} m)^2 = 1.400x10^{-12} m/s^2. It is seen that the Newtonian gravitational term is about 10% of $4G_N K_{G5} M/r$, or that the first-field-mass makes up about 90% of the mass of the Milky Way within 80 kpc for the numbers used here. This is in reasonable agreement with present (and rather uncertain) estimates from experimental data.

It is also of interest to compare our formulas with the estimates of galactic mass given in the references cited. From Eq. (268), $\Delta M_{1OUT} = 4K_{G5}\gamma_M Mr - 3K_{G5}\gamma_M MR$, we see that the first-field-mass is expected to grow linearly with radius. Hence, if there were no other effects, and the Milky

Way was the only occupant of space, the mass inside 200 kpc should be about 2.5 times the mass inside 80 kpc. Multiplying $6.9\text{x}10^{11}$ M_{SUN} (the mass inside 80 kpc given by reference 8) by 2.5 leaves $1.725\text{x}10^{12}$ M_{SUN} as the mass within 200 kpc, which is larger than the masses of references 9 and 10. However, the Milky Way is not the only occupant of space and neighboring matter is expected to reduce the first-field-mass in regions far from the galactic center, as will be discussed in the next section. (Recall that the field-mass arises from $\mathbf{P_{GL}}$, and at a point centered between two equal masses $\mathbf{P_{GL}}$ will be zero, so other massive bodies will reduce the field-mass.) As there is much neighboring matter within our Local Group, it is to be expected that the first-field-mass will depart significantly from linear growth at large distances from the galactic center. A further analysis is outside the scope of this paper.

D.11 – Why Gravitational Infinities At Large Radii Do Not Occur

Eq. (268) relates that the first-field-mass generated by a sphere of mass M and radius R is $4K_{G5}\gamma_M Mr - 3K_{G5}\gamma_M MR$ for $r > R$. Were this to hold indefinitely, the first-field-mass would become infinite as r does. But potentially even more problematically, without some limiting mechanism, the first-field-mass will itself act as a further source of $\mathbf{P_{GL}}$, and this will lead to more mass, leading to a further source of $\mathbf{P_{GL}}$, and so on, and each such term will tend toward infinity as r does. However, there is a limiting mechanism that prevents this, as will now be discussed.

The potential issue of infinities is best illustrated in a series of solutions to Eq. (239), $\partial^2\phi_G/\partial r^2 + (2/r)\partial\phi_G/\partial r = -\rho_G/\varepsilon_0$. We have seen that outside a spherical mass of radius R, in the region where $r > R$ and $\rho_M = 0$, that the first series solution to Eq. (239) is given by Eq. (246) $\phi_{G1} = \rho_{GS}R^3/3r\varepsilon_0 - \rho_{GS}R^2/2\varepsilon_0$. Now use Eq. (209), $\mathbf{P_{GL}} - \mathbf{N_{GL}} = -\varepsilon_0\nabla\phi_G/\rho_0$ and Eq. (211), $\mathbf{N_{GL}} = -\mathbf{P_{GL}}$, and hence $\mathbf{P_{GL}} - \mathbf{N_{GL}} = 2\mathbf{P_{GL}} = -\varepsilon_0\nabla\phi_G/\rho_0$ so that

$\mathbf{P_{GL}} = -\varepsilon_0\nabla\phi_G/2\rho_0$ (277)

For the present case of $\phi_{G1} = \rho_{GS}R^3/3r\varepsilon_0 - \rho_{GS}R^2/2\varepsilon_0$, $\mathbf{P_{GL1}} = -\varepsilon_0\nabla\phi_{G1}/2\rho_0 = (\rho_{GS}R^3/6r^2\rho_0)\hat{\mathbf{r}}$. Then, use of Eq. (262) gives the mass density from P_{GL1} as $\delta\rho_{M11} = K_{G3}P_{GL1} = K_{G3}\rho_{GS}R^3/6r^2\rho_0$.

For the second solution in our series, $\delta\rho_{M11}$ becomes the source mass in Eq. (239). With Eq. (193), $\rho_G = 3[K_{G1}-K_{G2}]\rho_E\rho_0/2$, and with $\rho_E = \delta\rho_{M11}c^2$, we obtain $\rho_{G11} = 3[K_{G1}-K_{G2}]\delta\rho_{M11}c^2\rho_0/2 = 3[K_{G1}-K_{G2}](K_{G3}\rho_{GS}R^3/6r^2\rho_0)c^2\rho_0/2 = (K_{G3}3[K_{G1}-K_{G2}]c^2/16)(16\rho_{GS}R^3/12r^2) = K_{G5}4\rho_{GS}R^3/3r^2$. (Recall Eq. (267), $K_{G5} = K_{G3}(3[K_{G1}-K_{G2}]c^2/16)$.) Eq. (239) becomes

$\partial^2\phi_{G2}/\partial r^2 + (2/r)\partial\phi_{G2}/\partial r = -4K_{G5}\rho_{GS}R^3/3\varepsilon_0 r^2$ (278)

The solution to Eq. (278) is $\phi_{G2}=-4K_{G5}\rho_{GS}R^3\ln(r)/3\varepsilon_0$. To see this, $\partial\phi_{G2}/\partial r=-4K_{G5}\rho_{GS}R^3/3\varepsilon_0 r$ so $(2/r)\partial\phi_{G2}/\partial r = -8K_{G5}\rho_{GS}R^3/3\varepsilon_0 r^2$, and $\partial^2\phi_{G2}/\partial r^2 = 4K_{G5}\rho_{GS}R^3/3\varepsilon_0 r^2$ so $\partial^2\phi_{G2}/\partial r^2 + (2/r)\partial\phi_{G2}/\partial r = -4K_{G5}\rho_{GS}R^3/3\varepsilon_0 r^2$. Eq. (277) informs that $\mathbf{P_{GL2}} = -\varepsilon_0\nabla\phi_{G2}/2\rho_0 = (4K_{G5}\rho_{GS}R^3/6\rho_0 r)\hat{\mathbf{r}}$. Then, Eq. (262) gives the mass density from P_{GL2} as $\delta\rho_{M12} = K_{G3}P_{GL2} = 4K_{G3}K_{G5}\rho_{GS}R^3/6\rho_0 r$.

For the third solution, $\delta\rho_{M12}$ is the source mass in Eq. (239), and $\rho_{G2} = 3[K_{G1}-K_{G2}]\delta\rho_{M12}c^2\rho_0/2 = 3[K_{G1}-K_{G2}](4K_{G3}K_{G5}\rho_{GS}R^3/6\rho_0 r)c^2\rho_0/2 = (3K_{G3}[K_{G1}-K_{G2}]c^2/16)(K_{G5}4[4]^2\rho_{GS}R^3/12r) = ([4K_{G5}]^2\rho_{GS}R^3/3r)$ so Eq. (239) becomes

$\partial^2\phi_{G3}/\partial r^2 + (2/r)\partial\phi_{G3}/\partial r = -[4K_{G5}]^2\rho_{GS}R^3/3\varepsilon_0 r$ (279)

$\phi_{G3} = -[4K_{G5}]^2\rho_{GS}R^3 r/6\varepsilon_0$ solves Eq. (279) since $\partial\phi_{G3}/\partial r = -[4K_{G5}]^2\rho_{GS}R^3/6\varepsilon_0$ so $(2/r)\partial\phi_{G3}/\partial r = -[4K_{G5}]^2\rho_{GS}R^3/3\varepsilon_0 r$, and $\partial^2\phi_{G3}/\partial r^2 = 0$ so $\partial^2\phi_{G3}/\partial r^2 + (2/r)\partial\phi_{G3}/\partial r = -[4K_{G5}]^2\rho_{GS}R^3/3\varepsilon_0 r$. Eq. (277) informs that $\mathbf{P_{GL3}} = -\varepsilon_0\nabla\phi_{G3}/2\rho_0 = ([4K_{G5}]^2\rho_{GS}R^3/12\rho_0)\hat{\mathbf{r}}$. Then, Eq. (262) gives the mass density from P_{GL3} as $\delta\rho_{M13} = K_{G3}P_{GL3} = K_{G3}[4K_{G5}]^2\rho_{GS}R^3/12\rho_0$.

The fourth solution, has $\delta\rho_{M13}$ as the source mass in Eq. (239); $\rho_{G3} = 3[K_{G1}–K_{G2}]\delta\rho_{M13}c^2\rho_0/2 = 3[K_{G1}–K_{G2}](K_{G3}[4K_{G5}]^2\rho_{GS}R^3/12\rho_0)c^2\rho_0/2 = (3K_{G3}[K_{G1}–K_{G2}]c^2/16)(4K_{G5}{}^2[4]^3\rho_{GS}R^3/24) = [4K_{G5}]^3\rho_{GS}R^3/6$ so Eq. (239) becomes

$$\partial^2\phi_{G4}/\partial r^2 + (2/r)\partial\phi_{G4}/\partial r = –[4K_{G5}]^3\rho_{GS}R^3/6\varepsilon_0 \tag{280}$$

$\phi_{G4} = –[4K_{G5}]^3\rho_{GS}R^3r^2/36\varepsilon_0$ solves Eq. (280). To see this, $\partial\phi_{G4}/\partial r = –2[4K_{G5}]^3\rho_{GS}R^3r/36\varepsilon_0$ so $(2/r)\partial\phi_{G4}/\partial r = –4[4K_{G5}]^3\rho_{GS}R^3/36\varepsilon_0$, and $\partial^2\phi_{G4}/\partial r^2 = –2[4K_{G5}]^3\rho_{GS}R^3/36\varepsilon_0$ so $\partial^2\phi_{G4}/\partial r^2 + (2/r)\partial\phi_{G4}/\partial r = –6[4K_{G5}]^3\rho_{GS}R^3/36\varepsilon_0 = –[4K_{G5}]^3\rho_{GS}R^3/6\varepsilon_0$. Eq. (277) leads to $\mathbf{P_{GL4}} = –\varepsilon_0\nabla\phi_{G4}/2\rho_0 = [4K_{G5}]^3\rho_{GS}R^3\mathbf{r}/36\rho_0$. Then, Eq. (262) gives the mass density from P_{GL4} as $\delta\rho_{M14} = K_{G3}P_{GL4} = K_{G3}[4K_{G5}]^3\rho_{GS}R^3r/36\rho_0$. Collecting the masses:

$$\delta\rho_{M11} = K_{G3}P_{GL1} = K_{G3}\rho_{GS}R^3/6r^2\rho_0 \tag{281}$$

$$\delta\rho_{M12} = K_{G3}P_{GL2} = K_{G3}4K_{G5}\rho_{GS}R^3/6\rho_0 r = [4K_{G5}r]\delta\rho_{M11} \tag{282}$$

$$\delta\rho_{M13} = K_{G3}P_{GL3} = K_{G3}[4K_{G5}]^2\rho_{GS}R^3/12\rho_0 = [4K_{G5}r]^2\delta\rho_{M11}/2 \tag{283}$$

$$\delta\rho_{M14} = K_{G3}P_{GL4} = K_{G3}[4K_{G5}]^3\rho_{GS}R^3r/36\rho_0 = [4K_{G5}r]^3\delta\rho_{M11}/6 \tag{284}$$

We see that each term in our series is $\delta\rho_{M1N} = [(4K_{G5}r)^{N-1}/ (N-1)!]\delta\rho_{M11}$ for the first-field-mass density. We could allow N to range from N = 1 to infinity by continuing our process, and we would then arrive at the series form of an exponential. Were there no limiting mechanism, the total first-field-mass density would be $\delta\rho = \delta\rho_{M11}\exp(4K_{G5}r)$. However, there is no evidence for this. Rather, only the effects of the term $\delta\rho_{M11} = K_{G3}P_{GL1} = K_{G3}\rho_{GS}R^3/6r^2\rho_0$ are seen. This empirically indicates a truncation of the power series to just one term, $\delta\rho = \delta\rho_{M11}$.

Why do the higher order solutions of the power series ($\delta\rho_{M12}$, $\delta\rho_{M13}$, etc.) not manifest themselves in nature? The answer can be found by recalling the physics of the situation, where it is helpful to review each term one at a time. For the primary term, we see above that the mass equivalent of extrinsic-energy ρ_M (ρ_M is most often a mass itself) leads to a displacement of the aether of $\mathbf{P}_{GL1} = -\varepsilon_0\nabla\phi_{G1}/2\rho_0 = (\rho_{GS}R^3/6r^2\rho_0)\hat{\mathbf{r}}$. For the secondary term, we see above that the aetherial-expansion-mass $\delta\rho_{M11}$ leads to a displacement of the aether of $\mathbf{P}_{GL2} = -\varepsilon_0\nabla\phi_{G2}/2\rho_0 = (4K_{G5}\rho_{GS}R^3/6\rho_0 r)\hat{\mathbf{r}}$. For both the primary and secondary terms, the aetherial displacements go to zero at infinite r. But for the tertiary term, were it to be allowed by nature, the aetherial-expansion-mass $\delta\rho_{M12}$ would lead to a displacement of the aether of $\mathbf{P}_{GL3} = -\varepsilon_0\nabla\phi_{G3}/2\rho_0 = ([4K_{G5}]^2\rho_{GS}R^3/12\rho_0)\hat{\mathbf{r}}$, and $\mathbf{P}_{GL3}$ does not go to zero at infinite r; rather, it is constant. If there is some sort of boundary condition at very large r that does not allow aetherial expansion, then $\mathbf{P}_{GL3}$ will be forced to zero by this effect. To force $\mathbf{P}_{GL3} = 0$, the source term leading to $\mathbf{P}_{GL3}$, $\delta\rho_{M12}$, must be cancelled by the boundary condition as well, and the power series will truncate after the source term, $\delta\rho_{M11}$, and the power series for $\mathbf{P}_{GL}$ will truncate after the first two terms $\mathbf{P}_{GL1}$ and $\mathbf{P}_{GL2}$.

The physical condition just described can arise from neighboring galaxies. Each galaxy will push the positive-attached-aether outward from its center, and in between galaxies these effects will tend to cancel out, with P being forced to zero at some intervening point. This boundary condition will lead to the cancellation of $\mathbf{P}_{GL3}$ at such intervening points. Recall that $\mathbf{P}_{GL3}$ is a constant outward radial expansion. The boundary condition ensures that this constant is zero by providing fixed points for attached-aether at the boundary. And with $\mathbf{P}_{GL3}$ now zero everywhere, there is no source mass for the higher order terms.

But what of the universe as a whole? Here there must be some sort of limiting condition on the aether at a very large distance, or it could result from an infinite aether that resists expansion due to its enormity, or perhaps galaxies also exist forever into the distance. We can only speculate on what reality exists at distances beyond our observational ability. In any case, the physical effect is that $\delta\rho_{M12}$ attempts to achieve a uniform, positive, outward, constant displacement term of the positive-attached-aether, but a reaction force from the boundary condition results in an additional condition that cancels $\delta\rho_{M12}$.

Since we can apply the above analysis to any ponderable mass, we arrive at the conclusion that we must neglect any first-field-mass contribution to $\mathbf{P_G}$ when using Eq. (262), $\delta\rho_{M1} = K_{G3}P_G$. (We must only use the $\mathbf{P_G}$ contribution originating from normal mass and second-field-mass when calculating $\delta\rho_{M1}$.)

D.12 – Intergalactic Forces.

D.12.1. Dark Matter Between Galaxies. As just mentioned in section D.11, each galaxy will push the positive-attached-aether outward from its center, and in between galaxies these effects will tend to cancel out, with $\mathbf{P_G}$ being forced to zero at some intervening point. Since it is the displacement of the aether by an amount $\mathbf{P_G}$ that leads to the field-masses, this tells us that there will be little first-field-mass in between galaxies. And since we have identified the first-field-mass with dark matter, we expect little dark matter in between galaxies, which is again consistent with observation.

Eq. (262) gives $\delta\rho_{M1} = K_{G3}P_G$ as a positive field-mass-density, telling us where any dark matter is located; it is located where P_G is. Dark matter is produced by the aetherial displacement $\mathbf{P_G}$ which is most prevalent surrounding the large masses that typically exist in the centers of galaxies. In such galaxies, a large central mass M dominates gravitational interactions and for individual stars of mass m within those galaxies, the condition m << M holds, and Eq. (275) applies along with its dark matter term. But when the fields of several such large central masses become relevant (such as between galaxies) then $\mathbf{P_G}$ is no longer solely determined by one large central mass, and the condition m << M for the Eq. (275) two-body interaction no longer applies.

So how do we calculate the forces between galaxies? In this case, $\mathbf{P_G}$ should be calculated at every point in the relevant spatial volume based upon the non-dark matter present, and then the dark matter should be calculated from that $\mathbf{P_G}$ field, and those dark matter effects should be included in the analysis. (The relevant spatial volume will include the galaxies we are calculating the forces for as well as any other matter near enough such that its gravitational effects are non-negligible.) At this point a review of the physics may be helpful. Any large galactic spherical central mass of radius R has a first-component of P_G originating from normal matter which decreases as $1/r^2$ for $r > R$. This first-component of P_G has a first-field-mass called dark matter, and the total dark mass within the radius r increases linearly with r. (See Eq. (268).) That dark matter gives rise to a second-component of P_G that decreases as $1/r$ (for $r > R$). That second-component of P_G would generate a further dark mass leading to a constant third-component of P_G, but boundary conditions preclude that from happening. (A constant P_G component would be a constant outward radial aetherial displacement, and this is what is not allowed by the boundary conditions.) As a result of this physics, dark matter is only created from the first-component of P_G, however both the first and second components of P_G are relevant to the inter-galactic forces, and there is no third-component of P_G due to boundary condition suppression of such a term.

D.12.2. Evaluation of the Second-Field-Mass Effects. So far we have neglected the second-field-mass when discussing distant stellar orbits and intra-galactic forces. It is of interest to verify that we can neglect this term. In section D.10.3 we look at the case where $r = 80$ kpc $= 2.469\text{x}10^{21}$ m and $M = 1.279\text{x}10^{41}$ kg. With those values we find that the first-field-mass term gives $4G_NK_{G5}M/r = 1.383\text{x}10^{-11}$ m/s^2 and that the Newtonian term gives $G_NM/r^2 = 1.400\text{x}10^{-12}$ m/s^2. The second-field-mass term from Eq. (275) (again setting $\gamma_M = 1$ and dividing by $\gamma_m m$) is $5G_NK_{G6}M^2/r^3$. Below in Eq. (293) we find $K_{G6} = 8.9167\text{x}10^{-28}$ m/kg, and hence $5G_NK_{G6}M^2/r^3 = 5(6.6743\text{x}10^{-11}$ m^3/kg-

s^2) $(8.9167 \times 10^{-28}$ m/kg$)(1.279 \times 10^{41}$ kg$)^2/(2.469 \times 10^{21}$ m$)^3 = 3.234 \times 10^{-19}$ m/s^2. We see that our neglect of the second-field-mass is entirely appropriate for distant stellar orbits within galaxies, and since the second-field-mass effect falls off with radius faster than both the Newtonian term and the first-field-mass term it is appropriate for us to ignore the second-field-mass when considering gravitational effects between galaxies as well.

D.12.3. Dark Energy. Section D.11 has just speculated that a boundary condition counteracts the effect of what would otherwise be a continuous, outward, uniform radial $\mathbf{P_G}$ displacement. This counteraction negates what would otherwise be an attractive dark mass effect. By taking this one step further, we can speculate that the boundary condition may apply a net negative effect. Such a speculation leads to an explanation of dark energy.

D.12.4. Far-Distant Phenomena. In far-distant reaches of the universe we cannot be assured that the aether will remain in a state nominally at rest with respect to its velocity near the earth. It is entirely possible that the aether is expanding or contracting, and at great distances these effects could build up significantly. Such effects may affect what defines, physically, the zero value of $\mathbf{P_G}$ in such regions. (Recall that the zero value of $\mathbf{P_G}$ is its nominal position.) Boundary conditions in our analysis of some particular region of the aether would also be affected by aetherial expansion or contraction. Motion of the aether could certainly affect the motion of physical bodies such as galaxies. Also, at extremely large distances we cannot be completely certain that our observations remain sound, since we rely on images from light, and light itself might be affected during its lengthy travel over the billions of light-years it takes to reach us, especially in regions where the aether itself is moving. The present paper is predominantly concerned with physical phenomena that occur closer to home such as in our own galaxy and those nearby. In such nearby regions, our derivations are consistent with an aether nominally at rest. The physics of far-distant realms should be the topic of future speculation, consideration and study.

D.13 – Perihelion Advances and an Estimate of K_{G6}

Recall now Eq. (275), $\mathbf{F_{GOUT}} = -G_N\gamma_m m(M_{EFF}/r^2 + 4K_{G5}\gamma_M M/r + 5K_{G6}\gamma_M{}^2M^2/r^3)\hat{\mathbf{r}}$. In sections D.10 and D.11 we have only dealt with the first two terms, due to the dominance of those terms term at relatively large r. At smaller r, which includes distances involving the orbits of the planets, the first term of Eq. (275) is dominant. However, we will now include the third term in our analysis. It will be seen that doing so allows for an understanding of the advance of the perihelions as well as an estimate of the constant K_{G6}.

D.13.1. Experimental Ephemerides Data. The ephemerides of the planets have been studied in detail. The results yielded by those studies are arrived at by a combination of measurements, fittings and calculations. An excellent set of results are presented by Pitjeva [11], and those results appear in the second column of table 1 below. It is important to note that the ephemerides results obtained by Pitjeva assume that general relativity is correct and use fittings for over 250 parameters to arrive at the results shown in table 1.

D.13.2. Analytic Approach to Calculating the Perihelion Advance. Price and Rush[12] have determined a useful method to calculate the angle Ψ swept out by the radius vector between two consecutive apsides of planetary orbits:

$$\Psi = \pi\, [3 + a\, \{\Phi'(a)/\Phi(a)\}]^{-1/2} \tag{285}$$

In Eq. (285) a is the radius of a circular orbit that most closely coincides with the planet being evaluated and $\Phi(a)$ is the total force on that planet. Referring to Eq. (275) $\Phi(a) = -G_N\gamma_m m(M_{EFF}/r^2 + 4K_{G5}\gamma_M M/r + 5K_{G6}\gamma_M{}^2M^2/r^3)$, or

$$\Phi(a) = -K_{P2}/a^2 - K_{P3}/a^3 - K_{P1}/a \tag{286}$$

In Eq. (286) we assign constants to aid us in our perihelion advance calculations:

$$K_{P1} = 4G_N\gamma_m mK_{G5}\gamma_M M \tag{287}$$

$$K_{P2} = G_N\gamma_m mM_{EFF} \tag{288}$$

$$K_{P3} = 5G_N\gamma_m mK_{G6}\gamma_M{}^2M^2 \tag{289}$$

From Eq. (286) we get $\Phi'(a) = 2K_{P2}/a^3 + 3K_{P3}/a^4 + K_{P1}/a^2$ and $\Phi'(a)/\Phi(a) = -[2K_{P2}/a^3 + 3K_{P3}/a^4 + K_{P1}/a^2]/[K_{P2}/a^2+K_{P3}/a^3+K_{P1}/a] = -[2K_{P2}/a^3 + 3K_{P3}/a^4 + K_{P1}/a^2]/[(K_{P2}/a^2)(1+K_{P3}/(K_{P2}a)+K_{P1}a/K_{P2}]$ $\approx -\{[2K_{P2}/a^3 + 3K_{P3}/a^4 + K_{P1}/a^2]/(K_{P2}/a^2)\}(1 - K_{P3}/(K_{P2}a) - K_{P1}a/K_{P2}) = -[2/a + 3K_{P3}/K_{P2}a^2 + K_{P1}/K_{P2}](1 - K_{P3}/(K_{P2}a) - K_{P1}a/K_{P2})$ which to first order in the small quantities $K_{P3}/(K_{P2}a)$ and $K_{P1}a/K_{P2}$ is $\Phi'(a)/\Phi(a) = -2/a - K_{P3}/K_{P2}a^2 + K_{P1}/K_{P2}$ and hence Eq. (285) becomes

$$\Psi = \pi[3 + a\,\{\Phi'(a)/\Phi(a)\}]^{-1/2} = \pi[3 - 2 - K_{P3}/K_{P2}a + K_{P1}a/K_{P2}]^{-1/2}$$

$$\approx \pi[1 + K_{P3}/2K_{P2}a - K_{P1}a/2K_{P2}] = \pi[1 + 5K_{G6}M/2a - 2K_{G5}a] \tag{290}$$

The final equation of Eqs. (290) makes use of Eqs. (287), (288) and (289), and also uses the approximations $M_{EFF} \approx M$, $\gamma_m \approx 1$, and $\gamma_M \approx 1$. The advance of the perihelion is that portion in excess of π, and the rate of advance of the perihelion is that excess divided by the time taken to go through the half orbit, which is T/2, where T is the period of the orbit. So the rate of perihelion advance is

$$d\Psi/dt = (\Psi - \pi)/(T/2) = 2\pi[5K_{G6}M/2a - 2K_{G5}a]/T \tag{291}$$

With Eq. (276) giving $K_{G5} \approx 10^{-21}$ m^{-1}, and with known values for M, a, and T, we can now use the known "anomalous" perihelion advance for one planet to solve for K_{G6}, and then use Eq. (291)

to determine the anomalous perihelion advances for the remaining planets. Manipulating Eq. (291), $(1/2)Td\Psi/dt = 5\pi K_{G6}M/2a - 2\pi K_{G5}a$, leading to

$$K_{G6} = [(1/2)Td\Psi/dt + 2\pi K_{G5}a](2a/5\pi M) \tag{292}$$

An estimate for the mass of the sun is M = 1.988e30 kg. Here, the data for the orbital radius a and the orbital period T will come from NASA's planetary fact sheet [13], and the data for the anomalous perihelion advance from Pitjeva[11]. For Mars, the period is T = 687 days, the average radius is a = 227.9e6 km and its $d\Psi/dt$ is 1.343 arc-seconds per century. There are 36,524 days per century, 2π radians per 360°, and 3600 arc-seconds per 1°, giving the conversion factor 1 arc-sec/century = 1 arc-sec/century x 2π radians/360 deg x 1 deg/3600 arc-sec x 1 century/36,524 days = 1.327×10^{-10} radians/day. Therefore, for Mars, $(1/2)Td\Psi/dt = (1/2) \times 687$ days $\times 1.343 \times 1.327 \times 10^{-10}$ radians/day $= 6.123\times10^{-8}$, $2\pi K_{G5}a = 2\pi \times 10^{-21}\ m^{-1} \times 227.9e9\ m = 1.432\times10^{-9}$ and $2a/5\pi M = (2\times227.9\times10^{9}\ m)/(5\pi\times1.998\times10^{30}\ kg)=1.459\times10^{-20}$ m/kg. Hence $K_{G6}=(6.123\times10^{-8} +1.432\times10^{-9})\ 1.459\times10^{-20}$ m/kg $= 9.145\times10^{-28}$ m/kg. This allows us to calculate the anomalous perihelion advances for all nine planets using Eq. (291) and the results of those calculations appear in Table 1. Despite the simplicity of the Price and Rush treatment, it is seen that it leads to a rough agreement with Pitjeva's results when Eq. (275) is used as the gravitational force law.

Of course it is to be expected that the results of Price and Rush will not be exact. Critically, Price and Rush assume that the deviation from a circular orbit is small, but for Mercury the orbital eccentricity (the ratio of the aphelion to the average radius minus one) is 0.205, and for Mars it is 0.094. Therefore, a more exact numerical approach will now be described.

D.13.3. First Numerical Integration Approach. Numerical integration allows for a more exact calculation of the anomalous perihelion advance than the simple treatment of Price and Rush. In a first numerical integration, the expression F = ma is used to calculate the advance of the perihelion for each of the nine planets. The routine: 1) calculates the force on the planet as given by Eq. (275) while setting $M_{EFF} \approx M$ and $\gamma_M \approx 1$; 2) divides the force by the planet's mass to obtain the acceleration; 3) multiplies the acceleration over a small time to find the change in velocity; 4) adds the change in velocity to find the new velocity; 5) multiplies the velocity over the same small time step to obtain the change in position; and 6) adds that change in position to find the new position. This is done for both of two cartesian coordinates. Then, this process is repeated. The process starts at the perihelion and ends at the aphelion. The aphelion is determined by the position the planet has when the radius first begins to decrease. In order to enable more rapid convergence, an estimate is made of the position of the planet at the end of the interval and that estimate is used to determine an estimated acceleration there. The acceleration used to arrive at the end-of-interval velocities is the average of the acceleration at the start and end of the interval. Likewise, the velocity used to determine the end-of-interval positions is the average of the velocities at the start and end of the interval.

Table 1. Competing Calculations of "Anomalous" Perihelion Advances

Planet	Radius a (10^6 km)	Period T (days)	Pitjeva	Eq. (291) (Price/Rush)	Eq. (275) W/O K_{G5}	Eq. (275) With K_{G5}
Mercury	57.9	88.0	42.976 +/- 0.005	42.171	42.977	42.977
Venus	108.2	224.7	8.644 +/- 0.033	8.805	8.623	8.589
Earth	149.6	365.2	3.846 +/- 0.007	3.900	3.837	3.804
Mars	227.9	687.0	1.343 +/- 0.007	1.343	1.351	1.322
Jupiter	778.6	4,331	0.067 +/- 0.093	0.0468	0.0622	0.0453
Saturn	1,433.5	10,747	-0.010 +/- 0.015	0.0013	0.0135	0.00104
Uranus	2,872.5	30,589	-3.89 +/- 3.90	-0.0064	0.00238	-0.00646
Neptune	4,495.1	59,800	-4.44 +/- 5.40	-0.0063	7.75e-4	-0.00630
Pluto	5,906.4	90,560	2.84 +/- 4.51	-0.0058	4.17e-4	-0.00565

The calculation routine is done twice. In the first pass through the calculation the predominant K_{P2}/r^2 gravitation term is the only term evaluated (K_{P1} and K_{P3} are set to zero). Since it is known analytically that a pure K_{P2}/r^2 force will lead to zero perihelion advance, the advance evaluated from the first pass is the numerical error during that pass. Next, the effect of the full force including all three terms (K_{P1}/r, K_{P2}/r^2 and K_{P3}/r^3) is evaluated in a second pass through the calculation. The advance calculated by the first pass (the numerical error) is then subtracted from the second pass to arrive at the final calculated perihelion advance. Results of the numerical integration are shown in the last two columns of Table 1, with one column showing the results when we include the effects of K_{G5} and the other column showing the results when K_{G5} is set to zero. (The last two columns both include the effects of the third term; K_{G6} is nonzero for both. See section I.3.6 in Appendix I for comments on the calculation.)

D.13.4. Second Numerical Integration Approach. The first numerical approach is a numerical integration of $\mathbf{F} = m\mathbf{a}$, but we know this expression is not exact. The exact expression is $\mathbf{F} = d\mathbf{p}/dt$. At velocities much less than the speed of light, such as planetary velocities, the two expressions differ by a small amount. For a force transverse to the planet's velocity the exact expression becomes $\mathbf{F} = \gamma m\mathbf{a}$, while for the force parallel to the planet's velocity $\mathbf{F} = \gamma^3 m\mathbf{a}$, where $\gamma = (1 - v^2/c^2)^{-1/2}$. To see this, with $\mathbf{p} = \gamma m\mathbf{v}$, transversely γ doesn't change and we get $d\mathbf{p}/dt|_T = \gamma m\mathbf{a}$. However longitudinally γ does change and $d\mathbf{p}/dt|_L = d\{\gamma m\mathbf{v}\}/dt = d\{(1-v^2/c^2)^{-1/2}m\mathbf{v}\}/dt = -(1/2)(1-v^2/c^2)^{-3/2}(-2v/c^2)(dv/dt)(m\mathbf{v}) + \gamma m(d\mathbf{v}/dt) = \gamma^3(v^2/c^2)m\mathbf{a} + \gamma m\mathbf{a} = \gamma m\mathbf{a}[1+\gamma^2(v^2/c^2)] = \gamma m\mathbf{a}[1 + \{(v^2/c^2)/(1-v^2/c^2)\}] = \gamma m\mathbf{a}[\{(1-v^2/c^2)/(1-v^2/c^2)\}+\{(v^2/c^2)/(1-v^2/c^2)\}] = \gamma m\mathbf{a}[\{(1-v^2/c^2+v^2/c^2)/(1-v^2/c^2)\}] = \gamma m\mathbf{a}[\{1/(1-v^2/c^2)\}] = \gamma^3 m\mathbf{a}$.

To do the second numerical integration we apply the force to a change in momentum and we must therefore extract the velocity from the momentum. This extraction begins with the equation for the momentum, $\gamma mv = (p_x^2 + p_y^2)^{1/2} = \beta\gamma mc$. Dividing by m, $[(p_x/m)^2 + (p_y/m)^2]^{1/2} = \beta\gamma c$. Squaring, $(p_x/m)^2 + (p_y/m)^2 = \beta^2\gamma^2c^2$, or $\beta^2\gamma^2 = (p_x/mc)^2 + (p_y/mc)^2$. Now, $\beta^2\gamma^2 = \beta^2/(1-\beta^2) = (p_x/mc)^2 + (p_y/mc)^2$, or $(1-\beta^2)[(p_x/mc)^2 + (p_y/mc)^2] = \beta^2$. Rearranging, $(p_x/mc)^2 + (p_y/mc)^2 = \beta^2 + \beta^2[(p_x/mc)^2 + (p_y/mc)^2] = \beta^2[1 + (p_x/mc)^2 + (p_y/mc)^2]$, or $\beta^2 = [(p_x/mc)^2 + (p_y/mc)^2]/[1 + (p_x/mc)^2 + (p_y/mc)^2]$

$= [(p_x/m)^2 + (p_y/m)^2]/[c^2 + (p_x/m)^2 + (p_y/m)^2]$ and hence $\gamma = (1-\beta^2)^{-1/2} = \{1 - [(p_x/m)^2 + (p_y/m)^2]/[c^2 + (p_x/m)^2 + (p_y/m)^2]\}^{-1/2}$.

The fastest moving planet, Mercury, has an orbital velocity near 1.6×10^{-4} c. Also, were γ constant, the effect of γ would just be an effective increase in mass and it would not affect the perihelion advance. What can affect the perihelion advance is a change in γ, and for Mercury the change in velocity is about an order of magnitude smaller than the velocity itself. With γ involving v^2/c^2, it is expected then that the change in force due to the effects of γ will be quite small. On the other hand, the advance of the perihelion is itself a very small effect, and so it is important to evaluate it precisely.

The numerical algorithm for the more exact case calculates the momentum increment over an interval by multiplying the average force over the interval by the time step. Next, the momentum increment is added to the momentum at the start of the interval to obtain the momentum at the end of the interval. Then the velocity at the end of the interval is found from the expression $\mathbf{p} = \gamma m\mathbf{v}$. Once $\mathbf{v}$ is found, the rest of the numerical integration routine is the same as the one using $\mathbf{F} = m\mathbf{a}$. The second numerical integration again sets $M_{EFF} \approx M$ and $\gamma_M \approx 1$ when evaluating the force. The results of the second numerical integration agree with the first numerical integration through the sixth decimal place. Hence the results shown in Table 1 are the same whether we use $\mathbf{F} = d\mathbf{p}/dt$ or $\mathbf{F} = m\mathbf{a}$ in our integration. (See section I.3.6 in Appendix I for comments on the calculation.)

D.13.5. Summary of Results and Comparison to Experiment. Table 1 presents various calculations for the "anomalous" perihelion advance of the planets, showing results from Pitjeva and the three analyses described here. The Price/Rush column shows results determined from evaluating Eq. (291) where the parameter $K_{G6} = 9.144784 \times 10^{-28}$ m/kg is set to make the Mars perihelion advance match the Pitjeva value. For the second-to-last column, the numerical integration of Eq. (275) was done assuming $K_{G5} = 0$ and a fit of $K_{G6} = 8.904 \times 10^{-28}$ m/kg to get the results shown. It is seen that numerical integration of Eq. (275) is in very good agreement with the Pitjeva results for that case. For the last column, the numerical integration of Eq. (275) was done assuming the value of K_{G5} as determined by Eq. (276), $K_{G5} = 10^{-21}$ m^{-1} and K_{G6} was set to the value

$$K_{G6} = 8.9167 \times 10^{-28} \text{ m/kg} \tag{293}$$

It is seen that numerical integration of Eq. (275) when K_{G5} is included results are now six standard deviations away from the quoted Pitjeva result for Earth, 1.7 standard deviations away for Venus and three standard deviations away for Mars. If this was the complete analysis, it would be somewhat counter-indicative of Eq. (275). However, there is an important caveat to the Pitjeva results. Notably, the work reported by Pitjeva assumes general relativity is correct and then uses that assumption to fit over 250 parameters in order to arrive at the published results. Also notably, the work reported by Pitjeva includes many effects not included in the treatment here, as Pitjeva includes the oblateness of the sun, the relativistic effect of Jupiter, and the effects of the planets upon one another as well as the effects of asteroids, moons, and Trans-Neptunian-Objects. Indeed, these latter effects are modeled with some of the fitted parameters. Also. the total perihelion advances are quite different than those presented in Table 1, as the effects of the planets and other objects is considerably larger than the "anomalous" advance now credited to relativity. When calculating the ephemerides it is important to have estimates for all these other effects, and it is hoped that Eq. (275) will soon be subjected to such an analysis. Since Table 1 shows that Eq. (275) yields results that are quite close to Pitjeva, a refitting of the over 250 fitted parameters will quite possibly lead to Eq. (275) agreeing with experiment just as well as general relativity does.

D.14 – Energy Flow Effects on the Analysis-cube

The effects of energy flow will now be considered. We now propose:

The Extrinsic-Energy Flow Speculation. When extrinsic-energy flows through the quantum, the tension in the positive (negative) attached-aether is reduced (increased) in the directions perpendicular to the flow with the magnitude of reduction proportional to the flow.

Here we have proposed an Extrinsic-Energy Flow Speculation rather than an Extrinsic-Energy Flow Law, because we simply do not have enough experimental guidance to grant this speculation the status of a Law. Below we will show that this speculation leads to the observations concerning light bending, and we will present two candidates for the specifics of our speculation. But other candidates may exist as well, and we could have also added the quantum-force into our speculation. Establishment of an Extrinsic-Energy Flow Law is an important topic for future work.

The Extrinsic-Energy Flow Speculation stipulates that energy flow modifies the tension, but does so only in the directions perpendicular to the flow. Without loss of generality we can assume here in section D.14 that the velocity of the energy is in the Z direction. With this coordinate choice, Eqs. (179) and (180) now become the equations

$$F_{TPZ} = K_{TP}X_Q = K_{T0}(1 - K_{G1}\rho_E)X_Q \tag{294}$$

$$F_{TNZ} = K_{TN}X_Q = K_{T0}(1 - K_{G2}\rho_E)X_Q \tag{295}$$

$$F_{TPX} = F_{TPY} = K_{T0}(1 - K_{G1}\rho_E)X_Q - K_{T0}K_{F4}\rho_{FE}X_Q \tag{296}$$

$$F_{TNX} = F_{TNY} = K_{T0}(1 - K_{G2}\rho_E)X_Q + K_{T0}K_{F4}\rho_{FE}X_Q \tag{297}$$

In Eqs. (296) and (297) K_{F4} is an arbitrary proportionality constant, ρ_{FE} is the flowing extrinsic-energy density, and the remainder of the symbols are defined in section D.1. The extrinsic-energy-flow induced-tension-force (which changes the total-tension) is

$$F_{TPXF} = F_{TPYF} = -F_{TNXF} = -F_{TNYF} = -K_{T0}K_{F4}\rho_{FE}X_Q \tag{298}$$

Of course, $F_{TPZF} = F_{TNZF} = 0$. The extrinsic-energy-flow induced tension-energy is found by integrating Eq. (298) in the same way the ρ_E induced tension-energy was found in section D.1 resulting in

$$E_{TPXF} = E_{TPYF} = -E_{TNXF} = -E_{TNYF} = -(1/2)K_{T0}K_{F4}\rho_{FE}X_Q^2 \quad (299)$$

Of course, $E_{TPZF} = E_{TNZF} = 0$.

The total energy of the positive-analysis-cube in the X dimension is $E_P = E_{QP} + E_{TP} + E_{TPXF}$, where $E_{TP} = (1/2)K_{T0}(1 - K_{G1}\rho_E)X_Q^2$ and $E_{QP} = K_{Q0}(1 - K_{G2}\rho_E)/X_Q^2$ are given in Eqs. (187) and (189), respectively. The extremum of E_P can be found by setting $dE_P/dX_Q = 0 = K_{T0}(1 - K_{G1}\rho_E)X_Q - 2K_{Q0}(1 - K_{G2}\rho_E)/X_Q^3 - K_{T0}K_{F4}\rho_{FE}X_Q$, or $K_{T0}(1 - K_{G1}\rho_E - K_{F4}\rho_{FE})X_Q = 2K_{Q0}(1 - K_{G2}\rho_E)/X_Q^3$ at the extremum. Defining X_3 as the value obtained at the extremum, $X_3^4 = 2K_{Q0}(1 - K_{G2}\rho_E)/K_{T0}(1 - K_{G1}\rho_E - K_{F4}\rho_{FE})$, or,

$$X_3 = [2K_{Q0}(1 - K_{G2}\rho_E)/K_{T0}(1 - K_{G1}\rho_E - K_{F4}\rho_{FE})]^{1/4} \quad (300)$$

To see whether the extremum is a minimum or a maximum, we take the second derivative of E_P:

$$(d^2E_P/dX_Q^2)_{X3} = [K_{T0}(1 - K_{G1}\rho_E - K_{F4}\rho_{FE}) + 6K_{Q0}(1 - K_{G2}\rho_E)/X_Q^4]_{X3}$$

$$= K_{T0}(1 - K_{G1}\rho_E - K_{F4}\rho_{FE}) + 6K_{Q0}(1 - K_{G2}\rho_E)/[2K_{Q0}(1 - K_{G2}\rho_E)/K_{T0}(1 - K_{G1}\rho_E - K_{F4}\rho_{FE})]$$

$$= 4K_{T0}(1 - K_{G1}\rho_E - K_{F4}\rho_{FE}) \quad (301)$$

Since $1 > K_{G1}\rho_E + K_{F4}\rho_{FE}$ (or else the cube will not be bound) Eq. (301) is always positive, indicating an energy minimum. A similar equation follows for the Y dimension. To achieve this energy minimum, the x and y sides of the attached-aether cube will have a length of X_3 as given by Eq. (300). The z side of the cube, unaffected by the energy flow, will have the length X_1 as given by Eq. (192), $X_1 \approx X_0(1 + [K_{G1}-K_{G2}]\rho_E/4)$.

Comparing Eq. (300) to X_1 we see that the presence of mass and energy flow increases the size of the positive-analysis-cube. Assuming the change in size $\delta X_3 = X_3 - X_0$ is small compared to X_0

we form $X_3 = [2K_{Q0}(1 - K_{G2}\rho_E)/K_{T0}(1 - K_{G1}\rho_E - K_{F4}\rho_{FE})]^{1/4} = [2K_{Q0}/K_{T0}]^{1/4}[(1 - K_{G2}\rho_E)/(1 - K_{G1}\rho_E - K_{F4}\rho_{FE})]^{1/4} = X_0[1 - K_{G2}\rho_E]^{1/4}[1 - K_{G1}\rho_E - K_{F4}\rho_{FE}]^{-1/4} \approx X_0[1 + K_{G1}\rho_E/4 - K_{G2}\rho_E/4 + K_{F4}\rho_{FE}/4]$, and since the energy flow only affects the dimension perpendicular to the velocity we obtain

$$\delta X_{3X}/X_0 = \delta X_{3Y}/X_0 = (X_3 - X_0)/X_0 \approx [K_{G1}-K_{G2}]\rho_E/4 + K_{F4}\rho_{FE}/4 \quad (302)$$

$$\delta X_{3Z}/X_0 = (X_1 - X_0)/X_0 \approx [K_{G1}-K_{G2}]\rho_E/4 \quad (303)$$

The total energy of the negative-analysis-cube in the X dimension is $E_N = E_{QN} + E_{TN} + E_{TNXF}$, where $E_{TN} = (1/2)K_{T0}(1 - K_{G2}\rho_E)X_Q^2$ and $E_{QN} = K_{Q0}(1 - K_{G1}\rho_E)/X_Q^2$ are given in Eqs. (188) and (190), respectively and $E_{TNXF} = (1/2)K_{T0}K_{F4}\rho_{FE}X_Q^2$ is given in Eq. (299). The extremum of E_N can be found by setting $dE_N/dX_Q = 0 = K_{T0}(1 - K_{G2}\rho_E)X_Q - 2K_{Q0}(1 - K_{G1}\rho_E)/X_Q^3 + K_{T0}K_{F4}\rho_{FE}X_Q$, or $K_{T0}(1 - K_{G2}\rho_E + K_{F4}\rho_{FE})X_Q = 2K_{Q0}(1 - K_{G1}\rho_E)/X_Q^3$ at the extremum. Defining X_4 as the value obtained at the extremum, $X_4^4 = 2K_{Q0}(1 - K_{G1}\rho_E)/K_{T0}(1 - K_{G2}\rho_E + K_{F4}\rho_{FE})$, or,

$$X_4 = [2K_{Q0}(1 - K_{G1}\rho_E)/K_{T0}(1 - K_{G2}\rho_E + K_{F4}\rho_{FE})]^{1/4} \quad (304)$$

To see whether the extremum is a minimum or a maximum, we take the second derivative of E_N:

$$(d^2E_N/dX_Q^2)_{X4} = [K_{T0}(1 - K_{G2}\rho_E + K_{F4}\rho_{FE}) + 6K_{Q0}(1 - K_{G1}\rho_E)/X_Q^4]_{X4}$$

$$= K_{T0}(1 - K_{G2}\rho_E + K_{F4}\rho_{FE}) + 6K_{Q0}(1 - K_{G1}\rho_E)/[2K_{Q0}(1 - K_{G1}\rho_E)/K_{T0}(1 - K_{G2}\rho_E + K_{F4}\rho_{FE})]$$

$$= 4K_{T0}(1 - K_{G2}\rho_E + K_{F4}\rho_{FE}) \quad (305)$$

With $1 > K_{G2}\rho_E - K_{F4}\rho_{FE}$ Eq. (305) is always positive, indicating an energy minimum. A similar equation follows for the Y dimension. To achieve this energy minimum, the x and y sides of the attached-aether cube will have a length X_4 given by Eq. (304). The z side of the cube, unaffected by the energy flow, will have the length X_2 as given by Eq. (196), $X_2 \approx X_0(1 + [K_{G2}-K_{G1}]\rho_E/4)$.

Comparing Eq. (304) to X_2 we see that the presence of mass and energy flow decreases the size of the negative-analysis-cube. Assuming the change in size $\delta X_4 = X_4 - X_0$ is small compared to X_0 we form $X_4 = [2K_{Q0}(1 - K_{G1}\rho_E)/K_{T0}(1 - K_{G2}\rho_E + K_{F4}\rho_{FE})]^{1/4} = [2K_{Q0}/K_{T0}]^{1/4}[(1 - K_{G1}\rho_E)/(1 - K_{G2}\rho_E + K_{F4}\rho_{FE})]^{1/4} = X_0[1 - K_{G1}\rho_E]^{1/4}[1 - K_{G2}\rho_E + K_{F4}\rho_{FE}]^{-1/4} \approx X_0[1 + K_{G2}\rho_E/4 - K_{G1}\rho_E/4 - K_{F4}\rho_{FE}/4]$, and since the energy flow only affects the dimension perpendicular to the velocity we obtain

$$\delta X_{4X}/X_0 = \delta X_{4Y}/X_0 = (X_4 - X_0)/X_0 \approx [K_{G2} - K_{G1}]\rho_E/4 - K_{F4}\rho_{FE}/4 \tag{306}$$

$$\delta X_{4Z}/X_0 = (X_2 - X_0)/X_0 \approx [K_{G2} - K_{G1}]\rho_E/4 = -\delta X_{3Z}/X_0 \tag{307}$$

D.15 – Energy Flow Effect on the Gravitational Force

Consider a region where there is an ambient attached-aether displacement $\mathbf{P}_{AG}$ (a gravitational field) not equal to zero. Section D.4 considers the case where there is a stationary mass in such a region, and now we will consider the case of extrinsic-energy-flow in a gravitational field. Without loss of generality we can assume that our sphere is centered at the origin, and that its motion is in the Z direction. For analysis we will again divide the sphere into slices of thickness ΔY centered at $y = Y$, and then further divide those slices into strips with thickness ΔZ centered at $z = Z$, as shown in Figures 15 and 16.

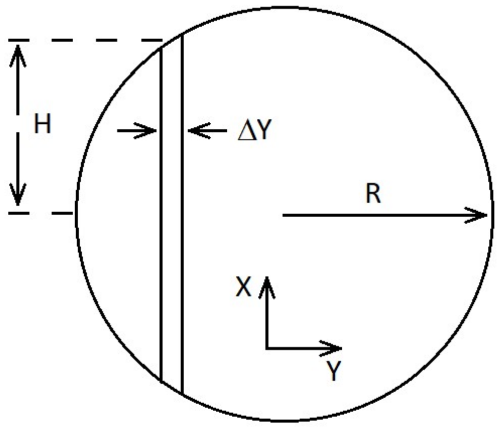

Figure 15. A sphere of attached-aether containing flowing extrinsic-energy, and a slice of width ΔY.

Figure 15 shows a view of a sphere of flowing extrinsic-energy with radius R. Examining a slice of thickness ΔY we can see that the half-height of that slice will be $H = (R^2 - Y^2)^{1/2}$.

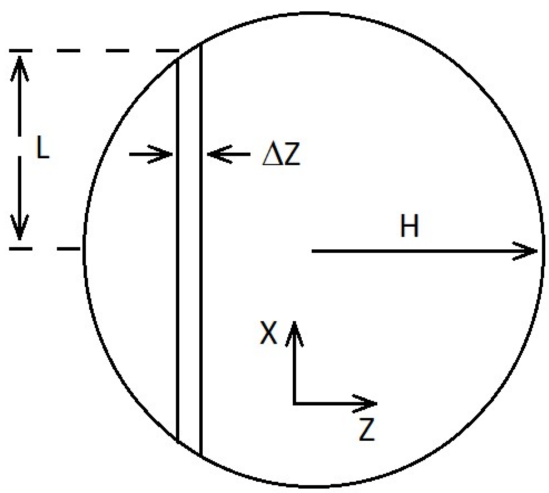

Figure 16. A slice of attached-aether containing flowing extrinsic-energy, and a strip of width ΔZ.

Figure 16 shows a view of the slice of attached-aether containing flowing extrinsic-energy. The slice has the half-height, H, that we found in Fig. 15. Examining the strip of thickness ΔZ centered at Z we can see that the half-length of that strip will be $L = (H^2 - Z^2)^{1/2}$. The strip shown in Fig. 16 has dimensions of ΔY by ΔZ by L.

We now observe that the sphere of flowing extrinsic-energy will cause its own longitudinal attached-aether displacement. Considering first the positive-attached-aether, Eqs. (302) and (303) give the increase in size of each analysis-cube due to mass and extrinsic-energy flow. A distance r will contain r/X_0 analysis-cubes, and this allows us to find the displacement of the aether due to mass and extrinsic-energy flow at the edge of the sphere, $\mathbf{P_{GF}}$. If $\mathbf{r}$ is in the X direction, Eq. (302) leads to $\mathbf{P_{GFX}} = (\mathbf{x}/X_0)\delta X_{3X} \approx [K_{G1}-K_{G2}]\rho_E x\mathbf{i}/4 + K_{F4}\rho_{FE} x\mathbf{i}/4$ with a similar equation for y. Eq. (303) relates that for Z there is no second term. We obtain

$$\mathbf{P_{GF}} \approx (x\mathbf{i} + y\mathbf{j} + z\mathbf{j})[K_{G1}-K_{G2}]\rho_E/4 + (x\mathbf{i} + y\mathbf{j})K_{F4}\rho_{FE}/4 = C_M\mathbf{r} + C_F(x\mathbf{i} + y\mathbf{j}) \qquad (308)$$

To simplify future manipulations Eq. (308) has made the following assignments:

$$C_M = [K_{G1}-K_{G2}]\rho_E/4 \qquad (309)$$

$$C_F = K_{F4}\rho_{FE}/4 \qquad (310)$$

(Notice that if we substitute Eq. (193), $\rho_G = 3[K_{G1}-K_{G2}]\rho_E\rho_0/2$, into Eq. (227), $\mathbf{P_{GS}} = (\rho_G/6\rho_0)\mathbf{r}$, we see that Eq. (227) becomes $\mathbf{P_{GS}} = (\rho_G/6\rho_0)\mathbf{r} = ([K_{G1}-K_{G2}]\rho_E/4)\mathbf{r}$ and hence the treatment here is consistent with section D.4.)

If there is now an ambient attached-aether displacement $\mathbf{P_{AG}}$, $\mathbf{P_{AG}} = P_{AGX}\mathbf{i} + P_{AGY}\mathbf{j} + P_{AGZ}\mathbf{k}$, then the total attached-aether displacement within the sphere becomes $\mathbf{P_G} = \mathbf{P_{AG}} + \mathbf{P_{GF}} = [P_{AGX} + (C_M + C_F)x]\mathbf{i} + [P_{AGY} + (C_M + C_F)y]\mathbf{j} + [P_{AGZ} + C_M z]\mathbf{k}$. The displacement of the attached-aether induced by the flowing extrinsic-energy ($\mathbf{P_{GF}}$) is purely longitudinal, and hence will only add to or subtract from the longitudinal ambient displacement, and therefore $\mathbf{P_{AG}}$ is defined here as the longitudinal ambient displacement of $\mathbf{P_G}$.

Returning to Figure 16, within the strip, and using Eq. (225), the energy of a small positive-attached-aether analysis-cube centered at x is:

$E_{PG}(x,y,z) = K_{T0}X_0^2(1 - 4K_c^2K_{GC}|\mathbf{P_G}|^2/\xi_0^2)$

$= K_{T0}X_0^2\{1 - 4K_c^2K_{GC}|[P_{AGX}+(C_M+C_F)x]\mathbf{i} + [P_{AGY}+(C_M+C_F)y]\mathbf{j} + [P_{AGZ}+C_Mz]\mathbf{k}|^2/\xi_0^2\}$

$= K_{T0}X_0^2\{1 - 4K_c^2K_{GC}[P_{AGX}^2 + 2P_{AGX}(C_M+C_F)x + (C_M+C_F)^2x^2 + P_{AGY}^2 + 2P_{AGY}(C_M+C_F)y +$

$(C_M+C_F)^2y^2 + P_{AGZ}^2 + 2P_{AGZ}C_Mz + C_M^2z^2]/\xi_0^2\}$ (311)

In Eq. (311) the subscript PG is for the case where the flowing extrinsic-energy is immersed in positive-attached-aether. We can now find the force in the X direction that is present on the strip of flowing extrinsic-energy depicted in Fig. 16. For a small additional (and virtual) displacement δx, the energy will become:

$E_{PG1}(x+\delta x,y,z) = K_{T0}X_0^2\{1 - 4K_c^2K_{GC}[P_{AGX}^2 + 2P_{AGX}(C_M+C_F)(x+\delta x) + (C_M+C_F)^2(x+\delta x)^2$

$+ P_{AGY}^2 + 2P_{AGY}(C_M+C_F)y + (C_M+C_F)^2y^2 + P_{AGZ}^2 + 2P_{AGZ}C_Mz + C_M^2z^2]/\xi_0^2\}$ (312)

In Eq. (312) the subscript 1 is for the case of a virtual displacement in the X direction. Subtracting Eq. (311) from Eq. (312) leaves the energy change resulting from the additional displacement:

$\delta E_{GP1}(x,y,z) = \Delta E_{GP}(x+\delta x,y,z) - \Delta E_{GP}(x,y,z)$

$= K_{T0}X_0^2\{-4K_c^2K_{GC}[2P_{AGX}(C_M+C_F)\delta x + (C_M+C_F)^2(2x\delta x+ \delta x^2)]/\xi_0^2\}$ (313)

In the above expression we can drop the term that is second order in the small quantity δx. We can now evaluate the force on the strip by considering the sum of all volume elements within the strip. We can drop the term proportional to $2x\delta x$ because for every value of positive x on our strip there is a value of negative x of equal magnitude. The surviving term, which is proportional to $2P_{AGX}(C_M+C_F)\delta x$, is independent of x, y or z. Recalling that Eq. (313) refers to the change in energy for a small cube within the strip, we can form the relation for the force on the whole strip by summing over all of the analysis-cubes within the strip (Σ_{strip} is the symbol for that sum). The volume of the strip is $2L\Delta y\Delta z$, and therefore the number of analysis-cubes within the strip is $2L\Delta y\Delta z/X_0^3$, and the magnitude of the force on the strip is

$$F_{stripPGFX} = \Sigma_{strip}\,\{\delta E_{PG1}(x,y,z)/\delta x\} = [4K_{T0}X_0^2K_c^2K_{GC}2P_{AGX}(C_M+C_F)/\xi_0^2][2L\Delta y\Delta z/X_0^3]$$

$$= 16L\Delta y\Delta z K_{T0}K_c^2K_{GC}P_{AGX}(C_M+C_F)/X_0\xi_0^2 \qquad (314)$$

The force on the strip shown in Fig. 16 is proportional to the volume of the strip ($2L\Delta y\Delta z$) but independent of x, y and z. The sum of the volume of all of the strips will be the volume of the sphere, V_{sphere}. Hence, we can sum the forces from all such strips to arrive at:

$$F_{spherePGFX} = 8V_{sphere}K_{T0}K_c^2K_{GC}P_{AGX}(C_M+C_F)/X_0\xi_0^2 \qquad (315)$$

Mass and flowing extrinsic-energy will also have an effect on negative-attached-aether. Both the ambient negative-attached-aether disturbance and the disturbance due to mass and flowing extrinsic-energy are the negative of what they are for the positive-attached-aether, $\mathbf{N_{GF}} = -\mathbf{P_{GF}}$ and $\mathbf{N_{AG}} = -\mathbf{P_{AG}}$. We also see that Eq. (226), $E_{NG} = K_{T0}X_0^2(1 - 4K_c^2K_{GC}|\mathbf{N_G}|^2/\xi_0^2)$, coupled with $\mathbf{N_G} = -\mathbf{P_G}$ from Eq. (211), shows that the force back on the sphere of flowing extrinsic-energy will be the same from each attached-aether component, $F_{spherePGFX} = F_{sphereNGFX}$. The total force magnitude is thus:

$$F_{sphereGFX} = F_{spherePGFX} + F_{sphereNGFX} = 16V_{sphere}K_{T0}K_c^2K_{GC}P_{AGX}(C_M+C_F)/X_0\xi_0^2 \qquad (316)$$

Observe that the force is aligned with the direction of **x** in the above analysis, since the energy varies with δx. Hence both $F_{sphereGFX}$ and P_{AGX} are aligned with x. Inside the sphere shown in Fig. 16 the total aetherial displacement P_{GLX} increases in the direction of x as a result of the mass and flowing extrinsic-energy pushing the attached-aether out, and hence, the attached-aether will be forced in the direction of $\mathbf{P_{AGX}}$ as lower energy is preferred. (Eq. (225), $E_{PG} = K_{T0}X_0^2(1 - 4K_c^2K_{GC}|\mathbf{P_G}|^2/\xi_0^2)$ shows that larger values of $\mathbf{P_G}$ lead to lower energy.) The force on the extrinsic-energy will be in the opposite direction as the force on the positive-attached-aether. As mentioned in section C.13, this can be thought of as analogous to pushing a ball downward through a tub of water: as the disturbance (the ball, or the extrinsic-energy) is forced down, the substance it is pushed through (the water, or the attached-aether) is forced in the opposite direction. Hence, the force on the extrinsic-energy (the ball) is in the direction opposite to that on $\mathbf{P_G}$ (the water). (Similar comments apply to $\mathbf{N_G}$.) This leaves

$$\mathbf{F_{sphereGFX}} = -16V_{sphere}K_{T0}K_c^2K_{GC}\mathbf{P_{AGX}}(C_M+C_F)/X_0\xi_0^2 \quad (317)$$

We now recall Eq. (209), $\mathbf{P_{GL}} - \mathbf{N_{GL}} = -\varepsilon_0\nabla\phi_G/\rho_0$, and recall $\mathbf{P_{AGX}} = \mathbf{P_{GL}}$ and Eq. (211) $\mathbf{N_{GL}} = -\mathbf{P_{GL}}$ and hence $\mathbf{P_{GL}} - \mathbf{N_{GL}} = 2\mathbf{P_{GL}} = 2\mathbf{P_{AGX}} = -\varepsilon_0(\nabla\phi_G)_\mathbf{x}/\rho_0$, or $\mathbf{P_{AGX}} = -\varepsilon_0(\nabla\phi_G)_\mathbf{x}/2\rho_0$, where here $(\nabla\phi_G)_\mathbf{x}$ is the x component of $\nabla\phi_G$. Thus we arrive at

$$\mathbf{F_{sphereGFX}} = 8V_{sphere}K_{T0}K_c^2K_{GC}\varepsilon_0(\nabla\phi_G)_\mathbf{x}(C_M+C_F)/\rho_0X_0\xi_0^2 \quad (318)$$

Next, we will evaluate the force in the Z direction. For a small additional (and virtual) displacement δz, the energy of Eq. (311) will become:

$$E_{PG2}(x,y,z+\delta z) = K_{T0}X_0^2\{1 - 4K_c^2K_{GC}[P_{AGX}^2 + 2P_{AGX}(C_M+C_F)x + (C_M+C_F)^2x^2 + P_{AGY}^2 + 2P_{AGY}(C_M+C_F)y + (C_M+C_F)^2y^2 + P_{AGZ}^2 + 2P_{AGZ}C_M(z+\delta z) + C_M^2(z+\delta z)^2]/\xi_0^2\} \quad (319)$$

In Eq. (319) the subscript 2 is for case 2 which is our evaluation of force in the Z direction. Subtracting Eq. (311) from Eq. (319) leaves the energy change resulting from the additional displacement:

$$\delta E_{PG2}(x,y,z) = E_{GP2}(x,y,z+\delta z) - E_{GP}(x,y,z) =$$
$$-K_{T0}X_0^24K_c^2K_{GC}\{2P_{AGZ}C_M\delta z + C_M^2(2z\delta z + \delta z^2)]/\xi_0^2\} \quad (320)$$

In the above expression we can drop the term that is second order in the small quantity δz. We can now evaluate the force on the strip by considering the sum of all volume elements within the strip. We can drop the term $2C_M^2z\delta z$ because for every value of positive z on our strip there is a value of negative z of equal magnitude. The surviving term, $2P_{AGZ}C_M\delta z$, is independent of x, y or z.

Recalling that Eq. (320) refers to the change in energy for a small cube within the strip, we can form the relation for the force on the whole strip by summing over all of the analysis-cubes within the strip (Σ_{strip} is the symbol for that sum). The volume of the strip is $2L\Delta y\Delta z$, and therefore the number of analysis-cubes within the strip is $2L\Delta y\Delta z/X_0^3$, and the force magnitude on the strip is

$F_{stripPGFZ} = \Sigma_{strip}\{\delta E_{PG2}(x,y,z)/\delta z\} = [4K_{T0}X_0^2K_c^2K_{GC}2P_{AGZ}C_M/\xi_0^2][2L\Delta y\Delta z/X_0^3]$

$= 16L\Delta y\Delta zK_{T0}K_c^2K_{GC}P_{AGZ}C_M/X_0\xi_0^2$ (321)

The force on the strip shown in Fig. 16 is proportional to the volume of the strip ($2L\Delta y\Delta z$) but independent of x, y and z. The sum of the volume of all of the strips will be the volume of the sphere, V_{sphere}. Hence, we can sum the forces from all such strips to arrive at:

$F_{spherePGFZ} = 8V_{sphere}K_{T0}K_c^2K_{GC}P_{AGZ}C_M/X_0\xi_0^2$ (322)

Mass and flowing extrinsic-energy will also have an effect on negative-attached-aether. Both the ambient negative-attached-aether disturbance and the disturbance due to mass and flowing extrinsic-energy are the negative of what they are for the positive-attached-aether, $\mathbf{N_{GF}} = -\mathbf{P_{GF}}$ and $\mathbf{N_{AG}} = -\mathbf{P_{AG}}$. From Eq. (226), $E_{NG} = K_{T0}X_0^2(1 - 4K_c^2K_{GC}|\mathbf{N_G}|^2/\xi_0^2)$, coupled with $\mathbf{N_G} = -\mathbf{P_G}$ from Eq. (211), we also see that the force back on the sphere of flowing extrinsic-energy will be the same from each attached-aether component, $F_{spherePGFZ} = F_{sphereNGFZ}$. The total force is thus:

$F_{sphereGFZ} = F_{spherePGFZ} + F_{sphereNGFZ} = 16V_{sphere}K_{T0}K_c^2K_{GC}P_{AGZ}C_M/X_0\xi_0^2$ (323)

Observe that the force is aligned with the direction of **z** in the above analysis, since the energy varies with δz. Hence both $F_{sphereGFZ}$ and P_{AGZ} are aligned with z. Inside the sphere shown in Fig. 16 the total aetherial displacement P_{GLZ} increases in the direction of z as a result of the mass pushing the attached-aether out, and hence, the attached-aether will be forced in the direction of $\mathbf{P_{AGZ}}$ as lower energy is preferred. (Eq. (225), $E_{PG} = K_{T0}X_0^2(1 - 4K_c^2K_{GC}|\mathbf{P_G}|^2/\xi_0^2)$ shows that larger values of $\mathbf{P_G}$ lead to lower energy. While there is no flowing extrinsic-energy effect in the z direction, the stationary extrinsic-energy remains.) The force on the extrinsic-energy will be in the opposite direction as the force on the positive-attached-aether. This leaves

$$\mathbf{F}_{\mathbf{sphereGFZ}} = -16V_{sphere}K_{T0}K_c^2K_{GC}\mathbf{P}_{\mathbf{AGZ}}C_M/X_0\xi_0^2 \quad (324)$$

We now recall Eq. (209), $\mathbf{P}_{\mathbf{GL}} - \mathbf{N}_{\mathbf{GL}} = -\varepsilon_0\nabla\phi_G/\rho_0$, and recall $\mathbf{P}_{\mathbf{GL}} = \mathbf{P}_{\mathbf{AGZ}}$ and Eq. (211) $\mathbf{N}_{\mathbf{GL}} = -\mathbf{P}_{\mathbf{GL}}$ and hence $\mathbf{P}_{\mathbf{GL}} - \mathbf{N}_{\mathbf{GL}} = 2\mathbf{P}_{\mathbf{GL}} = 2\mathbf{P}_{\mathbf{AGZ}} = -\varepsilon_0(\nabla\phi_G)_{\mathbf{z}}/\rho_0$, or $\mathbf{P}_{\mathbf{AGZ}} = -\varepsilon_0(\nabla\phi_G)_{\mathbf{z}}/2\rho_0$, where here $(\nabla\phi_G)_{\mathbf{z}}$ is the z component of $\nabla\phi_G$. Thus we arrive at

$$\mathbf{F}_{\mathbf{sphereGFZ}} = 8V_{sphere}K_{T0}K_c^2K_{GC}\varepsilon_0(\nabla\phi_G)_{\mathbf{z}}C_M/\rho_0X_0\xi_0^2 \quad (325)$$

Next, we observe that the expression for $F_{sphereGFY}$ will follow the same derivation as that for $F_{sphereGFX}$ except with y taking the place of x. This leaves us with:

$$\mathbf{F}_{\mathbf{sphereGFY}} = 8V_{sphere}K_{T0}K_c^2K_{GC}\varepsilon_0(\nabla\phi_G)_{\mathbf{y}}(C_M+C_F)/\rho_0X_0\xi_0^2 \quad (326)$$

We can now combine Eqs. (318), (325) and (326) to arrive at the total force on the moving sphere of flowing extrinsic-energy

$$\mathbf{F}_{\mathbf{sphereGF}} = \mathbf{F}_{\mathbf{sphereGFX}} + \mathbf{F}_{\mathbf{sphereGFY}} + \mathbf{F}_{\mathbf{sphereGFZ}} =$$

$$8V_{sphere}K_{T0}K_c^2K_{GC}\varepsilon_0(\nabla\phi_G)_{\mathbf{x}}(C_M+C_F)/\rho_0X_0\xi_0^2 + 8V_{sphere}K_{T0}K_c^2K_{GC}\varepsilon_0(\nabla\phi_G)_{\mathbf{y}}(C_M+C_F)/\rho_0X_0\xi_0^2 +$$

$$8V_{sphere}K_{T0}K_c^2K_{GC}\varepsilon_0(\nabla\phi_G)_{\mathbf{z}}C_M/\rho_0X_0\xi_0^2 =$$

$$(8V_{sphere}K_{T0}K_c^2K_{GC}\varepsilon_0/\rho_0X_0\xi_0^2)\{[(\nabla\phi_G)_{\mathbf{x}}+(\nabla\phi_G)_{\mathbf{y}}](C_M+C_F)] + (\nabla\phi_G)_{\mathbf{z}}C_M\} =$$

$$(8V_{sphere}K_{T0}K_c^2K_{GC}\varepsilon_0/\rho_0X_0\xi_0^2)[(C_M+C_F)\nabla\phi_G - C_F(\nabla\phi_G)_{\mathbf{z}}] \quad (327)$$

Now realize that we have chosen $\mathbf{k} = \hat{\mathbf{v}}$ and hence $(\hat{\mathbf{v}} \cdot \nabla\phi_G) = |(\nabla\phi_G)_{\mathbf{z}}|$ and we arrive at:

$$\mathbf{F}_{\mathbf{sphereGF}} = (8V_{sphere}K_{T0}K_c^2K_{GC}\varepsilon_0/\rho_0X_0\xi_0^2)[(C_M+C_F)\nabla\phi_G - C_F(\hat{\mathbf{v}} \cdot \nabla\phi_G)\hat{\mathbf{v}}] \quad (328)$$

Now we will manipulate Eq. (328) to make for easy comparison with Eq. (237) of section D.4. Recall Eq. (193), $\rho_G = 3[K_{G1}-K_{G2}]\rho_E\rho_0/2$, from which, $[K_{G1}-K_{G2}]\rho_E = 2\rho_G/3\rho_0$. Next recall Eq. (309), $C_M = [K_{G1}-K_{G2}]\rho_E/4$, and hence $C_M = \rho_G/6\rho_0$. We then have $[(C_M+C_F)\nabla\phi_G - C_F(\hat{\mathbf{v}} \cdot \nabla\phi_G)\hat{\mathbf{v}}] = C_M[(1+C_F/C_M)\nabla\phi_G - (C_F/C_M)(\hat{\mathbf{v}} \cdot \nabla\phi_G)\hat{\mathbf{v}}] = (\rho_G/6\rho_0)[(1+C_F/C_M)\nabla\phi_G - (C_F/C_M)(\hat{\mathbf{v}} \cdot \nabla\phi_G)\hat{\mathbf{v}}]$ Hence,

$$\mathbf{F}_{\mathbf{sphereGF}} = (4V_{sphere}\rho_G K_{T0} K_c^2 K_{GC}\varepsilon_0/3\rho_0^2 X_0\xi_0^2)[(1+C_F/C_M)\nabla\phi_G - (C_F/C_M)(\hat{\mathbf{v}}\cdot\nabla\phi_G)\hat{\mathbf{v}}] \quad (329)$$

We also see that with $C_M = \rho_G/6\rho_0$, and with Eq. (310), $C_F = K_{F4}\rho_{FE}/4$, $C_F/C_M = 3K_{F4}\rho_{FE}\rho_0/2\rho_G$, and with Eq. (193), $\rho_G = 3[K_{G1}-K_{G2}]\rho_E\rho_0/2$,

$$C_F/C_M = K_{F4}\rho_{FE}/[K_{G1}-K_{G2}]\rho_E \quad (330)$$

We now recall $K_{T0} = 3\rho_0^2 X_0\xi_0^2/4K_c^2\varepsilon_0$ from Eq. (141) and $Q_G = K_{GC}V_{sphere}\rho_G$ from Eq. (236) to get

$$\mathbf{F}_{\mathbf{sphereGF}} = Q_G[(1+C_F/C_M)\nabla\phi_G - (C_F/C_M)(\hat{\mathbf{v}}\cdot\nabla\phi_G)\hat{\mathbf{v}}] \quad (331)$$

Recall that Eq. (237) is $\mathbf{F_G} = Q_G\nabla\phi_G$, and we see that Eq. (331) reduces to (237) in the case where the energy flow density ρ_{FE} is zero (when ρ_{FE} is zero, C_F is zero).

D.16 – Bending of Starlight in a Gravitational Field; Setting K_{F4}

Eq. (331) can now be used to calculate the bending of starlight in the gravitational field of the sun. As a first step, with the sun treated as a stationary mass M_{SUN}, Eq. (247) informs that $\nabla\phi_{SUN}$ from M_{SUN} is $\nabla\phi_{SUN} = -(\rho_{GSUN}R_{SUN}^3/3r^2\varepsilon_0)\hat{\mathbf{r}}$. Then, with Eq. (193), $\rho_G = 3[K_{G1}-K_{G2}]\rho_E\rho_0/2$, $\nabla\phi_{SUN} = -([K_{G1}-K_{G2}]\rho_{ESUN}\rho_0 R_{SUN}^3/2r^2\varepsilon_0)\hat{\mathbf{r}}$, and with $\gamma_{SUN}M_{SUN}c^2 = (4/3)\pi R_{SUN}^3\rho_{ESUN}$, $R_{SUN}^3\rho_{ESUN} = 3\gamma_{SUN}M_{SUN}c^2/4\pi$, and

$$\nabla\phi_{SUN} = -(3[K_{G1}-K_{G2}]\gamma_{SUN}M_{SUN}c^2\rho_0/8\pi r^2\varepsilon_0)\hat{\mathbf{r}} \quad (332)$$

Next, the photon will be treated as a moving sphere of radius R_{PH} and energy E_{PH}. In this modeling, recalling Eq. (236), $Q_G = K_{GC}V_{sphere}\rho_G$, $Q_{GPH} = K_{GC}(4/3)\pi R_{PH}^3\rho_{GPH} = 2\pi K_{GC}R_{PH}^3[K_{G1}-K_{G2}]\rho_{EPH}\rho_0$ where we again use Eq. (193). And with $\rho_{EPH} = E_{PH}/[(4/3)\pi R_{PH}^3]$,

$$Q_{GPH} = 3K_{GC}[K_{G1}-K_{G2}]E_{PH}\rho_0/2 \quad (333)$$

Combining Eqs. (332) and (333), $\nabla\phi_{SUN}Q_{GPH} = -(9K_{GC}[K_{G1}-K_{G2}]^2E_{PH}\gamma_{SUN}M_{SUN}c^2\rho_0^2/16\pi r^2\varepsilon_0)\hat{\mathbf{r}}$ and recalling Eq. (251), $G_N = 9K_{GC}[K_{G1}-K_{G2}]^2\rho_0^2c^4/16\pi\varepsilon_0$, this leaves us with $\nabla\phi_{SUN}Q_{GPH} = -(G_NE_{PH}\gamma_{SUN}M_{SUN}/c^2r^2)\hat{\mathbf{r}}$. Substituting this into Eq. (331) leaves:

$$\mathbf{F_{GF\_PH\_SUN}} = -(G_NE_{PH}\gamma_{SUN}M_{SUN}/c^2r^2)[(1+C_F/C_M)\hat{\mathbf{r}} - (C_F/C_M)(\hat{\mathbf{v}}\cdot\hat{\mathbf{r}})\hat{\mathbf{v}}] \quad (334)$$

Now, recall Eq. (330), $C_F/C_M = K_{F4}\rho_{FE}/[K_{G1}-K_{G2}]\rho_E$, and observe that ρ_{FE} and ρ_E refer to the photon in this case. (The photon is the small sphere within the attached-aether in Figs. 15 and 16; while the sun leads to $\nabla\phi$.) At this point in our development we must now define what is meant by "energy flow"; we have two obvious choices, and other choices exist as well. One choice is that energy flow is the kinetic energy (the energy of motion) with $\mathbf{E_{FKE}} = E_{KE}\hat{\mathbf{v}} = (\gamma - 1)Mc^2\hat{\mathbf{v}}$. (The subscript KE is for kinetic energy). The second choice for the energy flow is $\mathbf{E_{FMO}} = \gamma Mc^2\mathbf{v}$, which is the amount of total energy, γMc^2, multiplied by the velocity $\mathbf{v}$ that the total energy flows with, and we notice that $\gamma Mc^2\mathbf{v}$ is the momentum multiplied by c^2. (The subscript MO is for momentum). If we take the first choice, for the photon, the kinetic energy equals the total energy, $\rho_{FEKE} = \rho_{EKE}$, so for a photon $C_{FKE}/C_{MKE} = K_{F4KE}\rho_{FEKE}/[K_{G1}-K_{G2}]\rho_{EKE} = K_{F4KE}/[K_{G1}-K_{G2}]$. This allows Eq. (334) to be rewritten as

$$\mathbf{F_{GF\_PH\_SUN\_KE}} =$$

$$-(G_NE_{PH}\gamma_{SUN}M_{SUN}/c^2r^2)[(1 + K_{F4KE}/[K_{G1}-K_{G2}])\hat{\mathbf{r}} - (K_{F4KE}/[K_{G1}-K_{G2}])(\hat{\mathbf{v}}\cdot\hat{\mathbf{r}})\hat{\mathbf{v}}] \quad (335)$$

If we take the second choice, for the photon, the energy γMc^2 equals the momentum times c, $\rho_{EMO} = \rho_{M}c^2$, and multiplying this by the light velocity c, $\rho_{FEMO} = \rho_{EMO}c = \rho_{M}c^3$, so for a photon $C_{FMO}/C_{MMO} = K_{F4MO}\rho_{FEMO}/[K_{G1}-K_{G2}]\rho_{EMO} = cK_{F4MO}/[K_{G1}-K_{G2}]$. This allows Eq. (334) to be rewritten as

$\mathbf{F}_{\mathbf{GF\_PH\_SUN\_MO}} =$

$$-(G_N E_{PH}\gamma_{SUN}M_{SUN}/c^2r^2)[(1 + cK_{F4MO}/[K_{G1}-K_{G2}])\hat{\mathbf{r}} - (cK_{F4MO}/[K_{G1}-K_{G2}])(\hat{\mathbf{v}}\cdot\hat{\mathbf{r}})\hat{\mathbf{v}}] \quad (336)$$

We can now evaluate the force on the photon both parallel and perpendicular to $\hat{\mathbf{r}}$. For the parallel case, $(\hat{\mathbf{v}}\cdot\hat{\mathbf{r}}) = 1$ and $\hat{\mathbf{v}} = \hat{\mathbf{r}}$, and Eqs. (335) and (336) each reduce to Newton's Law of Universal Gravitation, Eq. (250), with E_{PH}/c^2 as the equivalent mass of the photon:

$$\mathbf{F}_{\mathbf{GF\_PH\_SUN\_PARALLEL}} = -(G_N E_{PH}\gamma_{SUN}M_{SUN}/c^2r^2 \quad (337)$$

Hence, for photons moving directly away from the sun, the effects of energy flow do not enter. This is of course obvious from the Extrinsic-Energy Flow Speculation, which relates that the energy flow only effects the aether in directions perpendicular to the flow. This is also why we were able to derive the gravitational redshift in section D.7 prior to any consideration of the energy flow effects.

For the perpendicular case $(\hat{\mathbf{v}}\cdot\hat{\mathbf{r}}) = 0$ and Eq. (335) becomes:

$$\mathbf{F}_{\mathbf{GF\_PH\_SUN\_KE\_PERP}} = -(G_N E_{PH}\gamma_{SUN}M_{SUN}/c^2r^2)[(1 + K_{F4KE}/[K_{G1}-K_{G2}])\hat{\mathbf{r}}\ ] \quad (338)$$

Eq. (338) is a force that is the Newtonian force multiplied by a factor $(1 + K_{F4KE}/[K_{G1}-K_{G2}])$, and it is famously known that experimental results of light bending around the sun show that this factor is 2. This allows us to set the constant K_{F4KE}

$$K_{F4KE} = [K_{G1}-K_{G2}] \quad (339)$$

In a similar way, Eq. (336) becomes:

$$\mathbf{F}_{\mathbf{GF\_PH\_SUN\_MO\_PERP}} = -(G_N E_{PH}\gamma_{SUN}M_{SUN}/c^2r^2)[(1 + cK_{F4MO}/[K_{G1}-K_{G2}])\hat{\mathbf{r}}] \quad (340)$$

Setting the factor $(1 + cK_{F4MO}/[K_{G1}-K_{G2}])$ to 2 allows us to set the constant K_{F4MO}

$$K_{F4MO} = [K_{G1}-K_{G2}]/c \quad (341)$$

Here we have presented two possible energy flow definitions that lead to a factor of two for the light bending around the sun. Other definitions are possible. In appendix I we will explore these possibilities more under the assumption that the aether is at rest with respect to the cosmic background radiation.

D.17 – Gravitational Slowing of Light: The Shapiro Effect and Assignment of Inertial-mass

Rearrangement of Eq. (107), $T_0 = m_0c^2$, leads to the expression $c = (T_0/m_0)^{1/2}$ where T_0 is the nominal aetherial tension per unit area and m_0 is the inertial-mass density. We have seen that the tension can vary and we must also admit that the inertial-mass density may vary. This leads us to a more general expression for the speed of light

$V_{LIGHT} = (T/m)^{1/2}$ (342)

It remains to define T and m in Eq. (342). For the positive-aether case, Eq. (66) relates that $F_{TPP} = K_{T0}X_0[1 + 2K_cx/\xi_0]$. We also see from Eq. (218) that the gamma-force tension component is $\mathbf{F}_{\boldsymbol{\gamma}\mathbf{PT}} = 4K_c(1 + K_{GC})K_{T0}(X_0/\xi_0)x\hat{\mathbf{r}}$. The gamma-force tension and nominal tension are in opposite directions, leaving the total tension per unit area for the positive aether as $T_P = (F_{TPP} – F_{P\gamma T})/X_0^2 = (K_{T0}/X_0)[1 + 2K_cx/\xi_0] – (K_{T0}/X_0)(1 + K_{GC})[4K_cx/\xi_0] = (K_{T0}/X_0)[1 – 2K_cx/\xi_0 – 4K_{GC}K_cx/\xi_0]$, or,

$T_P = (K_{T0}/X_0)[1 – 2K_c|\mathbf{P}_G|/\xi_0 – 4K_cK_{GC}|\mathbf{P}_G|/\xi_0]$ (343)

In Eq. (343) $x = |\mathbf{P}_G|$. ($|\mathbf{P}_G|$ is the distance the aetherial cube moves from its nominal position due to gravitational effects. An electromagnetic effect can also exist, but it is typically neutralized while gravitational effects can accumulate.)

For the negative-aether case, Eq. (G1) is $F_{TN} = K_{T0}X_0[1 – 2K_cx/\xi_0]$, while Eq. (G7) gives the gamma-force tension component $\mathbf{F}_{\boldsymbol{\gamma}\mathbf{NT}} = 4K_cK_{GC}K_{T0}(X_0/\xi_0)x\hat{\mathbf{r}}$. The gamma-force tension and nominal tension are in opposite directions, leaving the total tension per unit area as $T_N = (F_{TN} - F_{\gamma NT})/X_0^2 = (K_{T0}/X_0)[1 - 2K_c x/\xi_0 - 4K_{GC}K_c x/\xi_0]$, or,

$$T_N = (K_{T0}/X_0)[1 - 2K_c|\mathbf{N_G}|/\xi_0 - 4K_cK_{GC}|\mathbf{N_G}|/\xi_0] \qquad (344)$$

Since by Eq. (211) $|\mathbf{N_{GL}}| = |\mathbf{P_{GL}}|$, and also assuming $|\mathbf{N_{GT}}| = |\mathbf{P_{GT}}|$ we can see that the tension per unit area T is the same for both the positive-aether and negative-aether cases.

Turning now to the inertial-mass density m, we propose:

The Aetherial Inertial-Mass Assignment: The aetherial inertial-mass density equals the field energy density plus the gravitational potential energy density divided by c^2.

The total field energy for the positive-attached-aether is given by Eq. (225), $E_{PG} = K_{T0}X_0^2(1 - 4K_c^2K_{GC}|\mathbf{P_G}|^2/\xi_0^2)$, and for the negative-attached-aether it is given by Eq. (226) $E_{NG} = K_{T0}X_0^2(1 - 4K_c^2K_{GC}|\mathbf{N_G}|^2/\xi_0^2)$, and with Eq. (211) $|\mathbf{N_{GL}}| = |\mathbf{P_{GL}}|$, and also assuming $|\mathbf{N_{GT}}| = |\mathbf{P_{GT}}|$ we have $E_{PG} = E_{NG}$. To obtain the field energy densities we must of course divide by the volume of the analysis-cube to arrive at E_{PG}/X_0^3.

The gravitational potential energy density is the aetherial density ρ_0 times the potential ϕ and it is the same for both the negative and positive aether:

$$E_{P\phi} = E_{N\phi} = \rho_0\phi \qquad (345)$$

From Eq. (246), $\phi_{SO} = \rho_{GS}R_S^3/3r\varepsilon_0 - \rho_{GS}R_S^2/2\varepsilon_0$ and until now we have taken the gradient of ϕ_{SO} for our quantities of interest. Since the gradient of a constant is zero we have so far been unconcerned with further setting of the constant. Now however we have a physical effect entering in and we must set the constant appropriately. We will apply the boundary condition that at infinity the gravitational potential is zero (so that objects sufficiently far away do not affect the aetherial inertial-mass). This leaves us with $\phi = \rho_{GS}R_S^3/3r\varepsilon_0$. Eq. (238) gives us $\rho_{GS} = 3[K_{G1}-K_{G2}]\rho_{ES}\rho_0/2 = 9[K_{G1}-K_{G2}]\gamma M_S c^2\rho_0/8\pi R_S^3$, so $\phi = 3[K_{G1}-K_{G2}]\gamma M_S c^2\rho_0/8\pi\varepsilon_0 r = K_\phi/\rho_0 r$, where we have defined

$K_\phi = 3[K_{G1}–K_{G2}]\gamma M_S c^2\rho_0^2/8\pi\varepsilon_0$ (346)

We obtain

$E_{P\phi} = E_{N\phi} = \rho_0\phi = K_\phi/r$ (347)

Dimensional analysis. Below Eq. (271) we see that $[K_{G1}–K_{G2}]$ has dimensions of kg^{-1} s^2 m. ρ_0 has dimensions of C divided by m^3. ε_0 has dimensions m^{-3} kg^{-1} s^4 A^2. So we have K_ϕ/r with units of (kg^{-1} s^2 m) kg m^2 s^{-2} (C^2/m^6)/(m^{-3} kg^{-1} s^4 C^2 s^{-2}) m, which is kg m^2 s^{-2} /m^3, which is an energy density, as expected.

Applying the aetherial inertial-mass assignment we now get an expression for the attached-aether inertial-mass m by adding the field energy density to the gravitational potential energy density:

$m = [(K_{T0}/X_0)(1 – 4K_c^2K_{GC}|\mathbf{P_G}|^2/\xi_0^2) + K_\phi/r]/c^2$ (348)

We can now substitute Eqs. (343) and (348) into Eq. (342) to form the expression for the speed of light as it passes near the sun:

$V_{LIGHT} =$ (349)

$c\{(K_{T0}/X_0)[1–2K_c|\mathbf{P_G}|/\xi_0–4K_cK_{GC}|\mathbf{P_G}|/\xi_0]/[(K_{T0}/X_0)(1–4K_c^2K_{GC}|\mathbf{P_G}|^2/\xi_0^2)+K_\phi/r]\}^{1/2}$

Near the sun we assume $K_{T0}/X_0 >> K_\phi/r >> (K_{T0}/X_0)2K_c|\mathbf{P}|/\xi_0 >> (K_{T0}/X_0)4K_c^2K_{GC}|\mathbf{P_G}|^2/\xi_0^2$. (The assumption $K_{T0}/X_0 >> K_\phi/r$ is verified in Section E below Eq. (369), and the remaining assumptions follow by assuming K_c is sufficiently small.) This allows us to drop the terms $4K_c^2K_{GC}|\mathbf{P_G}|^2/\xi_0^2$, $2K_c|\mathbf{P}|/\xi_0$ and $4K_cK_{GC}|\mathbf{P_G}|/\xi_0$ from Eq. (349). This leaves us with $V_{LIGHT} = c\{(K_{T0}/X_0)/[(K_{T0}/X_0)+K_\phi/r]\}^{1/2} = c\{1/[1+K_\phi/(K_{T0}/X_0)r]\}^{1/2}$ or

$V_{LIGHT} \approx c[1 – K_\phi/2(K_{T0}/X_0)r]$ (350)

We can now use Eq. (350) to calculate the transit time of a photon between the earth and a planet. Figure 17 shows the geometry of relevance for the calculation.

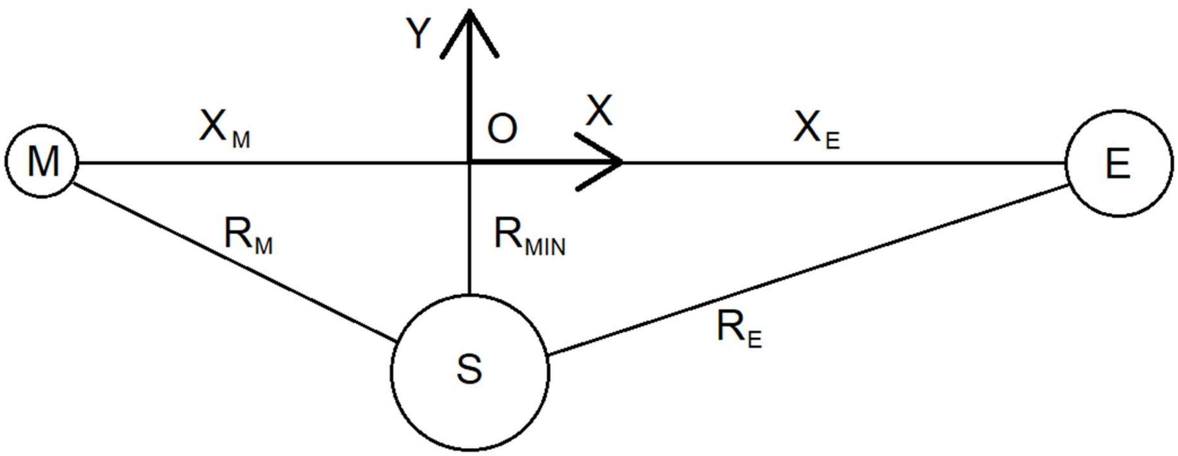

Figure 17. Geometry of Relevance for Calculating the Shapiro Effect.

In Figure 17, M is the planet Mercury, E is the earth and S is the sun. We will assume that the deflection of the light by the sun is small enough that we can consider the path to be predominantly along the x axis of the figure. We then have $dx/dt = V_{LIGHT} \approx c[1 - K_\phi/2(K_{T0}/X_0)r]$, which for a small incremental spatial advance dx we can rearrange to $dt \approx (dx/c)[1 + K_\phi/2(K_{T0}/X_0)r]$. From this, and noting that $r = (x^2+R_{MIN}^2)^{1/2}$ all along the path, the time for light to go from Mercury to Earth is $t = \int dt = (1/c) \int [1 + K_\phi/2(K_{T0}/X_0)(x^2 + R_{MIN}^2)^{1/2}]dx$, and evaluating the integral,

$$T_{ME} = x/c + (K_\phi X_0/2cK_{T0})\ln[(x^2 + R_{MIN}^2)^{1/2} + x] \qquad (351)$$

We can see that T_{ME} is the integral by differentiating it to find the integrand: $dT_{ME}/dx = 1/c + (K_\phi X_0/2cK_{T0})\{(1/ [(x^2 + R_{MIN}^2)^{1/2} + x]\}(1 + (1/2)(x^2 + R_{MIN}^2)^{-1/2}2x) = 1/c + (K_\phi X_0/2cK_{T0})[1 + x(x^2 + R_{MIN}^2)^{-1/2}]/[(x^2 + R_{MIN}^2)^{1/2} + x]$. Now $[1+x(x^2+R_{MIN}^2)^{-1/2}] = \{[(x^2+R_{MIN}^2)^{1/2}+x]/(x^2+R_{MIN}^2)^{1/2}\}$ so $dT_{ME}/dx = 1/c + (K_\phi X_0/2cK_{T0})\{[(x^2+R_{MIN}^2)^{1/2}+x]/(x^2+R_{MIN}^2)^{1/2}\}/[(x^2 + R_{MIN}^2)^{1/2} + x] = 1/c + (K_\phi X_0/2cK_{T0})/(x^2+R_{MIN}^2)^{1/2}$, which is the integrand.

We now evaluate Eq. (351) between $-X_M$ and X_E, $T_{ME} = X_E/c + (K_\phi X_0/2cK_{T0})\ln[(X_E^2 + R_{MIN}^2)^{1/2} + X_E] - \{-X_M\}/c - (K_\phi X_0/2cK_{T0})\ln[(\{-X_M\}^2 + R_{MIN}^2)^{1/2} - X_M] = (X_E + X_M)/c + (K_\phi X_0/2cK_{T0})\ln[R_E$

$+ X_E] - (K_\phi X_0/2cK_{T0})\ln[R_M - X_M]$. Here we have used $(X_E^2 + R_{MIN}^2)^{1/2} = R_E$ and $(X_M^2 + R_{MIN}^2)^{1/2} = R_M$. $(X_E + X_M)/c$ is the nominal flight time. Hence the gravitational delay is

$$T_{DELAY} = (K_\phi X_0/2cK_{T0})\ln[(R_E + X_E)/(R_M - X_M)] \quad (352)$$

Eq. (352) is the same equation as that given by general relativity in the low field limit, which is in agreement with experimental data for all values of R_{MIN}. We can now set Eq. (352) for the case when $R_{MIN} = R_S$, where R_S is the radius of the sun, and include the factor of 2 for the round-trip delay:

$$T_{SHAPIRO} = (K_\phi X_0/cK_{T0})\ln\{[(X_E^2 + R_S^2)^{1/2} + X_E]/[(X_M^2 + R_S^2)^{1/2} - X_M]\} \approx 200\ \mu s \quad (353)$$

Eq. (353) uses the terminology $T_{SHAPIRO}$ for the gravitational delay as it is presently called the Shapiro time delay. (Shapiro predicted the delay[14] as a result of the General Theory of Relativity.) The value of 200 μs in Eq. (353) is that used in the original Shapiro prediction and it has been shown to be consistent with experiment up to the level of accuracy desired for our work here. (Here we are interested in the theoretical formulation of the equations as well as approximate evaluations of the constants; more exact specifications of the constants are of course welcome for future efforts.)

D.18 – The Aetherial Gravitational Equation of Motion: Gravitational Waves

Recall that for electromagnetism we have derived Eq. (104), $m_0(\partial^2\mathbf{P}/\partial t^2) = T_0\nabla^2\mathbf{P} - K_{F3}\mathbf{J_T}$, where $m_0(\partial^2\mathbf{P}/\partial t^2)$ is the inertial-mass multiplied by the acceleration, $T_0\nabla^2\mathbf{P}$ is the tension force and $K_{F3}\mathbf{J_T}$ is the flow force. Following the same reasoning and derivation, for gravitational disturbances we get

$$m(\partial^2\mathbf{P_G}/\partial t^2) = T_0\nabla^2\mathbf{P_G} + \mathbf{F_{GFP}} \quad (354)$$

$$m(\partial^2\mathbf{N_G}/\partial t^2) = T_0\nabla^2\mathbf{N_G} + \mathbf{F_{GFN}} \quad (355)$$

In Eqs. (354) and (355) $\mathbf{F_{GFP}}$ ($\mathbf{F_{GFN}}$) is the flow force on the positive (negative) attached-aether. In sections D.14 through D.16 we have only introduced a speculation for the gravitational flow force to arrive at the bending of light, and we have seen that more than one speculation may be consistent with experiment: we do not have enough experimental data to know the specific form of $\mathbf{F_{GFP}}$ and $\mathbf{F_{GFN}}$.

However, we notice that when the flow force is zero (as it is in free space) and when $m = m_0$, we arrive at the same wave equation for both gravity and electromagnetism. The constants T_0 and m_0 are the same for both gravitational and electromagnetic disturbances. Hence, provided $\mathbf{F_{GFP}} = \mathbf{F_{GFN}} = 0$ in free space, gravitational waves are predicted to exist and they will move with the velocity of light.

Note that if we knew $\mathbf{F_{GFP}}$ and $\mathbf{F_{GFN}}$ we could derive a set of equations for gravitation similar to Maxwell's equations, but there is no need to do so. The fundamental equations are Poisson's Equation longitudinally and F = ma transversely for both gravity and electromagnetism. Due to the prevalence of Maxwell's Equations within electromagnetism, we have provided derivations for them above. For gravitation there is no need to derive similar equations, as there are no prevailing equivalents of Maxwell's Equations for gravity in the literature.

E – Remaining Topics and Concluding Remarks

E.1 – Identification of the Free Parameters

In the work above we have introduced numerous constants, and we have made several assignments of those constants. In this section E.1 we will relate the constants to each other and refine the expressions until we arrive at the remaining free parameters of the theory. Additionally, we will ensure that none of the assignments made above are in contradiction with each other.

E.1.1. Relating the Analysis-Cube-Constants to the Aetherial-Quantum-Constants.

We have defined four analysis-cube constants, X_0, K_{T0}, K_{Q0} and n. These constants are related to the aetherial quantum constants in a simple way. A special case of Eq. (10), $X_Q = \xi_Q/n$, is

$$n = \xi_0/X_0 \quad (356)$$

Next, we use Eq. (13), $K_{T0} = K_{T\xi0}/n$, and Eq. (14), $K_{Q0} = K_{Q\xi0}/n^5$, along with Eq. (356) to obtain

$$K_{T0} = K_{T\xi0}/n = K_{T\xi0}X_0/\xi_0 \quad (357)$$

$$K_{Q0} = K_{Q\xi0}/n^5 = K_{Q\xi0}(X_0/\xi_0)^5 \quad (358)$$

Or,

$$K_{T\xi0} = K_{T0}\xi_0/X_0 \quad (359)$$

$$K_{Q\xi0} = K_{Q0}(\xi_0/X_0)^5 \quad (360)$$

And we can form

$$K_{T0}/X_0 = K_{T\xi0}/\xi_0 \quad (361)$$

$$K_{Q0}/X_0^5 = K_{Q\xi0}/\xi_0^5 \quad (362)$$

Next, let us manipulate Eq. (7), $\xi_0 = (2K_{Q\xi0}/K_{T\xi0})^{1/4}$, using Eqs. (359) and (360)

$$\xi_0 = (2K_{Q\xi0}/K_{T\xi0})^{1/4} = [2K_{Q0}(\xi_0/X_0)^5/(K_{T0}\xi_0/X_0)]^{1/4} = [2K_{Q0}/K_{T0}]^{1/4}(\xi_0/X_0) \quad (363)$$

From Eq. (363) we see

$$X_0 = [2K_{Q0}/K_{T0}]^{1/4} \quad (364)$$

E.1.2. Evaluating the Electromagnetic Flow Force Constants.

Above we have introduced the electromagnetic flow force constants K_{F1}, K_{F2} and K_{F3}. Eq. (158), $K_{F3}^2 = \mu_0T_0$ allows us to find K_{F3}:

$$K_{F3} = (\mu_0T_0)^{1/2} \quad (365)$$

Now recall Eq. (144), $\mu_0 T_0 = 2K_{F3}K_{F1}$, which leads to $K_{F1} = \mu_0 T_0/2K_{F3}$ or, using Eq. (365)

$$K_{F1} = (\mu_0 T_0)^{1/2}/2 \qquad (366)$$

Lastly, use Eq. (98), $K_{F3} = K_{F2} – K_{F1}$, or $K_{F2} = K_{F3} + K_{F1}$ to obtain

$$K_{F2} = 3(\mu_0 T_0)^{1/2}/2 \qquad (367)$$

E.1.3. Manipulation of the Gravitational Constants.

We have introduced six gravitational constants K_{G1}, K_{G2}, K_{G3}, K_{G4}, K_{G5} and K_{G6} as well as the constants K_ϕ, K_c and K_{GC}. There is also Newton's gravitational constant which has a known value. These constants are related to each other, other constants, and numerical values via Eqs. (251), (264), (265), (267), (270), (276), (293), (346) and (353). In this section we will find useful relations by refining those equations.

Three of the ten gravitational constants are already determined. Eq. (276) gives $K_{G5} = 10^{-21}$ m^{-1}. By Eq. (293) we have $K_{G6} = 8.9167\text{x}10^{-28}$ m/kg. The known value of Newton's gravitational constant is $G_N = 6.6743\text{x}10^{-11}$ Nm^2/kg^2. Next, we observe from Eq. (264) that $K_{G3} = 4K_{T0}K_c/\xi_0 X_0 c^2$ and using Eq. (361), $K_{T0}/X_0 = K_{T\xi 0}/\xi_0$, $K_{G3} = 4K_{T\xi 0}K_c/\xi_0^2 c^2$ so we see that K_{G3} is not a free parameter; it is a combination of other constants. Eq. (267), $K_{G5} = K_{G3}(3[K_{G1}–K_{G2}]c^2/16)$, enables us to substitute in the value of K_{G3} from Eq. (264) and with $K_{G5} = 10^{-21}$ m^{-1}, $K_{G5} = (4K_{T\xi 0}K_c/\xi_0^2 c^2)3[K_{G1}–K_{G2}]c^2/16 = 3K_{T\xi 0}K_c[K_{G1}–K_{G2}]/4\xi_0^2 = 10^{-21}$ m^{-1}, or

$$K_{G1}–K_{G2} = 4\xi_0^2\text{x}10^{-21}\ \text{m}^{-1}/3K_{T\xi 0}K_c \qquad (368)$$

Next we use Eq. (270), $K_{G6} = 9K_{G4}[K_{G1}–K_{G2}]^2c^4/320\pi$, and substituting in Eq. (265), $K_{G4} = 4K_c^2K_{T0}/X_0\xi_0^2c^2 = 4K_c^2K_{T\xi 0}/\xi_0^3c^2$ leaves $K_{G6} = 9K_c^2K_{T\xi 0}[K_{G1}–K_{G2}]^2c^2/80\pi\xi_0^3$, and then substituting in Eq. (368), $K_{G6} = 9K_c^2K_{T\xi 0}[4\xi_0^2\text{x}10^{-21}\ \text{m}^{-1}/3K_{T\xi 0}K_c]^2c^2/80\pi\xi_0^3 = \xi_0\text{x}10^{-42}$ m^{-2} $c^2/5\pi K_{T\xi 0} = 8.9167\text{x}10^{-28}$ m/kg, where the last equality makes use of Eq. (293). Hence,

$$K_{G6} = \xi_0\text{x}10^{-42}\ \text{m}^{-2}\ c^2/5\pi K_{T\xi 0} = 8.9167\text{x}10^{-28}\ \text{m/kg} \qquad (369)$$

Next, we consider the gravitational flow constant K_{F4}, which has two possible values as expressed in Eqs. (339) and (341) above:

$$K_{F4KE} = [K_{G1}–K_{G2}] \tag{339}$$

$$K_{F4MO} = [K_{G1}–K_{G2}]/c \tag{341}$$

Recall now Eq. (353), $T_{SHAPIRO} = (K_\phi X_0/cK_{T0})\ln\{[(X_E^2 + R_S^2)^{1/2} + X_E]/[(X_M^2 + R_S^2)^{1/2} – X_M]\} \approx 200\ \mu s$. With $X_M = 57.9 \times 10^9$ m, $X_E = 149.6 \times 10^9$ m and $R_S = 0.6963 \times 10^9$ m the value of the logarithm is $\ln\{[(149.6^2 + 0.6963^2)^{1/2} + 149.6]/[(57.9^2 + 0.6963^2)^{1/2} – 57.9]\} \approx \ln(71457) = 11.177$. This leaves $K_\phi \approx cK_{T0}\,(200\ \mu s)/11.177X_0 = (2.998\times10^8\ m/s)K_{T0}(200\ \mu s)/11.177X_0$, or

$$K_\phi \approx (5365\ m)\ K_{T0}/X_0 \tag{370}$$

Eq. (370) shows that at the sun's surface $K_\phi/r = K_\phi/R_S = (5365\ m)\ K_{T0}/[X_0\ (6.963\times10^8\ m)]$ and hence $K_{T0}/X_0 >> K_\phi/R_S$, validating our assumption following Eq. (349) above.

And now with Eq. (346), $K_\phi = 3[K_{G1}–K_{G2}]\gamma M_S c^2\rho_0^2/8\pi\varepsilon_0$, we get $(5365\ m)\ K_{T0}/X_0 = 3[K_{G1}–K_{G2}]\gamma M_S c^2\rho_0^2/8\pi\varepsilon_0$. Rearranging, $K_{T\xi0}/\xi_0 = K_{T0}/X_0 = 3[K_{G1}–K_{G2}]\gamma M_S c^2\rho_0^2/(5365\ m)8\pi\varepsilon_0$, and we now substitute in Eq. (368), $K_{G1}–K_{G2} = 4\xi_0^2 \times 10^{-21}\ m^{-1}/3K_{T\xi0}K_c$, to get $K_{T\xi0}/\xi_0 = 3[4\xi_0^2 \times 10^{-21}\ m^{-1}/3K_{T\xi0}K_c]\gamma M_S c^2\rho_0^2/(5365\ m)8\pi\varepsilon_0$, or $K_{T\xi0}/\xi_0 = [2.967 \times 10^{-26}\ m^{-2}\ \xi_0^2/K_{T\xi0}K_c]\gamma M_S c^2\rho_0^2/\varepsilon_0$. With $\varepsilon_0 = 8.8542\times10^{-12}\ m^{-3}\ kg^{-1}\ s^4\ A^2 = 8.8542\times10^{-12}\ m^{-3}\ kg^{-1}\ s^2\ C^2$, $\gamma = 1$, $M_S = 1.988 \times 10^{30}$ kg and $c = 2.998 \times 10^8$ m/s this becomes $K_{T\xi0}^2/\xi_0^2 = [2.967 \times 10^{-26}\ m^{-2}\ \xi_0/K_c](1.988 \times 10^{30}\ kg)(2.998 \times 10^8\ m/s)^2\rho_0^2/(8.8542\times10^{-12}\ m^{-3}\ kg^{-1}\ s^2\ C^2)$, or

$$K_{T\xi0}^2/\xi_0^2 = (\xi_0\rho_0^2/K_c)5.988 \times 10^{32}\ m^3\ kg^2\ s^{-4}\ C^{-2} \tag{371}$$

Dimensional analysis. K_c is dimensionless and with ρ_0 in C/m^3 and ξ_0 in m we see that the right-hand side has units of $m(C^2/m^6)\ m^3\ kg^2\ s^{-4}\ C^{-2} = (kg^2/m^2s^4)$. $K_{T\xi0}\xi_0$ is a force, which is in N, or kg-m/s^2. ξ_0 is a length, so $(K_{T\xi0}/\xi_0)$ has the units of kg/m-s^2, and the dimensions are as expected.

Next we turn to Eq. (251), $G_N = 9K_{GC}[K_{G1}–K_{G2}]^2\rho_0{}^2c^4/16\pi\varepsilon_0$. Substituting in $K_{G1}–K_{G2}$ from Eq. (368), $G_N = 9K_{GC}[4\xi_0{}^2x10^{-21}\ m^{-1}/3K_{T\xi0}K_c]^2\rho_0{}^2c^4/16\pi\varepsilon_0 = K_{GC}[\xi_0{}^2x10^{-21}\ m^{-1}/K_{T\xi0}K_c]^2\rho_0{}^2c^4/\pi\varepsilon_0$. Rearranging and taking the square root $\rho_0 = (G_N\pi\varepsilon_0/K_{GC})^{1/2}K_{T\xi0}K_c/\{\xi_0{}^2c^2x10^{-21}\ m^{-1}\}$. Now $G_N = 6.6743x10^{-11}\ Nm^2/kg^2 = 6.6743x10^{-11}\ m^3/kg\text{-}s^2$, $\varepsilon_0 = 8.8542\times10^{-12}\ m^{-3}\ kg^{-1}\ s^4\ A^2 = 8.8542\times10^{-12}\ m^{-3}\ kg^{-1}\ s^2\ C^2$ and hence $G_N\pi\varepsilon_0 = (6.6743x10^{-11}\ m^3/kg\text{-}s^2)\pi(8.8542\times10^{-12}\ m^{-3}\ kg^{-1}\ s^2\ C^2) = 1.857x10^{-21}\ C^2/kg^2$, and $(G_N\pi\varepsilon_0)^{1/2} = 4.309x10^{-11}$ C/kg. Also, with $c = 2.998x10^8$ m/s, $c\ x10^{-21}m^{-1} = [(2.998x10^8\ m/s)^2x10^{-21}m^{-1}] = 8.988x10^{-5}\ m/s^2$ And so we obtain $\rho_0 = \{K_{T\xi0}K_c4.309x10^{-11}\ C/kg\}/\{K_{GC}{}^{1/2}\xi_0{}^28.988x10^{-5}\ m/s^2\}$ or

$$\rho_0 = [4.794x10^{-7}\ K_{T\xi0}K_c/\xi_0{}^2K_{GC}{}^{1/2}]\ C\ s^2/kg\ m \qquad (372)$$

Dimensional analysis. K_c and K_{GC} are dimensionless. $K_{T\xi0}\xi_0$ is a force, which is in N, or kg m/s^2. ξ_0 is a length, so $K_{T\xi0}/\xi_0{}^2$ has the units of kg/m^{2}s^2, and hence ρ_0 described in Eq. (371) has the dimensions of (kg/m^{2}s^2)(C s^2/kg m) = C/m^3 as expected.

E.1.4. Evaluating the Six Aetherial Quantum Constants.

We have six constants associated with the aetherial quantum: ξ_0, $K_{Q\xi0}$, $K_{T\xi0}$, T_0, m_0, and ρ_0. These constants are related through Eqs. (7), (91), (107), (141) and (361). Hence, we will reduce these constants to relations involving at most a single free aetherial quantum constant, ξ_0.

To begin, recall Eq. (372) and square it:

$$\rho_0{}^2 = [2.298x10^{-13}\ K_{T\xi0}{}^2K_c{}^2/K_{GC}\xi_0{}^4]\ C^2\ s^4/kg^2\ m^2 \qquad (373)$$

Now use Eq. (141), $K_{T0} = 3\rho_0{}^2X_0\xi_0{}^2/4K_c{}^2\varepsilon_0$, which, with Eq. (361), $K_{T0}/X_0 = K_{T\xi0}/\xi_0$, we can rewrite as $K_{T0}/X_0 = K_{T\xi0}/\xi_0 = 3\rho_0{}^2\xi_0{}^2/4K_c{}^2\varepsilon_0$. Then substitute in Eq. (373) to get $K_{T\xi0}/\xi_0 = 3([2.298x10^{-13}\ K_{T\xi0}{}^2K_c{}^2/K_{GC}\xi_0{}^4]\ C^2\ s^4/kg^2\ m^2)\xi_0{}^2/4K_c{}^2\varepsilon_0 = 3([2.298x10^{-13}\ K_{T\xi0}{}^2/K_{GC}\xi_0{}^2]\ C^2\ s^4/kg^2$

$m^2)/4\varepsilon_0$. With $\varepsilon_0 = 8.8542\times10^{-12}$ m^{-3} kg^{-1} s^2 C^2, and bringing $K_{T\xi0}{}^2/K_{GC}\xi_0{}^2$ to the left side, this becomes $K_{GC}\xi_0/K_{T\xi0} = (3\text{x}2.298\text{x}10^{-13}\ C^2\ s^4/kg^2\ m^2)/(4\text{x}8.8542\times10^{-12}\ m^{-3}\ kg^{-1}\ s^2\ C^2)$, or

$K_{GC}\xi_0/K_{T\xi0} = 1.947\text{x}10^{-2}$ m s^2/kg (374)

or

$K_{T\xi0} = (51.36\ kg/m\text{-}s^2)\xi_0 K_{GC}$ (375)

Now, to resolve $K_{Q\xi0}$, simply manipulate Eq. (7), $\xi_0 = (2K_{Q\xi0}/K_{T\xi0})^{1/4}$, to get $K_{Q\xi0} = \xi_0{}^4 K_{T\xi0}/2$, and using Eq. (375) we get

$K_{Q\xi0} = (51.36\ kg/m\text{-}s^2)\xi_0{}^5 K_{GC}/2 = (25.68\ kg/m\text{-}s^2)\xi_0{}^5 K_{GC}$ (376)

Next we use Eq. (91), $T_0 = K_{T0}/X_0$, and with Eq. (361), $K_{T0}/X_0 = K_{T\xi0}/\xi_0$ we obtain $T_0 = K_{T\xi0}/\xi_0$, and rearranging Eq. (375) to $K_{T\xi0}/\xi_0 = (51.36\ kg/m\text{-}s^2)K_{GC}$, we have

$T_0 = (51.36\ kg/m\text{-}s^2)K_{GC} = 51.36\ K_{GC}\ N/m^2$ (377)

And now we use Eq. (107), $T_0 = m_0c^2$, to get $m_0 = T_0/c^2$, and using Eq. (377) this is $m_0 = (51.36\ K_{GC}\ N/m^2)/(2.998\text{x}10^8\ m/s)^2$ or

$m_0 = 5.715\text{x}10^{-16}\ K_{GC}\ kg/m^3$ (378)

E.1.5. Expressing the Constants in their Reduced Forms.

With the results obtained in the above sections, we can now list all of the constants in their reduced forms. First, we have the flow constants. Eq. (377) gives $K_{F1} = (\mu_0T_0)^{1/2}/2$, Eq. (367) gives $K_{F2} = 3(\mu_0T_0)^{1/2}/2$ and Eq. (365) gives $K_{F3} = (\mu_0T_0)^{1/2}$. With Eq. (366), $T_0 = 51.36\ K_{GC}\ kg/m\text{-}s^2$, and with $\mu_0 = 1.257\ \text{x}10^{-6}$ m-kg-C^{-2}, $(\mu_0T_0)^{1/2} = 8.034\text{x}10^{-3}\ K_{GC}{}^{1/2}$ kg / s C and $(\mu_0T_0)^{1/2}/2 = 4.017\text{x}10^{-3}\ K_{GC}{}^{1/2}$ kg / s C we are left with

$K_{F1} = (\mu_0T_0)^{1/2}/2 = 4.017\text{x}10^{-3}\ K_{GC}{}^{1/2}$ kg / s C (379)

$K_{F2} = 3(\mu_0T_0)^{1/2}/2 = 1.2051\text{x}10^{-2}\ K_{GC}{}^{1/2}$ kg / s C (380)

$K_{F3} = (\mu_0T_0)^{1/2} = 8.034\text{x}10^{-3}\ K_{GC}{}^{1/2}$ kg / s C (381)

We now use Eq. (368), $K_{G1}–K_{G2} = 4\xi_0{}^2x10^{-21}$ m^{-1}/3$K_{T\xi0}K_c$ and Eq. (375), $K_{T\xi0}$ = (51.36 kg/m-s^2)ξ_0K_{GC}, to get $K_{G1}–K_{G2} = 4\xi_0{}^2x10^{-21}$ m^{-1}/3(51.36 kg/m-s^2)$\xi_0K_{GC}K_c$ or

$K_{G1}–K_{G2}$ = (2.596x10^{-23} s^2/kg)$\xi_0/K_{GC}K_c$ (382)

From Eq. (264) $K_{G3} = 4K_{T0}K_c/\xi_0X_0c^2$ and using Eq. (361), $K_{T0}/X_0 = K_{T\xi0}/\xi_0$, $K_{G3} = 4K_{T\xi0}K_c/\xi_0{}^2c^2$ and combining with Eq. (375), $K_{T\xi0}$ = (51.36 kg/m-s^2)ξ_0K_{GC}, we obtain K_{G3} = 4(51.36 kg/m-s^2)$K_{GC}K_c/\xi_0c^2$, or

K_{G3} = (205.5 kg/m-s^2)$K_{GC}K_c/\xi_0c^2$ (383)

From Eq. (265), $K_{G4} = 4K_c{}^2K_{T0}/X_0\xi_0{}^2c^2$ and using Eq. (361), $K_{T0}/X_0 = K_{T\xi0}/\xi_0$, $K_{G4} = 4K_c{}^2K_{T\xi0}/\xi_0{}^3c^2$ and combining with Eq. (375), $K_{T\xi0}$ = (51.36 kg/m-s^2)ξ_0K_{GC}, we obtain $K_{G4} = 4K_c{}^2$(51.36 kg/m-s^2)$K_{GC}/\xi_0{}^2c^2$, or

K_{G4} = (205.5 kg/m-s^2)$K_c{}^2K_{GC}/\xi_0{}^2c^2$ (384)

Above we have Eq. (339), $K_{F4KE} = [K_{G1}–K_{G2}]$, and Eq. (341), $K_{F4MO} = [K_{G1}–K_{G2}]/c$, and using Eq. (382), $K_{G1}–K_{G2}$ = (2.596x10^{-23} s^2/kg)$\xi_0/K_{GC}K_c$, these become:

K_{F4KE} = (2.596x10^{-23} s^2/kg)$\xi_0/K_{GC}K_c$ (385)

K_{F4MO} = (2.596x10^{-23} s^2/kg)$\xi_0/K_{GC}K_cc$ (386)

Next we manipulate Eq. (370), $K_\phi \approx$ (5365 m) K_{T0}/X_0, by using Eq. (361), $K_{T0}/X_0 = K_{T\xi0}/\xi_0$, to get $K_\phi \approx$ (5365 m) $K_{T\xi0}/\xi_0$, and then use Eq. (375), $K_{T\xi0}$ = (51.36 kg/m-s^2)ξ_0K_{GC}, to get $K_\phi \approx$ (5365 m) (51.36 kg/m-s^2)ξ_0K_{GC}/ξ_0 or

$K_\phi \approx 2.755x10^5\ K_{GC}$ kg/s^2 (387)

And now we gather some earlier evaluations of constants:

$K_{G5} \approx 10^{-21}$ m^{-1} (276)

K_{G6} = 8.9167x10^{-28} m/kg (293)

$G_N = 6.6743 \times 10^{-11}\ Nm^2/kg^2$ (251)

$K_{T\xi 0} = (51.36\ kg/m\text{-}s^2)\xi_0 K_{GC}$ (375)

$K_{Q\xi 0} = (51.36\ kg/m\text{-}s^2)\xi_0^5 K_{GC}/2 = (25.68\ kg/m\text{-}s^2)\xi_0^5 K_{GC}$ (376)

$T_0 = (51.36\ kg/m\text{-}s^2)K_{GC} = 51.36\ K_{GC}\ N/m^2$ (377)

$m_0 = 5.715 \times 10^{-16}\ K_{GC}\ kg/m^3$ (378)

And now take Eq. (372), $\rho_0 = [4.794 \times 10^{-7}\ K_{T\xi 0} K_c/\xi_0^2 K_{GC}^{1/2}]\ C\ s^2/kg\ m$, and then use Eq. (375), $K_{T\xi 0} = (51.36\ kg/m\text{-}s^2)\xi_0 K_{GC}$ to obtain $\rho_0 = [4.794 \times 10^{-7}\ (51.36\ kg/m\text{-}s^2)\xi_0 K_{GC}\ K_c/\xi_0^2 K_{GC}^{1/2}]\ C\ s^2/kg\ m$, or

$\rho_0 = (2.463 \times 10^{-5}\ C/m^2) K_c K_{GC}^{1/2}/\xi_0$ (388)

The analysis-cube constants are only used for our analysis and are not fundamental constants. Instead, we just divide up the aetherial quantum cube into n^3 smaller analysis-cubes and have used n to obtain Eq. (10), $X_Q = \xi_Q/n$, Eq. (13), $K_{T0} = K_{T\xi 0}/n$ and Eq. (14), $K_{Q0} = K_{Q\xi 0}/n^5$.

We have also used several previously well-known fundamental physical constants:

$c = 2.998 \times 10^8\ m/s$ (389)

$\varepsilon_0 = 8.8542 \times 10^{-12}\ C^2 s^2/kg\text{-}m^3$ (390)

$\mu_0 = 1.2566 \times 10^{-6}\ m\text{-}kg\text{-}C^{-2}$ (391)

We see that none of the assignments have resulted in contradictions. At this point we now have only four free parameters remaining in the theory, K_{GC}, K_c, K_{G1} and ξ_0, as can be seen by looking at the numbered equations here in section E.1.5. (Eq. (382) gives us a relationship for K_{G1}–K_{G2}. Here we assign K_{G2} as the dependent variable and K_{G1} as the free parameter.) While n is technically a free parameter, it is merely an artifact of the analysis, not a fundamental constant.

E.1.6. A Determination of K_{G1} and K_{G2}.

It has been noted above in section D.3.5.1 that the two components of the gamma-force originate from the Extrinsic-Energy Force-Reduction Law, a law which is repeated here:

The Extrinsic-Energy Force-Reduction Law. The presence of extrinsic-energy decreases the positive (negative) attached-aether tension and the negative (positive) attached-aether quantum-force by an amount proportional to the amount of extrinsic-energy present with a constant of proportionality K_{G1} (K_{G2}).

Now recall Eqs. (218) and (219)

$$\mathbf{F}_{\gamma PT} = 4K_c(1+ K_{GC})K_{T0}(X_0/\xi_0)x\hat{\mathbf{r}} \tag{218}$$

$$\mathbf{F}_{\gamma PQ} = -4K_cK_{GC}K_{T0}(X_0/\xi_0)x\hat{\mathbf{r}} \tag{219}$$

Also recall that $\mathbf{F}_{\gamma PT}$ and $\mathbf{F}_{\gamma PQ}$ act to reduce the tension and quantum-force, respectively. Hence, $F_{\gamma PT}$ and $F_{\gamma PQ}$ are the force reductions K_{G1} and K_{G2} of the Extrinsic-Energy Force-Reduction Law. We can form the ratio of the force reductions as $K_{G1}/K_{G2} = F_{\gamma PT}/F_{\gamma PQ} = (1+ K_{GC})/K_{GC} = 1 + 1/K_{GC}$. This leads to $1/K_{GC} = K_{G1}/K_{G2} - 1 = (K_{G1} - K_{G2})/K_{G2}$ or $K_{GC} = K_{G2}/(K_{G1} - K_{G2})$. Now, recall Eq. (382) $K_{G1}-K_{G2} = (2.596\text{x}10^{-23}\ s^2/kg)\xi_0/K_{GC}K_c$. Substituting $K_{GC} = K_{G2}/(K_{G1} - K_{G2})$ into Eq. (382) leaves $K_{G1}-K_{G2} = (K_{G1}-K_{G2})(2.596\text{x}10^{-23}\ s^2/kg)\xi_0/K_{G2}K_c$, or

$$K_{G2} = (2.596\text{x}10^{-23}\ s^2/kg)\xi_0/K_c \tag{392}$$

We now rearrange Eq. (382) to get $K_{G1} = (2.596\text{x}10^{-23}\ s^2/kg)\xi_0/K_{GC}K_c + K_{G2}$. We then substitute in the expression for K_{G2} from Eq. (392) to get $K_{G1} = (2.596\text{x}10^{-23}\ s^2/kg)\xi_0/K_{GC}K_c + (2.596\text{x}10^{-23}\ s^2/kg)\xi_0/K_c$, or

$$K_{G1} = (2.596\text{x}10^{-23}\ s^2/kg)\xi_0(1/K_{GC}K_c + 1/K_c) = (1 + 1/K_{GC})(2.596\text{x}10^{-23}\ s^2/kg)\xi_0/K_c \tag{393}$$

E.1.7. The New Parameters Needed by our Theory.

Eqs. (392) and (393) express K_{G1} and K_{G2} in terms of ξ_0, K_{GC} and K_c. We now see that we have needed only three free parameters for our theory, the aetherial quantum size ξ_0 and the two coupling parameters K_{GC} and K_c. Here "free parameters" means those for which we have no estimated value. We have also needed two parameters with values that are (approximately) determined by experiment, K_{G5} and K_{G6}.

E.2 – Neutron Stars & Super Massive Objects (There are no Black Holes)

Eqs. (276) and (293) give estimates of $K_{G5} = 10^{-21}$ m^{-1} and $K_{G6} = 8.9167 \times 10^{-28}$ m/kg $\approx 9 \times 10^{-28}$ m/kg, respectively, which allows evaluation of Eq. (274), $M_{EFF} = M - 3K_{G5}MR - 6K_{G6}M^2/R$ for various stellar objects of interest. (Here we are evaluating for $\gamma_M = 1$.) As a first example consider M_S and R_S to be the mass and radius of the sun, respectively, $M_S \approx 2 \times 10^{30}$ kg, $R_S \approx 7 \times 10^8$ m. We get $M_{SEFF} \approx M_S - 3K_{G5}M_SR_S - 6K_{G6}M_S^2/R_S = M_S - (3\times10^{-21}\ \text{m}^{-1}\times7\times10^8\ \text{m})M_S - [(6\times9\times10^{-28}\ \text{m/kg}\ \times2\times10^{30}\ \text{kg})/7\times10^8\ \text{m}]M_S = M_S - (2.1 \times 10^{-12})M_S - (1.54 \times 10^{-5})M_S$, or

$$M_{SEFF} = M_S - (1.54 \times 10^{-5})M_S \qquad (394)$$

With Eq. (394) we see that our earlier approximations of $M_{SEFF} \approx M_S$ are valid.

Now we'll evaluate M_{EFF} for a neutron star, with a radius of $R_N = 10^4$ m and a mass of $M_N = 1.5\ M_S \approx 3 \times 10^{30}$ kg. In this case, $M_{NEFF} = M_N - 3K_{G5}M_NR_N - 6K_{G6}M_N^2/R_N = M_N - (3\times10^{-21}\ \text{m}^{-1}\times10^4\ \text{m})M_N - [(6\times9\times10^{-28}\ \text{m/kg}\ \times3\times10^{30}\ \text{kg})/10^4\ \text{m}]M_N = M_N - (3\times10^{-17})M_N - 1.62M_N$, or

$$M_{NEFF} \approx M_N - 1.62M_N = -0.62M_N \qquad \text{(Under Overly Simplistic Assumptions)} \qquad (395)$$

Of course with a negative mass, Eq. (395) is untenable. This indicates that our present simplified treatment breaks down for the case of neutron stars. The reason for the failure of Eq. (395) to accurately handle the case of neutron stars is that the negative second-field-mass has now become large enough that it will significantly affect the determination of $\mathbf{P}_{GL}$. Thus far in our analysis we've neglected the negative second-field-mass effect on $\mathbf{P}_{\mathbf{GL}}$, and we see that neutron stars are out of scope for such an analysis. In Appendix J below we develop a treatment for hydrostatic equilibrium of stars that includes the effects of the negative second-field-mass. This treatment allows for an extension of our theory into the cases of neutron stars and other super dense objects. It is noteworthy that our theory differs from general relativity in that it does not have a singularity, and hence our theory has no black holes. Appendix J will show that super massive objects will instead be objects of a finite size.

E.3 – Aetherial Speculations and a Proposed Experiment

E.3.1. Speculations on the Aetherial Control. While theoretical physics advances are interesting in their own right, the goal of science is of course is to serve mankind and make lives better. Toward this aim, we wish not to simply understand our world, but also to control it. It is known that bodies moving through the aether experience clock retardation. If we can obtain the control necessary to move the aether past bodies (and generate an "aether wind") we should be able to cause a clock retardation that way as well. Such an advance would have tremendous advantages, as we could put aetherial wind generators inside of ambulances to buy precious time for medical personnel, or, we could send patients to the future where cures or medical staff and treatments might be available. (Clock retardation is essentially travel of a clock forward in time.) Many other practical advances are possible.

Also important would be the realm of travel. Since it is the aetherial dynamics that imposes the speed of light limit, if we have aetherial control we may be able to achieve travel at faster than the speed of light.

Once we fully understand gravitation, we may also be able to control gravity.

However, while aetherial control would have tremendous promise, it may also prove difficult to achieve. Section E.3.5 discusses a first possible test aimed toward achieving aetherial control.

E.3.2. Speculations on the Remaining Aetherial Parameters.

A rather obvious speculation is that the positive-aether quanta are single positrons and that the negative-aether quanta are single electrons. From such a speculation we can derive relations relating the remaining aetherial parameters. Recall Eqs. (378) and (388):

$$m_0 = 5.715\text{x}10^{-16}\ K_{GC}\ \text{kg/m}^3 \tag{378}$$

$$\rho_0 = (2.463\text{x}10^{-5}\ \text{C/m}^2)K_cK_{GC}^{1/2}/\xi_0 \tag{388}$$

With each aetherial quantum being an electron or a positron, it will contain a single charge of $1.602\text{x}10^{-19}$ C, and the aetherial charge density will be $\rho_0 = 1.602\text{x}10^{-19}\ \text{C}/\xi_0^3$. Setting that equal to the value given in Eq. (388) leads to $1.602\text{x}10^{-19}\ \text{C}/\xi_0^3 = (2.463\text{x}10^{-5}\ \text{C/m}^2)K_cK_{GC}^{1/2}/\xi_0$, or

$$K_cK_{GC}^{1/2}\xi_0^2 = (1.602\text{x}10^{-19}\ \text{C})/(2.463\text{x}10^{-5}\ \text{C/m}^2) = 6.504\ \text{x}10^{-15}\ \text{m}^2 \tag{396}$$

In our present speculation, each aetherial quantum has a mass equal to that of the electron and we obtain $m_0 = m_e/\xi_0^3 = 9.109\text{x}10^{-31}\ \text{kg}/\xi_0^3$. Setting that equal to the value in Eq. (378) leads to $9.109\text{x}10^{-31}\ \text{kg}/\xi_0^3 = 5.715\text{x}10^{-16}\ K_{GC}\ \text{kg/m}^3$, or

$$K_{GC} = 9.109\text{x}10^{-31}\ \text{kg}/(5.715\text{x}10^{-16}\ \xi_0^3\ \text{kg/m}^3) = 1.594\text{x}10^{-15}\ \text{m}^3/\xi_0^3 \tag{397}$$

Substituting (397) into (396), $K_c(1.594\text{x}10^{-15}\ \text{m}^3/\xi_0^3)^{1/2}\xi_0^2 = 6.504\ \text{x}10^{-15}\ \text{m}^2$, or

$$K_c\xi_0^{1/2} = 6.504\text{x}10^{-15}\ \text{m}^2/(1.594\text{x}10^{-15}\ \text{m}^3)^{1/2} = 1.629\text{x}10^{-7}\text{m}^{1/2} \tag{398}$$

We have assumed K_c is small to get convergence of K_T and K_Q when analysis-cubes move (see Appendices F, G and H and portions of sections C and D) so let's assume $K_c = 1.629\text{x}10^{-5}$. In that case Eq. (398) leads to $\xi_0^{1/2} = 10^{-2}\ \text{m}^{1/2}$, or $\xi_0 = 10^{-4}$ m, and with that value Eq. (397) informs that

K_{GC} = 1.594x10^{-3}, which is small. (Notice that with K_c = 1.629x10^{-4} we obtain $\xi_0 = 10^{-6}$ m, and K_{GC} = 1.594x10^{3} is no longer small.) If $\xi_0 = 10^{-4}$ m and with each quantum containing a single charge, we could possibly isolate an aetherial volume. (See section E.3.5 below.) Under these assumptions we also have a charge density of ρ_0 = 1.602x10^{-7} C/m^3, a mass density of m_0 = 9.109x10^{-19} kg/m^3 and recalling Eq. (377) we find a tension of T_0 = 51.36 K_{GC} N/m^2 = (51.36 K_{GC} N/m^2)(1.594x10^{-3})(1 PSI/[6895 N/m^2])=1.187x10^{-5} PSI. Note that water has a mass density of 10^3 kg/m^3. For these assumptions, the aether is vanishingly light and the tension quite small.

However, if $\xi_0 = 10^{-4}$ m, then an atom is much smaller than ξ_0, and the detached-aether inside the atom (the charges in the nucleus and electron cloud) have a much higher density than the attached-aether, violating one of our assumptions above. Such a condition may exist, but then our analysis above will need modification in the near field. We'll now look briefly at that issue.

E.3.3. Derivation of Poisson's Equation in the Near Field When the Detached-Aether density Exceeds the Attached-Aether Density.

In section E.3.2 we assume that one aetherial quantum contains one electric charge, and therefore a bare proton contains one quantum of positive-detached-aether. The positive-detached-aether of the proton has a density far larger than the positive-attached-aether. To achieve neutrality, positive-attached-aether will be repelled from, and negative-attached-aether attracted to, the region $r < R_P$, where R_P is the radius of the proton. (Here, the origin of our coordinate system is at the center of the proton.) We will now assume that the proton is a uniform sphere of charge for this analysis. We will divide the detached-aether of the proton up into spherical shells, each with a volume of $4\pi r_D^2 dr_D$ where r_D is the distance from the center of the shell to the origin and dr_D is the thickness of the shell.

The total negative-attached-aether needed to neutralize the positive-detached-aether will be one quantum minus the amount of positive-attached-aether that leaves the proton. (Prior to the proton occupying the region, there is equality of the negative and positive attached-aether density in that region. When a proton enters the region it will expel some or all of the positive-attached-aether.) The amount of positive-attached-aether originally in the region $r < R_P$ is $(R_P/R_{\xi 0})^3 q_0$, where q_0 is one quantum of aether and $R_{\xi 0}$ is the radius of an undisturbed attached-aetherial sphere that contains one quantum of aether. Hence, the total negative-attached-aether needed to neutralize the proton's detached-aether is $q_0[1 - f(R_P/R_{\xi 0})^3]$ where f is the fraction of positive-attached-aether that leaves the proton. This negative-attached-aether will come from the region between R_P and some boundary radius R_B. Before compression, the aether density within the region $R_P < r < R_B$ is the undisturbed aether value of $\rho_0 = q_0/(4/3)\pi R_{\xi 0}^3$. This allows us to find R_B, as we have $q_0[1 - f(R_P/R_{\xi 0})^3] = \rho_0[(4/3)\pi R_B^3 - (4/3)\pi R_P^3] = [q_0/(4/3)\pi R_{\xi 0}^3][(4/3)\pi R_B^3 - (4/3)\pi R_P^3] = [q_0/R_{\xi 0}^3][R_B^3 - R_P^3]$, or $1 - f(R_P/R_{\xi 0})^3 = R_B^3/R_{\xi 0}^3 - R_P^3/R_{\xi 0}^3$, or $1 - f(R_P/R_{\xi 0})^3 + R_P^3/R_{\xi 0}^3 = R_B^3/R_{\xi 0}^3$, or $R_B = R_{\xi 0}[1 - f(R_P/R_{\xi 0})^3 + R_P^3/R_{\xi 0}^3]^{1/3} = R_{\xi 0}[1 + (1 - f)(R_P/R_{\xi 0})^3]^{1/3} \approx R_{\xi 0}[1 + (1 - f)(R_P/R_{\xi 0})^3/3]$. We see that when all the positive-attached-aether leaves the proton, $f = 1$, we get $R_B = R_{\xi 0}$, and when none does $f = 0$, we get $R_B = R_{\xi 0}[1 + (R_P/R_{\xi 0})^3/3]$.

The negative-attached-aether within the sphere of radius R_B will be pulled into the proton, a sphere of radius R_P. Each spherical shell originally within the sphere of radius R_B will compress radially by the ratio of the spherical volumes, $dR_2 = dR_1(R_P/R_B)^3$, and there is a similar compression in the transverse directions also. A thin, negative-attached-aether spherical shell originally just inside radius R_B is pulled in to a radius just inside R_P, and $\mathbf{N}(R_P) = -(R_B - R_P)$ for that spherical shell. A negative-attached-aether spherical shell originally at radius gR_B (where g is a variable between 0 and 1) is pulled in to a radius $r = gR_P$, and $\mathbf{N}(r) = -g(R_B - R_P)$ for that spherical shell. Now, g is a variable that ranges from 0 to 1 as r ranges from 0 to R_P, or $g = r/R_P$, and hence we obtain

$$\mathbf{N}(r) = -[(R_B - R_P)/R_P]\mathbf{r} \qquad \text{for } r < R_P \tag{399}$$

To evaluate $\mathbf{N}(r)$ in the region $r > R_P$, we notice that any spherical shell will maintain its volume since there is no detached-aether in that region. We consider a spherical shell with inner radius R_P and outer radius r. If we do a thought experiment wherein the proton is gradually removed, that shell will expand to an inner radius at R_B and an outer radius R_O determined by the shell volume. We will have $(4/3)\pi(r^3 - R_P^3)$ for the volume of the shell when the proton is there and $(4/3)\pi(R_O^3 - R_B^3)$ when the proton is missing. Equating these, since the density is always constant outside of R_P, $(4/3)\pi(r^3 - R_P^3) = (4/3)\pi(R_O^3 - R_B^3)$, or $r^3 - R_P^3 = R_O^3 - R_B^3$, or $R_O^3 - r^3 = R_B^3 - R_P^3$. We also have $N = R_O - r$, or $R_O = N + r$. Substituting in we get $(N + r)^3 - r^3 = N^3 + 3N^2r + 3Nr^2 = R_B^3 - R_P^3$. And now, since $r >> N$ the higher order terms in N can be neglected and we obtain

$$\mathbf{N}(r) = -[(R_B^3 - R_P^3)/3r^2]\hat{\mathbf{r}} \qquad \text{for } r > R_P \tag{400}$$

Inspection of Eqs. (399) and (400) shows that $\mathbf{N}(r)$ again obeys Poisson's Equation. In this situation, $\mathbf{N}$ no longer equals $-\mathbf{P}$, but Poisson's Equation is obtained even in the near field. As long as the flow and tension forces are still described as in section C above, the derivation of Maxwell's Equations and the Lorentz Force Equation should follow in the near field as well, although a more rigorous derivation of these matters should be undertaken.

E.3.4. Comment on the Arbitrariness of the Assumptions of Section E.3.2.

Of course in section E.3.2 the assumptions we make are quite arbitrary. It is entirely possible that the elemental aetherial quantum is not made of electrons and positrons and that the coupling constant differs substantially from what is assumed here. In section E.3.2, we are just briefly exploring one possible parametric set for the aetherial constituents.

E.3.5. A Possible Initial Experiment to Look for Aetherial Isolation.

It may be possible to experimentally isolate a small volume of aether and do tests on it. Since we have identified detached-aether as electric charge, if we make two hollow beams, one of electrons and one of positrons, then the hollow region inside the beams may be isolated from the region outside the beams. This proposition follows provided the beams completely expel the ambient attached-aether in the regions they occupy while also detaching the attached-aether in the hollow region inside the beams from the attached-aether in the region outside of the beams. Next, the experiment should focus the beams so that the volume of the isolated attached-aether is in the shape of an American-football, pinching off the ends of the isolated aether. And then, the beams should be made to move that isolated aether by changing the focal points of the beams. It is desired that this focal change move the aether at a great speed, approaching that of light. Then, a beam of fast-decaying particles should be shot across the moving aether to determine if the moving aether causes a dilation in decay time of those particles. If the decay time is dilated, that would serve as proof that moving aether can be used to "send things to the future". Of course, the more speculative benefits of such forward time travel identified in section E.3.1 would require considerable inventiveness before they will come to fruition.

E.4 – The Precursor Works

An earlier work [15] deriving Maxwell's Equations laid foundations for some of the present work. Notably, the present work improves upon a weakness of the earlier work. (See Eqs. (116) through (119) above for the improved treatment.)

This work also builds upon other prior works of the author which show how we can once again understand our world by returning to physical models based on realist and absolute physics. The initial work on absolute theory [16] was concerned with a "special" absolute theory as an alternative to special relativity. The ABC Preon Model [17, 18] has shown how a realist physical model made up of three preons and three anti-preons can be used to model all known particles, providing an answer to the generational problem (why there are three generations of quarks and leptons) while also reducing the number of forces thought to exist. (The weak force is identified as a radioactive decay.) Another work [19] shows how an underlying philosophy of realism and absolute theory can be used to derive a new formulation of high velocity quantum mechanics equations. By returning to absolute theory we can easily understand all quantum behavior since instantaneous collapse of a real physical wave function is now allowed, unlike the situation required under relativity. Wave particle duality is understood as an instantaneous collapse of a wave of large physical size to one of smaller size. All of these advances are made possible by one simple step – a return to a philosophy of physical models, realism and absolute theory. That earlier philosophy, which included an aether, has been set aside for about a century because of the illusory simplicity of a philosophy of relativity. However, since the author's works, including especially the present work, have shown that returning to the older philosophy leads to answers to the most significant open questions of physics, it should be clear that it is the philosophy of relativity that should be set aside.

E.5 – Acknowledgements and Appreciations

While this is a single-author effort (the use of "we" in the above exposition refers to you, the reader, and I) there are still many who have contributed in non-author capacities. First, I wish to thank members of my immediate family, wife Geri, son Nate and son Ben for listening to and occasionally commenting on this work. Thanks to commenters at the site "Above Top Secret" for their online discussions [20]. Thanks also to Emilio Panarella, the editor of Physics Essays, for allowing review and publication of my precursor works through the years. Thanks to Al Zeller for critical comments and objections that led to improvements. And special thanks to my son Nick and daughter Catherine who each carefully read through significant portions of the work to verify the math and logic, finding several items in need of improvement.

And lastly, I wish to thank the Father, the Son and the Holy Spirit for continual guidance and assistance [21]. Frequently during my work as a physicist ideas and inspiration have followed prayers for guidance.

E.6 – References

[1] A.A. Michelson and E.W. Morley, Am. J. Sci. **s3-34**, 333 (1887).

[2] (Cambridge University Press, Cambridge, 1900).

[3] H.A. Lorentz, Proc. R. Acad. Amsterdam **6**, 809 (1904).

[4] A. Einstein, Ann. Phys. **17**, 891 (1905).

[5] A. Einstein, B. Podolsky, and N. Rosen, Phys. Rev. **47**, 777 (1935).

[6] J.S. Bell, Physics (NY) **1**, 195 (1965).

[7] A. Aspect, J. Dalibard, and G. Roger, Phys. Rev. Lett. **49**, 1804 (1982).

[8] Gnedin, O. Y., et al. The Astrophysical Journal Letters, **720** (1), L108–L112 (2010).

[9] McMillan, P. J., Monthly Notices of the Royal Astronomical Society **414** (3): 2446–2457 (2011).

[10] Karukes, E. V., et al., https://arxiv.org/abs/1912.04296.

[11] Pitjeva, E. V., "EPM ephemerides and relativity", Relativity in Fundamental Astronomy, Proceedings IAU Symposium, No. 261, 170-178 (2009).

http://adsabs.harvard.edu/full/2010IAUS..261..170P

[12] Price, M. P. and Rush, W. F., Am. J. Phys., Vol. **47**, No. 6: 531–534 (June 1979).

[13] https://nssdc.gsfc.nasa.gov/planetary/factsheet/

[14] I. I. Shapiro, Phys. Rev. Lett. **13** (26): 789–791, 1964.

[15] "A Derivation of Maxwell's Equations from a Simple Two-Component Solid-Mechanical Aether", D.J. Larson, Physics Essays, vol. 11, 524-530 (1998).

[16] "An Absolute Theory for the Electrodynamics of Moving Bodies", D.J. Larson, Physics Essays, vol. 7, 476-489, (1994).

[17] "The A-B-C Preon Model", D.J. Larson, Physics Essays, vol. 10, 27-34, (1997).

[18] "Additional Evidence and Predictions for the A-B-C Preon Model", D.J. Larson, Physics Essays, Volume 30: Pages 347-355, (2017).

[19] "An Empirical and Classical approach for Non-Perturbative, High Velocity, Quantum Mechanics", D.J. Larson, Physics Essays, Volume 30: Pages 264-268, 2017.

[20] See http://www.abovetopsecret.com/ for discussions that proved helpful for this work, one thread of which is http://www.abovetopsecret.com/forum/thread1021214/pg394#pid24635493.

[21] The Holy Bible (available in several versions).

Appendices.

Appendices F, G and H contain derivations that are extremely similar to other derivations found in the main body of this work; these appendices are included here for completeness. Appendix I investigates issues related to the case where the sun is moving with respect to the aether. Appendix J contains analysis of dense stellar objects, and Appendix K contains some concluding thoughts.

Appendix F – The Tension, Quantum-pressure and Delta-forces Within a Spherical Region Containing Unlike-Kind Detached-Aether.

F.1. A Spherical Attached-Aether Region Containing Unlike-Kind Detached-Aether. Consider now the injection of a sphere of negative-detached-aether into the positive-attached-aether. In this case, with $\rho_{PD} = 0$, Eq. (50) yields $\nabla \cdot \mathbf{P} = -\rho_{ND}/2\rho_0$, which has the solutions

$$\mathbf{P}_{\mathbf{IN}} = -(\rho_{ND}/6\rho_0)(x\mathbf{i} + y\mathbf{j} + z\mathbf{k}) = -(\rho_{ND}/6\rho_0)\mathbf{r} \qquad (r < R_0) \qquad \text{(F1)}$$

$$\mathbf{P}_{\mathbf{OUT}} = -P_0R_0^2\hat{\mathbf{r}}/r^2 \qquad (r > R_0) \qquad \text{(F2)}$$

In Eqs. (F1) and (F2) R_0 is the radius of the sphere and $P_0 = (\rho_{ND}/6\rho_0)R_0$ is the magnitude of P at R_0. Verifying Eqs. (F1) and (F2) are solutions to Eq. (50): $\nabla \cdot \mathbf{P}_{\mathbf{IN}} = (1/r^2)\partial(r^2[(-\rho_{ND}/6\rho_0)r])/\partial r =$

$(1/r^2)\partial[(-\rho_{ND}/6\rho_0)r^3]/\partial r = -\rho_{ND}/2\rho_0$ and $\nabla \cdot \mathbf{P}_{\mathbf{OUT}} = (1/r^2)\partial(r^2[-P_0R_0/r^2])/\partial r = -(1/r^2)\partial P_0R_0/\partial r = 0$.

Notice that immersion into attached-aether of unlike-kind detached-aether will lead to an inward compression of the attached-aether. Since tension is a force directed inward, the radially-inward motion of the attached-aether will do negative work against the tension. And since the quantum-pressure force is directed outward, the radially-inward motion of the attached-aether will do positive work against the quantum-pressure field.

F.2. The Tension-Force and Tension-Energy Inside a Spherical Attached-Aether Region Containing Unlike-Kind Detached-Aether. Eq. (F1) informs us that adding unlike-kind detached-aether into a spherical region will cause the attached-aether cubes within that region to compress equally in each cartesian direction. Focusing first on the tension, Eq. (15) informs us that a compression δX of an analysis-cube leads to a tension-force magnitude $F_{TP} = K_{T0}(X_0 - \delta X) = K_{T0}X_0(1 - \delta X_0/X_0)$ within that analysis-cube.

Consider cube-J, where J is some large integer with the center of cube-J separated from the center of the detached-aether-sphere by $-(J-1)X_0$. The energy to displace cube-J is given by Eq. (32), $W_{TD} = (X_0/\xi_0)\, K_c \int K_T X_0\, dx$, where now the displacement is $(J-1)\delta X$, with $\delta X/2$ coming from each of cubes 1 and J, and an additional δX coming from each of the cubes between cube-1 and cube-J. As a first order approximation, $K_T = K_{T0}$ and integrating Eq. (32) from 0 to $(J-1)\delta X$, the work done against the tension on cube-J from the displacement is $W_{TDJ} = -(J-1)(X_0/\xi_0)K_c K_{T0} X_0 \delta X$. (Compression means the cube originally at negative x moves in the positive direction. It moves $(J-1)\delta X$, so that is the upper limit of integration, and the work is negative as the energy is relaxed.) Setting the expansion effects aside to focus on the effect of displacement, the tension-energy of cube-J is $E_T = E_{T0} + W_{TDJ} = (1/2)K_{T0}X_0^2 - K_c K_{T0} X_0^2 (J-1)\delta X/\xi_0$. With $E_T = (1/2)K_T X_0^2$, we arrive at the expression $K_T = K_{T0}(1 - 2K_c(J-1)\delta X/\xi_0)$. The tension-force remains $F_T = K_T X_0$, but now K_T includes the next order correction. As detached-aether is slowly injected, the center of the Jth cube will move a distance $(J-1)\delta X$. At the beginning of the motion F_{TP} is of course just $K_{T0}X_0$, and it is at the end that $F_{TP} = K_{T0}X_0[1 - 2K_c(J-1)\delta X/\xi_0]$. With x now defined as the deviation of the cube center from its nominal center (which varies from 0 to $(J-1)\delta X$ during the detached-aether injection) and adding a subscript N for the **N**egative-detached-aether immersion, the full second-order expression for tension-force in the analysis-cube is

$F_{TPN} = K_{T0}X_0[1 - 2K_cx/\xi_0]$ (F3)

Eq. (F3) is seen to be the nominal value of $K_{T0}X_0$ when there is no injected detached-aether (and $x = 0$), and $K_{T0}X_0[1 - 2K_c(J-1)\delta X/\xi_0]$ when we reach full injection (and $x = (J-1)\delta X$). The linearity follows because δX (and hence $(J-1)\delta X$) is linear with the amount of detached-aether injected. With the second-order expression for the tension just derived, we can now include the second-order effect on the cube tension-energy. Eq. (32) gives the work done on the field by the cube displacement as $W_{TDJ} = -(X_0/\xi_0)\ K_c \int F_{TP}\ dx = -K_cK_{T0}X_0(X_0/\xi_0) \int [1-2K_cx/\xi_0]\ dx = -K_cK_{T0}X_0(X_0/\xi_0)x + K_c^2K_{T0}(X_0/\xi_0)^2x^2 = -K_cK_{T0}X_0(X_0/\xi_0)(J-1)\delta X + K_c^2K_{T0}(X_0/\xi_0)^2(J-1)^2\delta X^2 = -K_cK_{T0}X_0(X_0/\xi_0)P + K_c^2K_{T0}(X_0/\xi_0)^2P^2$. (For the displacement, the integral is evaluated between zero and its final displacement $(J-1)\delta X = P$.)

The compression energy is now calculated by including a minus sign in Eq. (63) since compression will release energy from the tension, $W_{TC} = -2 \int K_TX_0\ dw$. From Eq. (F3), $F_{TPN} = K_{T0}X_0[1 - 2K_cx/\xi_0]$. We now use Eq. (62), $x = (P/\delta X)w$, to obtain $K_TX_0 = K_{T0}X_0[1-2K_c(P/\delta X)w/\xi_0]$. As specified in Eq. (63) we integrate from $w = 0$ to $w = \delta X/2$. Hence $W_{TC1} = -2 \int K_TX_0\ dw = -2K_{T0}X_0 \int [1-2K_c(P/\delta X)w/\xi_0]\ dw = -2K_{T0}X_0w + 2K_{T0}X_0K_c(P/\delta X)w^2/\xi_0 = -K_{T0}X_0\delta X + K_{T0}X_0K_cP\delta X/2\xi_0$. Recall that Eq. (63) is for one dimension only, and that inside the sphere the compression will be the same in all three dimensions. Hence, $W_{TC3} = -3K_{T0}X_0\delta X + 3K_{T0}X_0K_cP\delta X/2\xi_0$. The total tension-energy of an arbitrary analysis-cube inside the spherical region is the undisturbed energy plus the displacement energy plus the compression energy, $E_{TPNI} = (1/2)K_{T0}X_0^2 - K_cK_{T0}X_0(X_0/\xi_0)P + K_c^2K_{T0}(X_0/\xi_0)^2P^2 - 3K_{T0}X_0\delta X + 3K_{T0}X_0K_cP\delta X/2\xi_0$, or, $E_{TPNI} = K_{T0}X_0^2[(1/2) - K_cP/\xi_0 +$

$K_c^2(P/\xi_0)^2 - 3\delta X/X_0 + 3K_cP\delta X/2X_0\xi_0]$. Here we have derived the force for positive P. For **P** at any angle we get the same value, so we will replace P by its absolute value, $|\mathbf{P}|$:

$$E_{TPNI} = K_{T0}X_0^2[(1/2) - K_c|\mathbf{P}|/\xi_0 + K_c^2(|\mathbf{P}|/\xi_0)^2 - 3\delta X/X_0 + 3K_c|\mathbf{P}|\delta X/2X_0\xi_0] \quad \text{(F4)}$$

In Eq. (F4) and the above paragraph, the subscript TPNI refers to Tension of the Positive-attached-aether due to immersed Negative-detached-aether in the Inside region of the sphere.

F.3. The Quantum-Force and Quantum-Energy Fields Inside a Spherical Attached-Aether Region Containing Unlike-Kind Detached-Aether. When slowly adding unlike-kind detached-aether into a spherical volume, the compression of the attached-aether will lead to an increased quantum-force and the displacement of any cube within the spherical region will do work against the quantum-force-field as given in Eq. (33), $W_{QD} = (X_0/\xi_0)\, K_c \int 2K_Q/X_0^3\, dx$. In this case, W_{QD} is positive, since the displacement caused by compression will result in an increase of the quantum-energy. As a first-order approximation, we use a quantum-force of $F_Q = F_{Q0} = 2K_{Q0}/X_0^3$. With the displacement of $(J–1)\delta X$, the work to displace cube-J is $W_{QDJ} = K_c(X_0/\xi_0)[2K_{Q0}(J–1)\delta X/X_0^3]$. Setting the expansion effects aside to focus on the effect of displacement, the quantum-energy of cube-J is $E_Q = E_{Q0} + W_{QDJ} = K_{Q0}/X_0^2 + (X_0/\xi_0)[2(J–1)K_cK_{Q0}/X_0^3]\delta X = (K_{Q0}/X_0^2)(1 + 2K_c(J–1)\delta X/\xi_0)$. With $E_Q = K_Q/X_0^2$, we arrive at the expression $K_Q = K_{Q0}(1 + 2K_c(J–1)\delta X/\xi_0)$. The quantum-force remains $F_Q = 2K_Q/X_0^3$, but now K_Q includes the next order correction. As detached-aether is slowly injected, the center of the Jth cube will move a distance $(J–1)\delta X$. At the beginning of the motion F_{QP} is of course just $2K_{Q0}/X_0^3$, and it is at the end that $F_{QP} = (2K_{Q0}/X_0^3)[1 + 2K_c(J–1)\delta X/\xi_0]$. Again defining x as the deviation of the cube center from its nominal center (x varies from 0 to $(J–1)\delta X$ during the detached-aether injection) the full expression for the quantum-force in the analysis-cube is

$$F_{QPN} = (2K_{Q0}/X_0^3)[1 + 2K_cx/\xi_0] \quad \text{(F5)}$$

With the expression for the second-order quantum-force just derived, we can now include the second order effect on the cube quantum-energy. The work done on the quantum-force-field due to the cube displacement is $W_{QDJ} = (X_0/\xi_0)K_c \int F_Q dx = K_c(X_0/\xi_0)(2K_{Q0}/X_0^3) \int [1+2K_c x/\xi_0]dx =$ $K_c(2K_{Q0}/X_0^2\xi_0)x + K_c^2(2K_{Q0}/X_0^2\xi_0^2)x^2 = K_c(2K_{Q0}/X_0^2\xi_0)(J-1)\delta X + K_c^2(2K_{Q0}/X_0^2\xi_0^2)(J-1)^2\delta X^2 =$ $K_c(2K_{Q0}/X_0^2\xi_0)P + K_c^2(2K_{Q0}/X_0^2\xi_0^2)P^2$. (For the displacement, the integral is evaluated between zero and $(J-1)\delta X = P$.)

The compression energy is calculated using Eq. (64) and with a positive sign, since compression will do work against the quantum-pressure, $W_{QC} = 4 \int K_Q/X_0^3\, dw$. From Eq. (F5), $F_{QPN} =$ $(2K_{Q0}/X_0^3)[1 + 2K_c x/\xi_0]$. We now use Eq. (62), $x = (P/\delta X)w$, to obtain $F_{QPN}/2 = K_Q/X_0^3 =$ $(K_{Q0}/X_0^3)[1+2K_c(P/\delta X)w/\xi_0]$. As specified in Eq. (64) we integrate from $w = 0$ to $w = \delta X/2$. Hence $W_{QC1} = 4 \int K_Q/X_0^3\, dw = 4(K_Q/X_0^3) \int [1+2K_c(P/\delta X)w/\xi_0]\, dw = 4(K_{Q0}/X_0^3)w +$ $4(K_{Q0}/X_0^3)K_c(P/\delta X)w^2/\xi_0 = 2(K_{Q0}/X_0^3)\delta X + (K_{Q0}/X_0^3)K_c P\delta X/\xi_0$. Recall that Eq. (64) is for one dimension only, and that inside the sphere the compression will be the same in all three dimensions. Hence, $W_{QC3} = 6(K_{Q0}/X_0^3)\delta X + 3(K_{Q0}/X_0^3)K_c P\delta X/\xi_0$. The total quantum-energy of an arbitrary analysis-cube inside the spherical region is the undisturbed energy plus the displacement energy plus the compression energy, $E_{QPNI} = K_{Q0}/X_0^2 + K_c(2K_{Q0}/X_0^2\xi_0)P + K_c^2(2K_{Q0}/X_0^2\xi_0^2)P^2 +$ $6(K_{Q0}/X_0^3)\delta X + 3(K_{Q0}/X_0^3)K_c P\delta X/\xi_0$, or, $E_{QPNI} = 2K_{Q0}/X_0^2[(1/2) + K_c P/\xi_0 + K_c^2(P/\xi_0)^2 + 3\delta X/X_0$ $+ 3K_c P\delta X/2X_0\xi_0]$. Here we have derived the force for positive P. For **P** at any angle the work done is same, so we will replace P by its absolute value, $|\mathbf{P}|$:

$$E_{QPNI} = (2K_{Q0}/X_0^2)[(1/2) + K_c|\mathbf{P}|/\xi_0 + K_c^2|\mathbf{P}|^2/\xi_0^2 + 3\delta X/X_0 + 3K_c|\mathbf{P}|\delta X/2X_0\xi_0] \qquad \text{(F6)}$$

F.4. The Delta-Force and Delta-Energy Fields Inside a Spherical Attached-Aether Region Containing Unlike-Kind Detached-Aether. Injection of detached-aether causes unlike-kind attached-aether to compress leading to the forces described in Eqs. (F3) and (F5), $F_{TPN} = K_{T0}X_0[1 - 2K_c x/\xi_0]$ and $F_{QPN} = (2K_{Q0}/X_0^3)[1 + 2K_c x/\xi_0]$, respectively. In order to achieve a force balance within the attached-aether, it has been proposed in section C.3 that presence of detached-aether leads to a balancing force called the delta-force obeying Eq. (59), $\mathbf{F_\delta} = -\mathbf{F_{TL}} - \mathbf{F_Q}$. At this point recall that the tension is directed inward (toward the center of the sphere) while the quantum-force is directed outward and $\mathbf{F_\delta}$ opposes them to arrive at $\mathbf{F_\delta} = K_{T0}X_0[1 - 2K_c x/\xi_0]\hat{\mathbf{r}} - (2K_{Q0}/X_0^3)[1 + 2K_c x/\xi_0]\hat{\mathbf{r}}$. And now recall Eq. (21), $K_{T0}X_0 = 2K_{Q0}/X_0^3$, leaving $\mathbf{F_\delta} = K_{T0}X_0[1 - 2K_c x/\xi_0 - 1 - 2K_c x/\xi_0]\hat{\mathbf{r}}$, or,

$$\mathbf{F_{\delta PN}} = -4K_c K_{T0}(X_0/\xi_0)x\hat{\mathbf{r}} \qquad \text{(F7)}$$

The delta-force $\mathbf{F_{\delta PN}}$ of Eq. (F7) is the total force on the area of the cube-face, like the tension and quantum-pressure forces of Eqs. (F3) and (F5), respectively. The work done on the delta-field due to the cube displacement is calculated from Eq. (32) as $W_{\delta PNI} = (X_0/\xi_0)\ K_c \int F_{\delta PN}\, dx$, and substituting in Eq. (F7) we obtain $W_{\delta PNI} = (X_0/\xi_0)\ K_c \int 4K_{T0}K_c(X_0/\xi_0)x\, dx = 2K_{T0}K_c^2(X_0/\xi_0)^2x^2 = 2K_{T0}K_c^2(X_0/\xi_0)^2P^2$. (For the displacement, the integral is evaluated between zero and P. For the sign of the energy see section C.3.10.)

The radial compression energy is now calculated by using Eq. (65), $W_{\delta C} = 2 \int F\, dw$. We now use Eq. (62), $x = (P/\delta X)w$ and use a negative sign since compression will decrease the delta-tension energy to obtain $F = -4K_{T0}K_cX_0(P/\delta X)w/\xi_0$. (Since the quantum-force exceeds the tension, the delta-force is a tension in this case, see section F.7.1.) As specified in Eq. (65) we integrate from $w = 0$ to $w = \delta X/2$. Hence $W_{\delta C1} = -2 \int 4K_{T0}K_cX_0(P/\delta X)w/\xi_0\, dw = -4K_{T0}X_0K_c(P/\delta X)w^2/\xi_0 = -K_{T0}X_0K_cP\delta X/\xi_0$. Recall that Eq. (65) is for one dimension only. Hence, $W_{\delta C3} = -3K_{T0}X_0K_cP\delta X/\xi_0$.

The total delta-energy of an arbitrary analysis-cube inside the spherical region is the displacement energy plus the compression energy, $E_{\delta PNI} = 2K_{T0}K_c^2(X_0/\xi_0)^2P^2 - 3K_{T0}X_0K_cP\delta X/\xi_0$. The calculation will hold for any angle so we can replace P by |**P**|, and therefore summing the displacement and expansion energies leaves

$$E_{\delta PNI} = 2K_{T0}K_c^2(X_0/\xi_0)^2|\mathbf{P}|^2 - 3K_{T0}X_0K_c|\mathbf{P}|\delta X/\xi_0 \tag{F8}$$

F.5. The Total Energy Inside a Spherical Attached-Aether Region Containing Like-Kind Detached-Aether. The total energy of the analysis-cube is found by summing the tension, quantum and delta energies from Eqs. (F4), (F6) and (F8). $E_{PNI} = E_{TPNI} + E_{QPNI} + E_{\delta PNI} = K_{T0}X_0^2[(1/2) - K_c|\mathbf{P}|/\xi_0 + K_c^2|\mathbf{P}|^2/\xi_0^2 - 3\delta X/X_0 + 3K_c|\mathbf{P}|\delta X/2X_0\xi_0] + (2K_{Q0}/X_0^2)[(1/2) + K_c|\mathbf{P}|/\xi_0 + K_c^2|\mathbf{P}|^2/\xi_0^2 + 3\delta X/X_0 + 3K_c|\mathbf{P}|\delta X/2X_0\xi_0] + 2K_{T0}K_c^2(X_0/\xi_0)^2|\mathbf{P}|^2 - 3K_{T0}X_0K_c|\mathbf{P}|\delta X/\xi_0$. Next, use Eq. (22), $K_{T0}X_0^2 = 2K_{Q0}/X_0^2$, to get $E_{PNI} = K_{T0}X_0^2[1 + 4K_c^2(P^2/\xi_0)^2]$.

We see that we have once again arrived at Eq. (72), $E_P = K_{T0}X_0^2[1 + 4K_c^2(P^2/\xi_0)^2]$, and therefore we need no second subscript on E_P in Eq. (72).

F.6. The Force and Energy Fields Outside a Spherical Attached-Aether Region Containing Unlike-Kind Detached-Aether. Outside of the sphere Eq. (F2) informs us that the analysis-cubes will be expanded radially, rather than compressed. A spherical shell originally located a distance r from the center of the sphere with thickness dr is pulled inward by a distance P_R. Since there is no detached-aether outside of the sphere, the density is a constant ρ_0 there and hence the volume of the shell remains constant during its inward displacement. With dr_P the shell thickness after it is displaced by P_R, volume invariance results in $4\pi r^2dr = 4\pi(r-P_R)^2dr_P$, or $dr_P/dr = r^2/(r-P_R)^2 = r^2/[r^2(1-P_R/r)^2] \approx 1 + 2P_R/r$. Now Eq. (F2) gives $P_R = P_0R_0^2/r^2$, and therefore $dr_P/dr \approx 1 + 2P_0R_0^2/r^3$, or $dr_P \approx dr + 2drP_0R_0^2/r^3$. This results in the amount of expansion being $\delta r_P = dr_P - dr \approx 2drP_0R_0^2/r^3$, where dr can be any small radial distance. Assigning dr to the size of an analysis-cube, $dr = X_0$, we obtain the expansion of the analysis-cubes, $\delta X = 2X_0P_0R_0^2/r^3$. At the edge of the sphere, the analysis-cubes are displaced radially inward by P_0 and the expansion will gradually lower the displacement of the more distant cubes to the displacement $P_R = P_0R_0^2/r^2$ given by Eq. (F2). In this case, all cubes are still displaced in the inward direction however, and so the tension-force-field is decreased, it is just that the amount of decrease falls off with r.

In the directions perpendicular to **r** the cubes will compress. The area of each spherical shell decreases to $A = 4\pi(r–P_R)^2$ from its original $4\pi r^2$. We have $(r–P_R)^2/r^2 = (1–P_R/r)^2 \approx 1–2P_R/r$, and we can see that the relative area compression is $\delta A/A = 2P_R/r$. Now we form $A = Y^2$ for our cubes and we see $\delta A/\delta Y = 2Y$, leading to $(\delta A/\delta Y)/A = (2Y)/Y^2$, or $\delta A/A = 2\delta Y/Y$ and hence $\delta Y/Y = P_R/r$. Therefore a cube of size $Y = X_0$ will compress by $\delta Y = X_0P_R/r = X_0P_0R_0^2/r^3$, or $\delta Y = \delta X/2$, which is Eq. (73).

Now consider cube-P, a cube that is displaced by a distance P. Integrating Eq. (32), $W_{TD} = (X_0/\xi_0) K_c \int K_TX_0\, dx$, from 0 to P, the first-order work (against the first-order force $F_{TP} = K_{T0}X_0$) done against the tension on cube-P from the displacement is $W_{TDP} = –(X_0/\xi_0)K_cK_{T0}X_0P$.

Setting the expansion effects aside to focus on the effect of displacement, the tension-energy of cube-P is $E_T = E_{T0} + W_{TDP} = (1/2)K_{T0}X_0^2 – K_cK_{T0}X_0^2P/\xi_0$. With $E_T = (1/2)K_TX_0^2$, we arrive at the expression $K_T = K_{T0}(1 – 2K_cP/\xi_0)$. The tension-force remains $F_T = K_TX_0$, but now K_T includes the next order correction. As detached-aether is slowly injected, the center of the Pth cube will move a distance P. At the beginning of the motion F_{TP} is of course just $K_{T0}X_0$, and it is at the end that $F_{TP} = K_{T0}X_0[1 – 2K_cP/\xi_0]$. With x now defined as the deviation of the cube center from its nominal center (which varies from 0 to P during the detached-aether injection) the full expression for tension-force in the analysis-cube is again given by Eq. (F3), $F_{TPN} = K_{T0}X_0[1 – 2K_cx/\xi_0]$.

The second-order effect on the cube tension-energy due to the cube displacement is now included by integrating Eq. (32) from 0 to P, $W_{TD} = -(X_0/\xi_0)\, K_c \int F_{TPN}\, dx = -K_c(X_0/\xi_0)K_{T0}X_0 \int [1-2K_c x/\xi_0]\, dx = -K_c(X_0/\xi_0)K_{T0}X_0 x + K_c^2(X_0/\xi_0)^2K_{T0}x^2 = -K_cK_{T0}X_0^2P/\xi_0 + K_c^2K_{T0}P^2X_0^2/\xi_0^2$.

The expansion energy is now calculated by Eq. (63) since expansion will increase the tension energy, $W_{TC} = 2 \int K_TX_0\, dw$. From Eq. (F3), $F_{TPN} = K_{T0}X_0[1 - 2K_cx/\xi_0]$. We now use Eq. (62), $x = (P/\delta X)w$, to obtain $K_TX_0 = K_{T0}X_0[1-2K_c(P/\delta X)w/\xi_0]$. As specified in Eq. (63) we integrate from $w = 0$ to $w = \delta X/2$. Hence $W_{TC1} = 2 \int K_TX_0\, dw = 2K_{T0}X_0 \int [1-2K_c(P/\delta X)w/\xi_0]\, dw = 2K_{T0}X_0w - 2K_{T0}X_0K_c(P/\delta X)w^2/\xi_0 = K_{T0}X_0\delta X - K_{T0}X_0K_cP\delta X/2\xi_0$.

Recall that Eq. (63) is for one dimension only. Eq. (73) informs that outside the sphere there is a compression $\delta Y = \delta X/2$ for each of the other two dimensions and for those dimensions we integrate from 0 to $\delta Y/2$, which is equal to 0 to $\delta X/4$. Since it is a compression we release some tension and our signs are reversed. For each transverse dimension then we obtain $W_{TC2} = -2K_{T0}X_0w + 2K_{T0}X_0K_c(P/\delta X)w^2/\xi_0 = -K_{T0}X_0\delta X/2 + K_{T0}X_0K_c(P/\delta X)\delta X^2/8\xi_0$, and the total compression energy will be twice that, since there are two dimensions. Hence, $W_{TC3} = W_{TC1} + 2W_{TC2} = K_{T0}X_0\delta X - K_{T0}X_0K_cP\delta X/2\xi_0 - K_{T0}X_0\delta X + K_{T0}X_0K_cP\delta X/4\xi_0 = -K_{T0}X_0K_cP\delta X/4\xi_0$. The total tension-energy of an arbitrary analysis-cube outside the spherical region is the undisturbed energy plus the displacement energy plus the expansion energy, $E_{TPNO} = (1/2)K_{T0}X_0^2 - K_cK_{T0}X_0^2P/\xi_0 + K_c^2K_{T0}P^2X_0^2/\xi_0^2 - K_{T0}X_0K_cP\delta X/4\xi_0$, or, $E_{TPNO} = K_{T0}X_0^2[(1/2) - K_cP/\xi_0 + K_c^2(P/\xi_0)^2 - K_cP\delta X/4X_0\xi_0]$. Here we have derived the force for positive P. For **P** at any angle the work done is the same. Hence we will replace P by its absolute value, $|\mathbf{P}|$:

$$E_{TPNO} = K_{T0}X_0^2[(1/2) - K_c|\mathbf{P}|/\xi_0 + K_c^2(|\mathbf{P}|/\xi_0)^2 - K_c|\mathbf{P}|\delta X/4X_0\xi_0] \qquad \text{(F9)}$$

In Eq. (F9) and the above paragraph, the subscript TPNO refers to Tension of the Positive-attached-aether due to immersed Negative-detached-aether in the region Outside of the sphere.

Turning to the quantum-field, the work done on cube-P during displacement is given by Eq. (33), $W_{QD} = (X_0/\xi_0)\, K_c \int 2K_Q/X_0^3\, dx$, and here the displacement is P and the first-order force parameter is K_{Q0}, leaving $W_{QDP} = 2K_cPK_{Q0}/X_0^2\xi_0$. Setting the expansion effects aside to focus on the effect of displacement, the quantum-energy of cube-P is $E_Q = E_{Q0} + W_{QDP} = K_{Q0}/X_0^2 + (X_0/\xi_0)[2PK_cK_{Q0}/X_0^3] = (K_{Q0}/X_0^2)(1 + 2K_cP/\xi_0)$. With $E_Q = K_Q/X_0^2$, we arrive at the expression $K_Q = K_{Q0}(1 + 2K_cP/\xi_0)$. The quantum-force remains $F_Q = 2K_Q/X_0^3$, but now K_Q includes the next order correction. As detached-aether is slowly injected, the center of the Pth cube will move a distance P. At the beginning of the motion F_{QP} is of course just $2K_{Q0}/X_0^3$, and it is at the end that $F_{QP} = (2K_{Q0}/X_0^3)[1 + 2K_cP/\xi_0]$. With x again defined as the deviation of the cube center from its nominal center (which varies from 0 to P during the detached-aether injection) the full expression for the quantum-force in the analysis-cube is again given by Eq. (F5), $F_{QPN} = (2K_{Q0}/X_0^3)[1 + 2K_cx/\xi_0]$. Including the second order effect on the cube quantum-energy due to the cube displacement, $W_{QDP} = (X_0/\xi_0)K_c \int (2K_{Q0}/X_0^3)[1+2K_cx/\xi_0]\, dx = (2K_cK_{Q0}/X_0^2\xi_0)x + (2K_c^2K_{Q0}/X_0^2\xi_0^2)x^2 = (2K_cK_{Q0}/X_0^2\xi_0)P + (2K_c^2K_{Q0}/X_0^2\xi_0^2)P^2$.

The radial expansion energy is now calculated by Eq. (64), and since expansion will decrease the quantum-energy, $W_{QC} = -4 \int K_Q/X_0^3\, dw$. From Eq. (F5), $F_{QPN} = (2K_{Q0}/X_0^3)[1 + 2K_cx/\xi_0]$. We now use Eq. (62), $x = (P/\delta X)w$, to obtain $K_Q/X_0^3 = (K_{Q0}/X_0^3)[1+2K_c(P/\delta X)w/\xi_0]$. As specified in Eq. (64) we integrate from $w = 0$ to $w = \delta X/2$. Hence $W_{QC1} = -4 \int K_Q/X_0^3\, dw = -4(K_{Q0}/X_0^3) \int [1+2K_c(P/\delta X)w/\xi_0]\, dw = -4(K_{Q0}/X_0^3)w - 4(K_{Q0}/X_0^3)K_c(P/\delta X)w^2/\xi_0 = -2(K_{Q0}/X_0^3)\delta X - (K_{Q0}/X_0^3)K_cP\delta X/\xi_0$.

Recall that Eq. (64) is for one dimension only. Eq. (73) informs that outside the sphere there is a compression $\delta Y = \delta X/2$ for each of the other two dimensions and for those dimensions we integrate from 0 to $\delta Y/2$, which is equal to 0 to $\delta X/4$. Since it is a compression we increase the quantum-force and our signs are positive. For each transverse dimension then we obtain $W_{QC2} = 4(K_{Q0}/X_0^3)w + 4(K_{Q0}/X_0^3)K_c(P/\delta X)w^2/\xi_0 = (K_{Q0}/X_0^3)\delta X + (K_{Q0}/X_0^3)K_cP\delta X/4\xi_0$, and the total compression energy will be twice that, since there are two dimensions. Hence, the full three dimension effect is $W_{QC3} = W_{QC1} + 2W_{QC2} = -2(K_{Q0}/X_0^3)\delta X - (K_{Q0}/X_0^3)K_cP\delta X/\xi_0 + 2(K_{Q0}/X_0^3)\delta X + (K_{Q0}/X_0^3)K_cP\delta X/2\xi_0 = -(K_{Q0}/X_0^3)K_cP\delta X/2\xi_0$. The total quantum-energy of an arbitrary analysis-cube outside the spherical region is the undisturbed energy plus the displacement energy plus the expansion energy, $E_{QPNO} = K_{Q0}/X_0^2 + (2K_cK_{Q0}/X_0^2\xi_0)P + (2K_c^2K_{Q0}/X_0^2\xi_0^2)P^2 - (K_{Q0}/X_0^3)K_cP\delta X/2\xi_0$, or, $E_{QPNO} = (2K_{Q0}/X_0^2)[(1/2) + K_cP/\xi_0 + K_c^2(P/\xi_0)^2 - K_cP\delta X/4X_0\xi_0]$. Here we have derived the force for positive P. For **P** at any angle the work done is the same. Hence we will replace P by its absolute value, $|\mathbf{P}|$:

$$E_{QPNO} = (2K_{Q0}/X_0^2)[(1/2) + K_c|\mathbf{P}|/\xi_0 + K_c^2(|\mathbf{P}|/\xi_0)^2 - K_c|\mathbf{P}|\delta X/4X_0\xi_0] \quad \text{(F10)}$$

In Eq. (F10) and the above paragraph, the subscript QPNO refers to Quantum-pressure of the Positive-attached-aether due to immersed Negative-detached-aether in the region Outside of the sphere.

For the delta-energy, since Eqs. (F3) and (F5) are the same both inside and outside of the spherical region, Eq. (F7), $\mathbf{F}_{\delta PN} = -4K_cK_{T0}(X_0/\xi_0)x\hat{\mathbf{r}}$, holds both inside and outside as well. For the displacement energy we use Eq. (32), $W_{TD} = (X_0/\xi_0)K_c \int F_{\delta PN}\, dx = (X_0/\xi_0)K_c \int 4K_cK_{T0}(X_0/\xi_0)x\, dx = 2K_c^2K_{T0}(X_0/\xi_0)^2x^2$, and evaluating at P leaves $W_{TDN} = 2K_c^2K_{T0}(X_0/\xi_0)^2P^2$. (For the sign of the energy, see section C.3.10.)

The radial expansion energy is now calculated by using Eq. (65), $W_C = 2\int F\,dw$. We now use Eq. (62), $x = (P/\delta X)w$ and use a positive sign since expansion will increase the delta energy (it is a tension) to obtain $F_{\delta PN} = 4K_{T0}K_cX_0(P/\delta X)w/\xi_0$. As specified in Eq. (65) we integrate from w = 0 to $w = \delta X/2$. Hence $W_{\delta C1} = 2\int 4K_{T0}K_cX_0(P/\delta X)w/\xi_0\,dw = 4K_{T0}X_0K_c(P/\delta X)w^2/\xi_0 = K_{T0}X_0K_cP\delta X/\xi_0$. Recall that Eq. (65) is for one dimension only. Eq. (73) informs that outside the sphere there is a compression $\delta Y = \delta X/2$ for each of the other two dimensions and for those dimensions we integrate from 0 to $\delta Y/2$, which is equal to 0 to $\delta X/4$. Also, since it is a compression the tension will decrease and our signs are negative. For each transverse dimension then we obtain $W_{\delta C2} = -4K_{T0}X_0K_c(P/\delta X)w^2/\xi_0 = -K_{T0}X_0K_cP\delta X/4\xi_0$, and the total compression energy will be twice that, since there are two dimensions. Hence, $W_{\delta P3} = W_{\delta P1} + 2W_{\delta P2} = K_{T0}X_0K_cP\delta X/\xi_0 - K_{T0}X_0K_cP\delta X/2\xi_0 = K_{T0}X_0K_cP\delta X/2\xi_0$. The total delta-energy of an arbitrary analysis-cube outside the spherical region is the displacement energy plus the expansion energy, $E_{\delta PNO} = 2K_{T0}K_c^2(X_0/\xi_0)^2P^2 + K_{T0}X_0K_cP\delta X/2\xi_0$. The calculation will hold for any angle so we can replace P by $|\mathbf{P}|$, and therefore summing the displacement and expansion energies leaves

$$E_{\delta PNO} = 2K_{T0}K_c^2(X_0/\xi_0)^2|\mathbf{P}|^2 + K_{T0}X_0K_c|\mathbf{P}|\delta X/2\xi_0 \qquad \text{(F11)}$$

The total energy of the analysis-cube is found by summing the tension, quantum and delta energies from Eqs. (F9), (F10) and (F11). $E_{PN0} = E_{TPNO} + E_{QPNO} + E_{\delta PNO} = K_{T0}X_0^2[(1/2) - K_c|\mathbf{P}|/\xi_0 + K_c^2|\mathbf{P}|^2/\xi_0^2 - K_c|\mathbf{P}|\delta X/4X_0\xi_0] + (2K_{Q0}/X_0^2)[(1/2) + K_c|\mathbf{P}|/\xi_0 + K_c^2|\mathbf{P}|^2/\xi_0^2 - K_c|\mathbf{P}|\delta X/4X_0\xi_0] + 2K_c^2K_{T0}(X_0/\xi_0)^2|\mathbf{P}|^2 + K_{T0}X_0K_c|\mathbf{P}|\delta X/2\xi_0$. Next, use Eq. (22), $2K_{Q0}/X_0^2 = K_{T0}X_0^2$, to get $E_P = K_{T0}X_0^2[1 + 4K_c^2(P^2/\xi_0)^2]$, which is Eq. (72). We see that Eq. (72) results whether we are inside or outside the sphere of charge, and whether positive-detached-aether or negative-attached-aether is the source for P.

F.7. The Physical Nature of the Delta-Force for Unlike-Kind Immersion. In the sections above we make three assumptions regarding our analysis of the case of negative-detached-aether within positive-attached-aether: 1) that the delta-force is a positive tension; 2) that displacement of the analysis-cube will not change its size; and 3) that the energies associated with the delta-force are positive for the displacement. This section will validate these assumptions while providing a physical understanding of the delta-force for unlike-kind immersion.

F.7.1. Delta-force Tension. The delta-force $\mathbf{F_\delta}$ is a force that balances against the sum of $\mathbf{F_T}$ and $\mathbf{F_Q}$. We have assigned a sign to $\mathbf{F_\delta}$ so that it adds to the weaker of $\mathbf{F_T}$ and $\mathbf{F_Q}$ as needed to provide the balancing force. Here, $\mathbf{F_T}$ is less than $\mathbf{F_Q}$, and $\mathbf{F_\delta}$ a positive tension providing an additional inward force.

The reason $\mathbf{F_\delta}$ adds to the weaker of the forces, rather than subtracting from the stronger, is because the delta-force originates from the forced expansion and compression of attached-aether as caused by the immersion of detached-aether. For the case of negative-detached-aether, a positive tension is exerted by the negative-detached-aether onto the positive-attached-aether and a positive quantum-force is exerted onto the negative-attached-aether. (This is a result of the positive-attached-aether compression needed to maintain the density postulate.) As the cubes are then displaced, the work done makes these forces grow, and this force is the delta-force. Therefore the delta-force is a positive tension (positive pressure) for the case of negative-detached-aether immersed into positive-attached-aether (negative-attached-aether).

F.7.2. Analysis-Cube Size During Displacement. When cube-P of positive-attached-aether is moved radially inward toward the center of the sphere due to injection of negative-detached-aether, cube-P has its tension decreased as described by Eq. (F3), $F_{TPN} = K_{T0}X_0[1 - 2K_cx/\xi_0]$, and cube-P has its quantum-force increased as described by Eq. (F5), $F_{QPN} = (2K_{Q0}/X_0^3)[1 + 2K_cx/\xi_0]$. If there were no delta-force, cube-P would therefore expand. However, the overlapping cube-N of negative-attached-aether has the opposite behavior and with no delta-force it would compress. (For the negative-attached-aether, Eqs. (F12) and (F13) below are $F_{TNN} = K_{T0}X_0[1 + 2K_cx/\xi_0]$ and $F_{QNN} = (2K_{Q0}/X_0^3)[1 - 2K_cx/\xi_0]$, respectively.) In order for the density postulate to hold as they displace, cube-P and cube-N must either both expand, both compress, or both retain their size. Since the sum of $\mathbf{F_T}$ and $\mathbf{F_Q}$ on cube-P, described by Eqs. (F3) and (F5), is equal and opposite to the sum of $\mathbf{F_T}$ and $\mathbf{F_Q}$ on cube-N, this symmetry informs us that the cube sizes will not change during their displacement, which is what is assumed above.

F.7.3. The Physics Leading to the Delta-Force Energies. We see that the delta-force arises between the positive-attached-aether and the negative-attached-aether in a way that leads to no displacement-induced size change of the displaced cubes. We also see that motion of positive-attached-aether inside a sphere containing negative-detached-aether causes $\mathbf{F_{TP}}$ (the positive-attached-aether tension) to recede and $\mathbf{F_{QP}}$ (the positive-attached-aether quantum-force) to grow, and hence $\mathbf{F_{\delta P}}$ (the positive-attached-aether delta-force) must grow to offset this growing difference. Since the displacement of the positive-attached-aether is inward and the displacement of the negative-attached-aether is outward in this case, and since $\mathbf{F_{\delta P}}$ is inward, it is the displacement of the outward-moving negative-attached-aether that does the work against $\mathbf{F_{\delta P}}$ to make it grow, since we must have motion against the force to do work against it to make it grow. (The inward-moving positive-attached-aether would do negative work on $\mathbf{F_{\delta P}}$ and therefore it cannot be the source of growth for $\mathbf{F_{\delta P}}$.) Therefore we see that $\mathbf{F_{\delta P}}$ is a positive tension that adds to the tension within the positive-attached-aether, and yet work is done against it by the motion of the negative-attached-aether. Similarly, the inward motion of the positive-attached-aether increases $\mathbf{F_{\delta N}}$ (the negative-attached-aether delta-force) which is a positive quantum-force within the negative-attached-aether. This is the mechanism by which each type of attached-aether exerts forces on the other in order to maintain the equal-density postulate.

It is the moving negative-attached-aether that is doing work against the positive-attached-aether delta force. Inside the spherical region the negative-attached-aether will move radially outward and expand. The outward motion does positive work against $\mathbf{F_{\delta P}}$ and hence the displacement term in Eq. (F8) is positive. Outside the spherical region negative-attached-aether will move radially outward and compress. The outward displacement does work against $\mathbf{F_{\delta P}}$ and hence the displacement term in the energy expression of Eq. (F11) is positive.

F.7.4. The Delta-Force in Other Cases. In section C.3 and here in Appendix F we have looked at the cases of detached-aether immersion into positive-attached-aether. For detached-aether immersion into negative-attached-aether, we need merely substitute N for P and P for N everywhere in our analysis. This follows because doing so will not affect the like or unlike character of our analysis when we do so. This substitution into Eqs. (66) and (68) leaves:

$$F_{TNN} = K_{T0}X_0[1 + 2K_c x/\xi_0] \quad \text{(F12)}$$

$$F_{QNN} = (2K_{Q0}/X_0^3)[1 - 2K_c x/\xi_0] \quad \text{(F13)}$$

The substitution into Eqs. (F3) and (F5) leaves:

$$F_{TNP} = K_{T0}X_0[1 - 2K_c x/\xi_0] \quad \text{(F14)}$$

$$F_{QNP} = (2K_{Q0}/X_0^3)[1 + 2K_c x/\xi_0] \quad \text{(F15)}$$

Appendix G – The Tension, Quantum-pressure and Gamma-forces Within a Spherical Negative-Attached-Aether Region Containing Extrinsic-Energy.

G.1. The Tension Inside a Spherical Negative-Attached-Aether Region Containing Extrinsic-Energy. Using Eq. (212), $\mathbf{P_{GIN}} = (\rho_G/6\rho_0)\mathbf{r}$, along with Eq. (211), $\mathbf{N_{GL}} = -\mathbf{P_{GL}}$, we see that adding extrinsic-energy into a spherical region will cause the negative-attached-aether cubes within that region to compress equally in each cartesian direction. As we slowly add extrinsic-energy into our spherical region, the displacement of any cube within that region will have work done on it by the tension, with the work given by Eq. (32), $W_{TD} = (X_0/\xi_0)\ K_c \int K_T X_0\, dx$, where the work will be negative for the inward (compression) displacements.

Consider a cube-J, where prior to injection of extrinsic-energy, the center of cube-J is separated from the center of the spherical region by $-(J-1)X_0$. The energy to displace cube-J is given by Eq. (32), and the displacement is $(J-1)\delta X$. (A displacement of $\delta X/2$ comes from each of cubes 1 and J and an additional δX comes from each of the cubes between cube-1 and cube-J.) The first order force is $F_T = K_{T0}X_0$, Integrating Eq. (32), $W_{TD} = (X_0/\xi_0)\ K_c \int K_{T0}X_0\, dx$, from $-(J-1)X_0$ to its final displacement $-(J-1)(X_0 - \delta X)$, the work done by the tension on cube-J from the displacement is $W_{TDJ} = -K_cK_{T0}(X_0^2/\xi_0)(J-1)\delta X$.

Setting the compression effects aside, the tension-energy of cube-J is $E_T = E_{T0} + W_{TDJ} = (1/2)K_{T0}X_0^2 - K_cK_{T0}(X_0^2/\xi_0)(J-1)\delta X$. With $E_T = (1/2)K_TX_0^2$, we arrive at the expression $K_T = K_{T0}[1 - 2K_c(J-1)\delta X/\xi_0]$. The tension-force remains $F_T = K_TX_0$, but now K_T includes the next order correction.

At the beginning of the extrinsic-energy injection (when no immersion has yet occurred) F_T is of course just $K_{T0}X_0$, and it is at the end of injection that $F_{TN\_END} = K_{T0}X_0[1 - 2K_c(J–1)\delta X/\xi_0]$. We will again use x as the variable tracking the cube motion away from its equilibrium position. With x varying from 0 to (J–1)δX during extrinsic-energy injection, we see that the force is given by

$$F_{TN} = K_{T0}X_0[1 - 2K_c x/\xi_0] \tag{G1}$$

To verify Eq. (G1), when there is no injection, x = 0 and $F_T = K_{T0}X_0$. When the injection is complete, x = (J–1)δX and $F_T = F_{T\_END} = K_{T0}X_0[1 - 2K_c(J–1)\delta X/\xi_0]$.

With the second order expression for the tension just derived for the displacement case, we can now include the second order effect in the cube tension-energy. The work done on the field due to the cube displacement is $W_{TDJ} = (X_0/\xi_0)\, K_c \int F_T\, dx = –K_{T0}X_0\, (X_0/\xi_0)\, K_c \int [1 - 2K_c x/\xi_0]\, dx = –K_cK_{T0}X_0^2x/\xi_0 + K_c^2K_{T0}(X_0/\xi_0)^2x^2$, or $W_{TDJ} = –K_cK_{T0}X_0^2(J–1)\delta X/\xi_0 + K_c^2K_{T0}(X_0/\xi_0)^2(J–1)^2\delta X^2 = –K_cK_{T0}X_0^2N_G/\xi_0 + K_c^2K_{T0}(X_0/\xi_0)^2N_G^2$.

For the displacement, the integral is evaluated between $–(J–1)X_0$ and its final displacement $–(J–1)(X_0 - \delta X) = –(J–1)X_0 + N_G$. The work done has a negative sign because the tension decreases in this case.

The compression energy is now calculated using Eq. (63), $W_{TC} = –2 \int K_TX_0\, dw$. From Eq. (G1), $K_TX_0 = K_{T0}X_0[1 - 2K_c x/\xi_0]$ where we add the minus sign in W_{TC} because compression will reduce the tension. We now use Eq. (62) with N_G for P, $x = (N_G/\delta X)w$, and we obtain $K_TX_0 = K_{T0}X_0[1–2K_c(N_G/\delta X)w/\xi_0]$. As specified in Eq. (63) we integrate from w = 0 to w = δX/2. Hence $W_{TC1} = –2 \int K_TX_0\, dw = –2K_{T0}X_0 \int [1–2K_c(N_G/\delta X)w/\xi_0]\, dw = –2K_{T0}X_0w + 2K_{T0}X_0K_c(N_G/\delta X)w^2/\xi_0 = –K_{T0}X_0\delta X + K_{T0}X_0K_cN_G\delta X/2\xi_0$. Recall that Eq. (63) is for one dimension only, and that inside the sphere the compression will be the same in all three dimensions. Hence, $W_{TC3} = –3K_{T0}X_0\delta X +$

$3K_{T0}X_0K_cN_G\delta X/2\xi_0$. The total tension-energy of an arbitrary analysis-cube inside the spherical region is the undisturbed energy plus the displacement energy plus the compression energy, $E_{TNGI} = (1/2)K_{T0}X_0^2 - K_cK_{T0}X_0(X_0/\xi_0)N_G + K_c^2K_{T0}(X_0/\xi_0)^2N_G^2 - 3K_{T0}X_0\delta X + 3K_{T0}X_0K_cN_G\delta X/2\xi_0$, or, $E_{TNGI} = K_{T0}X_0^2[(1/2) - K_cN_G/\xi_0 + K_c^2(N_G/\xi_0)^2 - 3\delta X/X_0 + 3K_cN_G\delta X/2X_0\xi_0]$. Here we have derived the force for positive N_G. For negative $\mathbf{N_G}$ or at any angle the work done is the same, so we will replace N_G by its absolute value, $|\mathbf{N_G}|$:

$$E_{TNGI} = K_{T0}X_0^2[(1/2) - K_c|\mathbf{N_G}|/\xi_0 + K_c^2(|\mathbf{N_G}|/\xi_0)^2 - 3\delta X/X_0 + 3K_c|\mathbf{N_G}|\delta X/2X_0\xi_0] \qquad \text{(G2)}$$

In Eq. (G2) and the above paragraph, the subscript TNGI refers to Tension of the Negative-attached-aether due to immersed Gravitational effects in the region Inside the sphere.

G.2. The Quantum-Force Inside a Spherical Negative-Attached-Aether Region Containing Extrinsic-Energy. Eq. (33) relates that the displacement of any cube within the sphere does work against the quantum-force-field of $W_{QD} = (X_0/\xi_0)\, K_c \int 2K_Q/X_0^3\, dx$. Before adding any extrinsic-energy to our sphere, $F_{QN} = 2K_{Q0}/X_0^3$, where $2K_{Q0}/X_0^3$ is the quantum-force within the analysis-cubes in their nominal state.

Consider again cube-J of the previous section, where J is an integer. The center of cube-J is separated from the center of the extrinsic-energy-sphere by $-(J-1)X_0$. The energy to displace cube-J is given by Eq. (33), and the displacement is $(J-1)\delta X$. (A displacement of $\delta X/2$ comes from each of cubes 1 and J and an additional δX comes from each of the cubes between cube-1 and cube-J.) To first order, $F_{QN} = 2K_{Q0}/X_0^3$. Integrating Eq. (33) from $-(J-1)X_0$ to its final displacement $-(J-1)(X_0 - \delta X)$, the quantum-force displacement-work is $W_{QDJ} = (X_0/\xi_0)K_c[2K_{Q0}/X_0^3](J-1)\delta X$. Here, the positive sign of the work results because we are compressing the sphere, and the displacement leads to a work that increases the quantum-energy.

Setting the compression effects aside, the quantum-energy of cube-J is $E_Q = E_{Q0} + W_{DQJ} = K_{Q0}/X_0^2 + (X_0/\xi_0)K_c[2K_{Q0}/X_0^3](J–1)\delta X = K_{Q0}/X_0^2[1 + 2K_c(J–1)\delta X/\xi_0]$. With $E_Q = K_Q/X_0^2$, we arrive at the expression $K_Q = K_{Q0}[1 + 2K_c(J–1)\delta X/\xi_0]$. The quantum-force remains $F_Q = 2K_Q/X_0^3$, but now K_Q includes the next order correction. At the beginning of the extrinsic-energy injection (when no immersion has yet occurred) F_Q is of course just $2K_{Q0}/X_0^3$, and it is at the end of immersion that $F_{QN\_END} = (2K_{Q0}/X_0^3)[1 + 2K_c(J–1)\delta X/\xi_0]$. Therefore we see that,

$$F_{QN} = (2K_{Q0}/X_0^3)[1 + 2K_c x/\xi_0] \qquad \text{(G3)}$$

To verify Eq. (G3), when there is no injection, $x = 0$ and $F_Q = 2K_{Q0}/X_0^3$. When the injection is complete, $x = (J–1)\delta X$ and $F_Q = F_{QN\_END} = (2K_{Q0}/X_0^3)[1 + 2K_c(J–1)\delta X/\xi_0]$.

With the second order expression for the quantum-force just derived for the displacement case, we can now include the second order effect in the cube quantum-energy. The work done on the field due to the cube displacement is $W_{QDJ} = (X_0/\xi_0)\, K_c \int F_Q\, dx = (2K_{Q0}/X_0^3)\, (X_0/\xi_0)\, K_c \int [1 + 2K_c x/\xi_0]\, dx = K_c(2K_{Q0}/X_0^2\xi_0)x + K_c^2(2K_{Q0}/X_0^2\xi_0^2)x^2$, or $W_{QDJ} = K_c(2K_{Q0}/X_0^2\xi_0)(J–1)\delta X + K_c^2(2K_{Q0}/X_0^2\xi_0^2)(J–1)^2\delta X^2 = K_c(2K_{Q0}/X_0^2\xi_0)N_G + K_c^2(2K_{Q0}/X_0^2\xi_0^2)N_G^2$.

For the displacement, the integral is evaluated between $–(J–1)X_0$ and its final displacement $–(J–1)(X_0 – \delta X) = –(J–1)X_0 + N_G$. The work done has a positive sign because the quantum-force increases in this case.

The compression energy is now calculated using Eq. (64), $W_{QC} = 4 \int K_Q/X_0^3\, dw$. From Eq. (G3), $F_{QN} = (2K_{Q0}/X_0^3)[1 + 2K_c x/\xi_0]$. We again use Eq. (62) with N_G for P, $x = (N_G/\delta X)w$, and we obtain $F_{QN}/2 = K_Q/X_0^3 = (K_{Q0}/X_0^3)[1+2K_c(N_G/\delta X)w/\xi_0]$. As specified in Eq. (64) we integrate from $w = 0$ to $w = \delta X/2$. Hence $W_{QC1} = 4 \int K_Q/X_0^3\, dw = 4K_{Q0}/X_0^3 \int [1+2K_c(N_G/\delta X)w/\xi_0]\, dw = (4K_{Q0}/X_0^3)w + (4K_{Q0}/X_0^3)K_c(N_G/\delta X)w^2/\xi_0 = (2K_{Q0}/X_0^3)\delta X + (K_{Q0}/X_0^3)K_c N_G \delta X/\xi_0$. Recall that Eq. (64) is for one dimension only, and that inside the sphere the compression will be the same in all three dimensions. Hence, $W_{TC3} = (6K_{Q0}/X_0^3)\delta X + (3K_{Q0}/X_0^3)K_cN_G\delta X/\xi_0$. The total quantum-energy of an arbitrary analysis-cube inside the spherical region is the undisturbed energy plus the displacement energy plus the compression energy, $E_{QNGI} = K_{Q0}/X_0^2 + K_c(2K_{Q0}/X_0^2\xi_0)N_G + K_c^2(2K_{Q0}/X_0^2\xi_0^2)N_G^2 + (6K_{Q0}/X_0^3)\delta X + (3K_{Q0}/X_0^3)K_cN_G\delta X/\xi_0$, or, $E_{QNGI} = (2K_{Q0}/X_0^2)[(1/2) + K_cN_G/\xi_0 + K_c^2N_G^2/\xi_0^2 + 3\delta X/X_0 + 3K_cN_G\delta X/2X_0\xi_0]$. Here we have derived the force for positive N_G. For $\mathbf{N_G}$ at any angle the work done is the same, so we will replace N_G by its absolute value, $|\mathbf{N_G}|$:

$$E_{QNGI} = (2K_{Q0}/X_0^2)[(1/2) + K_c|\mathbf{N_G}|/\xi_0 + K_c^2|\mathbf{N_G}|^2/\xi_0^2 + 3\delta X/X_0 + 3K_c|\mathbf{N_G}|\delta X/2X_0\xi_0] \qquad \text{(G4)}$$

In Eq. (G4) and the above paragraph, the subscript QNGI refers to Quantum-force of the Negative-attached-aether due to Gravitational effects in the region Inside the sphere.

G.3. The Gamma-Force and Gamma-Energy Fields Inside a Spherical Negative-Attached-Aether Region Containing Extrinsic-Energy. Injection of extrinsic-energy causes negative-attached-aether to compress leading to the forces described in Eqs. (G1) and (G3), $F_{TN} = K_{T0}X_0[1 - 2K_cx/\xi_0]$ and $F_{QN} = (2K_{Q0}/X_0^3)[1 + 2K_cx/\xi_0]$, respectively. In order to achieve a force balance within the attached-aether, we have proposed a balancing force called the gamma-force in Eq. (216), $\mathbf{F_\gamma} = -\mathbf{F_{TL}} - \mathbf{F_Q}$.

At this point recall that the tension $\mathbf{F_{TL}}$ is directed inward (toward the center of the sphere) while the quantum-force $\mathbf{F_Q}$ is directed outward to arrive at $\mathbf{F_{\gamma NI}} = K_{T0}X_0[1 - 2K_cx/\xi_0]\hat{\mathbf{r}} - (2K_{Q0}/X_0^3)[1 + 2K_cx/\xi_0]\hat{\mathbf{r}}$. And now recall Eq. (21) above, $2K_{Q0}/X_0^3 = K_{T0}X_0$, leaving $\mathbf{F_{\gamma NI}} = K_{T0}X_0[1 - 2K_cx/\xi_0 - 1 - 2K_cx/\xi_0]\hat{\mathbf{r}}$, or

$$\mathbf{F_{\gamma NI}} = -(4K_cK_{T0}X_0x/\xi_0)\hat{\mathbf{r}} \qquad \text{(G5)}$$

Since the quantum-force exceeds the tension for the negative-attached-aether case, the gamma-force could be a positive tension, a negative quantum-force or some combination of gamma-tension and gamma-quantum-force summing to the net force field given in Eq. (G5). We now use a coupling of gamma-force components similar to that employed in Eqs. (218) and (219):

$$\mathbf{F}_{\gamma\mathbf{NQ}} = -4K_c(1+K_{GC})K_{T0}(X_0/\xi_0)x\hat{\mathbf{r}} \tag{G6}$$

$$\mathbf{F}_{\gamma\mathbf{NT}} = 4K_cK_{GC}K_{T0}(X_0/\xi_0)x\hat{\mathbf{r}} \tag{G7}$$

It can be seen that Eq. (G6) is a negative quantum-force, as it is directed inward, while Eq. (G7) is a negative tension-force, as it is directed outward. The work done on the gamma-fields due to the cube displacement is calculated as Eq. (220), $W_{\gamma D} = (X_0/\xi_0)\ K_c \int F_\gamma\, dx$, or $W_{\gamma NDQ} = -2K_c^2(1+K_{GC})K_{T0}(X_0/\xi_0)^2(J-1)^2\delta X^2 = -2K_c^2(1+K_{GC})K_{T0}(X_0/\xi_0)^2N_G^2$ and $W_{\gamma NDT} = -2K_c^2K_{GC}K_{T0}(X_0/\xi_0)^2(J-1)^2\delta X^2 = -2K_c^2K_{GC}K_{T0}(X_0/\xi_0)^2N_G^2$.

(For the displacement, the integrals are evaluated between zero and $N_G = (J-1)\delta X$.)

The compression energy is now calculated using Eq. (63), $W_{TC} = 2 \int F_T\, dw$. From Eq. (G6), $F_{\gamma NQ} = -4K_c(1+K_{GC})K_{T0}(X_0/\xi_0)x$. We now use Eq. (62) with N_G for P, $x = (N_G/\delta X)w$, and for the quantum-force gamma-force component we have $F_{\gamma NQ} = -4K_c(1+K_{GC})K_{T0}(X_0/\xi_0)(N_G/\delta X)w$. As specified in Eq. (63) we integrate from $w = 0$ to $w = \delta X/2$ and the work is negative as we are compressing along a negative quantum-force. So $W_{\gamma NQC1} = -2 \int 4K_c(1+K_{GC})K_{T0}(X_0/\xi_0)(N_G/\delta X)w\, dw = -4K_c(1+K_{GC})K_{T0}(X_0/\xi_0)(N_G/\delta X)w^2 = -K_c(1+K_{GC})K_{T0}(X_0/\xi_0)N_G\delta X$. Recall that Eq. (63) is for one dimension only, and that inside the sphere the compression will be the same in all three dimensions. Hence, $W_{\gamma NQC3} = -3K_c(1+K_{GC})K_{T0}(X_0/\xi_0)N_G\delta X$. For the tension component of the gamma-force we have $F_{\gamma NT} = 4K_cK_{GC}K_{T0}(X_0/\xi_0)x$, and we again use $x = (N_G/\delta X)w$ to get $F_{\gamma NT} = 4K_cK_{GC}K_{T0}(X_0/\xi_0)(N_G/\delta X)w$. As specified in Eq. (63) we integrate from $w = 0$ to $w = \delta X/2$. Here the energy is positive as we compress against the negative gamma tension. Hence $W_{\gamma NTC1} = 2\int 4K_cK_{GC}K_{T0}(X_0/\xi_0)(N_G/\delta X)w\ dw = 4K_cK_{GC}K_{T0}(X_0/\xi_0)(N_G/\delta X)w^2 = K_cK_{GC}K_{T0}(X_0/\xi_0)N_G\delta X$. Recall that Eq. (63) is for one dimension only, and that inside the sphere the compression will be the same in all three dimensions. Hence, $W_{\gamma NTC3} = 3K_cK_{GC}K_{T0}(X_0/\xi_0)N_G\delta X$.

The total gamma-energy of an arbitrary analysis-cube inside the spherical region is the sum of the displacement and compression energies, $E_{\gamma NGI} = -2K_c^2(1+K_{GC})K_{T0}(X_0/\xi_0)^2N_G^2 - 2K_c^2K_{GC}K_{T0}(X_0/\xi_0)^2N_G^2 - 3K_c(1+K_{GC})K_{T0}(X_0/\xi_0)N_G\delta X + 3K_cK_{GC}K_{T0}(X_0/\xi_0)N_G\delta X = -2K_c^2(1+2K_{GC})K_{T0}(X_0/\xi_0)^2N_G^2 - 3K_cK_{T0}(X_0/\xi_0)N_G\delta X = -2K_c^2K_{T0}(X_0/\xi_0)^2N_G^2 - 4K_c^2K_{GC}K_{T0}(X_0/\xi_0)^2N_G^2 - 3K_cK_{T0}(X_0/\xi_0)N_G\delta X$. Here we have derived the force for positive N_G. For $\mathbf{N_G}$ at any angle the work done is same, so we will replace N_G by its absolute value, $|\mathbf{N_G}|$:

$$E_{\gamma NGI} = K_{T0}X_0^2[-2K_c^2(|\mathbf{N_G}|/\xi_0)^2 - 4K_c^2K_{GC}(|\mathbf{N_G}|/\xi_0)^2 - 3K_c|\mathbf{N_G}|\delta X/X_0\xi_0] \qquad \text{(G8)}$$

In Eq. (G8) and the above paragraph, the subscript γNGI refers to the γ-energy of the Negative-attached-aether due to immersed Gravitational effects in the region Inside the sphere.

The total energy of an analysis-cube is found by summing the tension, quantum and gamma energies found in Eqs. (G2), (G4) and (G8). $E_{NG} = E_{TNGI} + E_{QNGI} + E_{\gamma NGI} = K_{T0}X_0^2[(1/2) - K_c|\mathbf{N_G}|/\xi_0 + K_c^2|\mathbf{N_G}|^2/\xi_0^2 - 3\delta X/X_0 + 3K_c|N_G|\delta X/2X_0\xi_0] + (2K_{Q0}/X_0^2)[(1/2) + K_c|\mathbf{N_G}|/\xi_0 + K_c^2|\mathbf{N_G}|^2/\xi_0^2 + 3\delta X/X_0 + 3K_c|N_G|\delta X/2X_0\xi_0] + K_{T0}X_0^2[-2K_c^2(|\mathbf{N_G}|/\xi_0)^2 - 4K_c^2K_{GC}(|\mathbf{N_G}|/\xi_0)^2 - 3K_c|\mathbf{N_G}|\delta X/X_0\xi_0]$, or, with Eq. (22), $K_{T0}X_0^2 = 2K_{Q0}/X_0^2$

$$E_{NG} = K_{T0}X_0^2[1 - 4K_c^2K_{GC}(|\mathbf{N_G}|/\xi_0)^2] \qquad \text{(G9)}$$

G.4. The Physical Nature of the Gamma-Force Inside a Sphere of Negative-Attached-Aether Containing Extrinsic-energy. In the sections above we make four assumptions in our analysis regarding the case of extrinsic-energy within negative-attached-aether: 1) the gamma-force has a negative tension component; 2) the gamma-force has a negative quantum-pressure component; 3) displacement of the analysis-cube will not lead to a change in cube size; and 4) the energies associated with the gamma-force are negative for the displacement. This section will validate these assumptions while providing a physical understanding of the gamma-force.

G.4.1. The Negative Gamma-force Tension and Quantum-Pressure Components. The gamma-force $\mathbf{F}_\gamma$ is a force that balances against the sum of $\mathbf{F_T}$ and $\mathbf{F_Q}$. Above in Eqs. (G6) and (G7) we have assigned two components to $\mathbf{F}_\gamma$ and arranged that the sum of these components will subtract from the stronger of $\mathbf{F_T}$ and $\mathbf{F_Q}$ as needed to provide the balancing force. Since $\mathbf{F_T}$ is less than $\mathbf{F_Q}$ for the case discussed, the total $\mathbf{F}_\gamma$ is a negative quantum-force providing an additional inward force.

The reason $\mathbf{F}_\gamma$ has two components, and the reason the total $\mathbf{F}_\gamma$ subtracts from the stronger of the forces, rather than adding to the weaker, is because the gamma-force originates from the force reductions caused by the immersion of extrinsic-energy as specified in the Extrinsic-Energy Force-Reduction Law. As the cubes are then displaced, the work done makes these force reductions grow, and these negative forces are the two components of the gamma-force. These forces then balance against the sum of $\mathbf{F_T}$ and $\mathbf{F_Q}$ as described above in Eqs. (G6) and (G7).

G.4.2. Analysis-Cube Size During Displacement. As described above, when cube-N of negative-attached-aether is moved inward due to injection of extrinsic-energy, cube-N has its tension reduced as described by Eq. (G1), $F_{TN} = K_{T0}X_0[1 - 2K_cx/\xi_0]$, and cube-N has its quantum-force increased as described by Eq. (G3), $F_{QN} = (2K_{Q0}/X_0^3)[1 + 2K_cx/\xi_0]$. If there were no gamma-force, cube-N would therefore expand. However, the overlapping cube-P of positive-attached-aether has the opposite behavior (see section D.3.5.2), and with no gamma-force it would compress. (Eqs. (66) and (68) used in section D.3 are $F_{TPP} = K_{T0}X_0[1 + 2K_cx/\xi_0]$ and $F_{QPP} = (2K_{Q0}/X_0^3)[1 -$

$2K_cx/\xi_0$], respectively.) In order for the density postulate to hold as they displace, cube-N and cube-P must either both expand, both compress, or both retain their size. Since the sum of $\mathbf{F_T}$ and $\mathbf{F_Q}$ on cube-N, described by Eqs. (G1) and (G3), is equal and opposite to the sum of $\mathbf{F_T}$ and $\mathbf{F_Q}$ on cube-P, this symmetry informs that the cube sizes will not change during their displacement, which is what is assumed above.

G.4.3. The Physics Leading to the Gamma-Force Energies. We see that the gamma-force arises between the positive-attached-aether and the negative-attached-aether in a way that leads to no displacement-induced size change of the displaced cubes. We also see that motion of attached-aether causes $\mathbf{F_{QN}}$ (the negative-attached-aether quantum-force) to grow and $\mathbf{F_{TN}}$ (the negative-attached-aether tension) to recede, and hence $\mathbf{F_\gamma}$ (the total gamma-force) must grow in magnitude to offset this growing difference. In this case, $\mathbf{F_\gamma} = \mathbf{F_{\gamma NT}} + \mathbf{F_{\gamma NQ}}$, and inspection of Eqs. (G6) and (G7) reveals that $|\mathbf{F_{\gamma NQ}}| > |\mathbf{F_{\gamma NT}}|$.

First consider the gamma-force component $\mathbf{F_{\gamma NQ}}$. Since the displacement of the negative-attached-aether is inward and the displacement of the positive-attached-aether is outward in this case, and since $\mathbf{F_{\gamma NQ}}$ is inward, it is the displacement of the outward-moving positive-attached-aether that does the work against $\mathbf{F_{\gamma NQ}}$ to make its magnitude grow, since we must have motion against the force to do work against it to make it grow. (The inward-moving negative-attached-aether moves in the direction of the force and would do negative work on $\mathbf{F_{\gamma NQ}}$ and therefore it cannot be the source of growth for $\mathbf{F_{\gamma NQ}}$.) Therefore we see that $\mathbf{F_{\gamma NQ}}$ is a negative quantum-force that subtracts from the nominal quantum-force within the negative-attached-aether, and yet work is done against it by the motion of the positive-attached-aether.

Now let us look at the gamma-force component $\mathbf{F_{\gamma NT}}$. Since the displacement of the negative-attached-aether is inward and the displacement of the positive-attached-aether is outward in this case, and since $\mathbf{F}_{\gamma\mathbf{NT}}$ is outward, it is the displacement of the inward-moving negative-attached-aether that does the work against $\mathbf{F}_{\gamma\mathbf{NT}}$ to make it grow, since we must have motion against the force to do work against it to make it grow. (The outward-moving positive-attached-aether would do negative work on $\mathbf{F}_{\gamma\mathbf{NT}}$ and therefore it cannot be the source of growth for $\mathbf{F}_{\gamma\mathbf{NT}}$.) Therefore we see that $\mathbf{F}_{\gamma\mathbf{NT}}$ is a negative tension that subtracts from the nominal tension within the negative-attached-aether, and work is done against it by the motion of the negative-attached-aether.

Since $\mathbf{F}_{\gamma\mathbf{NT}}$ and $\mathbf{F}_{\gamma\mathbf{NQ}}$ are forces of a negative tension and negative pressure, respectively, they both contribute negative energies to (relax positive energies of) the aetherial cubes.

The leading $\mathbf{F}_{\gamma\mathbf{PT}}$ term in Eq. (218) from section D.3.4, $4K_cK_{T0}(X_0/\xi_0)x\hat{\mathbf{r}}$, and the leading $\mathbf{F}_{\gamma\mathbf{NQ}}$ term in Eq. (G6), $-4K_cK_{T0}(X_0/\xi_0)x\hat{\mathbf{r}}$, are what provide the mechanism by which each type of attached-aether exerts forces on the other to maintain the equal-density postulate. The leading $\mathbf{F}_{\gamma\mathbf{NQ}}$ term in Eq. (G6) cancels the sum of the forces of tension $\mathbf{F}_{\mathbf{TN}}$ and quantum-force $\mathbf{F}_{\mathbf{QN}}$, and when integrated over the displacement it also contributes a negative work to offset the work done by $\mathbf{F}_{\mathbf{TN}}$ and $\mathbf{F}_{\mathbf{QN}}$.

The trailing $\mathbf{F}_{\gamma\mathbf{NQ}}$ term, $-4K_cK_{GC}K_{T0}(X_0/\xi_0)x\hat{\mathbf{r}}$, in Eq. (G6) and $\mathbf{F}_{\gamma\mathbf{NT}}$ term, $4K_cK_{GC}K_{T0}(X_0/\xi_0)x\hat{\mathbf{r}}$ in Eq. (G7) are forces that balance each other while contributing negative work to reduce the total energy in the negative-attached-aether. It is this reduction in total energy that leads to Newtonian gravity, as discussed above.

Appendix H – The Tension, Quantum-pressure and Gamma-forces Outside a Spherical Region Containing Extrinsic-Energy.

H.1. Positive-Aetherial-Displacement Outside a Spherical Attached-Aether Region Containing Extrinsic-Energy. Eq. (213), $\mathbf{P_{GOUT}} = P_0R_0^2\hat{\mathbf{r}}/r^2$, informs us that outside of a sphere of extrinsic-energy the positive-attached-aether cubes will be compressed rather than expanded. To see this, notice that a spherical shell originally located a distance r from the center of the sphere with thickness dr is pushed outward by a distance $P_R = P_0R_0^2\hat{\mathbf{r}}/r^2$. Since there is no extrinsic-energy outside of the sphere, the density is a constant ρ_0 there and hence the volume of the shell remains constant during its outward displacement. With dr_D defined as the shell thickness after it is displaced by P_R, volume invariance results in $4\pi r^2dr = 4\pi(r+P_R)^2dr_D$, or $dr_D/dr = r^2/(r+P_R)^2 = r^2/[r^2(1+P_R/r)^2] = (1+P_R/r)^{-2} \approx 1 - 2P_R/r$. Now Eq. (213) gives the magnitude $P_R = P_0R_0^2/r^2$, and therefore $dr_D/dr \approx 1 - 2P_0R_0^2/r^3$, or $dr_D \approx dr - 2drP_0R_0^2/r^3$. This results in the amount of compression being $\delta r_D = dr_D - dr = -2drP_0R_0^2/r^3$, where dr can be any small radial distance. Assigning dr to the size of an analysis-cube, $dr = X_0$, we see that the analysis-cubes are compressed by $\delta X = 2X_0P_0R_0^2/r^3$. At the edge of the sphere, the analysis-cubes are displaced radially outward by P_0 and the radial compression will gradually lower the displacement of the more distant cubes to the displacement $P_R = P_0R_0^2/r^2$ given by Eq. (213). In this case, all cubes are still displaced in the outward direction however, and so the radial tension-force-field is increased, it is just that the amount of increase falls off as $1/r^3$.

In the directions perpendicular to **r** the cubes will expand. The area of each spherical shell increases to $4\pi(r+P_R)^2$ from its original $4\pi r^2$. We have $(r+P_R)^2/r^2 = (1+P_R/r)^2 \approx 1+2P_R/r$, and we can see that the relative area expansion is $\delta A/A = 2P_R/r$. Now we form $A = Y^2$, and we see $\delta A/\delta Y = 2Y$, leading to $(\delta A/\delta Y)/A = (2Y)/Y^2$, or $\delta A/A = 2\delta Y/Y$ and hence $\delta Y/Y = P_R/r$. Therefore a cube of size $Y = X_0$ will expand by $\delta Y = X_0 P_R/r = X_0 P_0 R_0^2/r^3$, or $\delta Y = \delta X/2$, which is Eq. (73).

Now consider cube-P_G, a cube that is displaced by a distance P_G. Integrating Eq. (32), $W_{TD} = (X_0/\xi_0)\, K_c \int K_T X_0\, dx$, from 0 to $|\mathbf{P_G}|$, the first-order work done against the tension on cube-P_G from the displacement is $W_{TD} = K_c K_{T0} X_0^2 |\mathbf{P_G}|/\xi_0$. (The work is positive for all cubes at any $\mathbf{P_G}$, since the spherical shell is displacing outward, against the tension. Hence, the absolute value is used in the expression.)

Setting the compression effects aside, the tension-energy of cube-P_G is $E_T = E_{T0} + W_{TD} = (1/2)K_{T0}X_0^2 + K_c K_{T0} X_0^2 |\mathbf{P_G}|/\xi_0$. With $E_T = (1/2)K_T X_0^2$, we arrive at the expression $K_T = K_{T0}[1 + 2K_c|\mathbf{P_G}|/\xi_0]$. The tension-force remains $F_T = K_T X_0$, but now K_T includes the next order correction.

As extrinsic-energy is slowly injected, cube-P_G will move a distance $|\mathbf{P_G}|$. At the beginning of the motion F_T is of course just $K_{T0}X_0$, and it is at the end that $F_{TP\_END} = K_{T0}X_0[1 + 2K_c|\mathbf{P_G}|/\xi_0]$. With x defined as the deviation of the cube center from its nominal center (which varies from 0 to $|\mathbf{P_G}|$ during the detached-aether injection) the full expression for tension-force in the analysis-cube is

$$F_{TP} = K_{T0}X_0[1 + 2K_c x/\xi_0] \qquad \text{(H1)}$$

In Eq. (H1) x varies from 0 to $|\mathbf{P_G}|$.

The second-order effect on the cube tension-energy is included by calculating the work done on the field due to the cube displacement which is $W_{TD} = K_c\,(X_0/\xi_0) \int F_{TP}\, dx = (X_0/\xi_0)\, K_{T0}X_0\, K_c \int [1+2K_c x/\xi_0]\, dx = K_c(X_0/\xi_0)K_{T0}X_0 x + K_c^2(X_0/\xi_0)^2 K_{T0} x^2 = K_c X_0^2 K_{T0}|\mathbf{P_G}|/\xi_0 + K_c^2(X_0/\xi_0)^2 K_{T0}|\mathbf{P_G}|^2$.

The compression energy is now calculated by including a minus sign in Eq. (63) since compression will release energy from the tension, $W_{TC} = -2 \int K_T X_0\, dw$. From Eq. (H1), $K_T X_0 = K_{T0}X_0[1 + 2K_c x/\xi_0]$. We now use Eq. (62) with P_G instead of P, $x = (P_G/\delta X)w$, to obtain $K_T X_0 =$

$K_{T0}X_0[1+2K_c(P_G/\delta X)w/\xi_0]$. As specified in Eq. (63) we integrate from w = 0 to w = $\delta X/2$. Hence $W_{TC1} = -2\int K_T X_0\, dw = -2K_{T0}X_0 \int [1+2K_c(P_G/\delta X)w/\xi_0]\, dw = -2K_{T0}X_0 w - 2K_{T0}X_0K_c(P_G/\delta X)w^2/\xi_0$ $= -K_{T0}X_0\delta X - K_{T0}X_0K_cP_G\delta X/2\xi_0$.

Recall that Eq. (63) is for one dimension only. Eq. (73) informs that outside the sphere there is an expansion $\delta Y = \delta X/2$ for each of the other two dimensions and for those dimensions we integrate from 0 to $\delta Y/2$, which is equal to 0 to $\delta X/4$. Also, since it is an expansion we do work against the tension and the work is positive. For each transverse dimension then we obtain $W_{TC2} = 2K_{T0}X_0w + 2K_{T0}X_0K_c(P_G/\delta X)w^2/\xi_0 = K_{T0}X_0\delta X/2 + K_{T0}X_0K_c(P_G/\delta X)\delta X^2/8\xi_0$, and the total expansion energy will be twice that, since there are two dimensions. Hence, $W_{TC3} = W_{TC1} + 2W_{TC2} = -K_{T0}X_0\delta X - K_{T0}X_0K_cP_G\delta X/2\xi_0 + K_{T0}X_0\delta X + K_{T0}X_0K_cP_G\delta X/4\xi_0 = -K_{T0}X_0K_cP_G\delta X/4\xi_0$. The total tension-energy of an arbitrary analysis-cube outside the spherical region is the undisturbed energy plus the displacement energy plus the expansion energy, $E_{TPGO} = (1/2)K_{T0}X_0^2 + K_cX_0^2K_{T0}|\mathbf{P_G}|/\xi_0 + K_c^2(X_0/\xi_0)^2K_{T0}|\mathbf{P_G}|^2 - K_{T0}X_0K_c|\mathbf{P_G}|\delta X/4\xi_0$, or,

$$E_{TPGO} = K_{T0}X_0^2[(1/2) + K_c|\mathbf{P_G}|/\xi_0 + K_c^2(|\mathbf{P_G}|/\xi_0)^2 - K_c|\mathbf{P_G}|\delta X/4X_0\xi_0] \qquad \text{(H2)}$$

Turning to the quantum-field, the energy freed as cube-P_G is displaced is given by Eq. (33), $W_{QD} = (X_0/\xi_0)K_c\int 2K_Q/X_0^3\, dx$, and the displacement is P_G. The first order work done against the quantum-force-field as cube-P_G is displaced is $W_{QDP} = -2P_GK_cK_{Q0}/X_0^2\xi_0$. Setting the compression effects aside to focus on the effect of displacement, the quantum-energy of cube-P_G is $E_Q = E_{Q0} + W_{QDP} = K_{Q0}/X_0^2 - (X_0/\xi_0)[2P_GK_cK_{Q0}/X_0^3] = (K_{Q0}/X_0^2)(1 - 2K_cP_G/\xi_0)$. With $E_Q = K_Q/X_0^2$, we arrive at the expression $K_Q = K_{Q0}(1 - 2K_cP_G/\xi_0)$. The quantum-force remains $F_Q = 2K_Q/X_0^3$, but now K_Q includes the next order correction. As detached-aether is slowly injected, the center of cube-P_G will move a distance P_G. At the beginning of the motion F_{QP} is of course just $2K_{Q0}/X_0^3$, and it is at the end that $F_{QP} = (2K_{Q0}/X_0^3)[1 - 2K_cP_G/\xi_0]$. With x again defined as the deviation of the cube center from its nominal center (which varies from 0 to P_G during the detached-aether injection) the full expression for quantum-force in the analysis-cube is

$$F_{QP} = (2K_{Q0}/X_0^3)[1 - 2K_cx/\xi_0] \tag{H3}$$

The second order effect on the cube quantum-energy due to the cube displacement is now included with $W_{QDP} = -(X_0/\xi_0)K_c \int F_{QP}\, dx = -(X_0/\xi_0)K_c \int (2K_{Q0}/X_0^3)[1-2K_cx/\xi_0]\, dx = -(2K_{Q0}K_c/X_0^2\xi_0)x + (2K_{Q0}K_c^2/X_0^2\xi_0^2)x^2 = -(2K_{Q0}K_c/X_0^2\xi_0)P_G + (2K_{Q0}K_c^2/X_0^2\xi_0^2)P_G^2$.

The compression energy is calculated using Eq. (64) and with a positive sign, since compression will do work against the quantum-pressure, $W_{QC} = 4 \int K_Q/X_0^3\, dw$. From Eq. (H3), $F_{QP} = (2K_{Q0}/X_0^3)[1 - 2K_cx/\xi_0]$. We now use Eq. (62) with P_G for P, $x = (P_G/\delta X)w$, to obtain $F_{QP}/2 = K_Q/X_0^3 = (K_{Q0}/X_0^3)[1-2K_c(P_G/\delta X)w/\xi_0]$. As specified in Eq. (64) we integrate from w = 0 to w = $\delta X/2$. Hence $W_{QC1} = 4 \int K_Q/X_0^3\, dw = 4(K_{Q0}/X_0^3) \int [1-2K_c(P_G/\delta X)w/\xi_0]\, dw = 4(K_{Q0}/X_0^3)w - 4(K_{Q0}/X_0^3)K_c(P_G/\delta X)w^2/\xi_0 = 2(K_{Q0}/X_0^3)\delta X - (K_{Q0}/X_0^3)K_cP_G\delta X/\xi_0$.

Recall that Eq. (64) is for one dimension only. Eq. (73) informs that outside the sphere there is an expansion $\delta Y = \delta X/2$ for each of the other two dimensions and for those dimensions we integrate from 0 to $\delta Y/2$, which is equal to 0 to $\delta X/4$. Also, since it is an expansion the quantum-pressure will decrease and our signs reverse. For each transverse dimension then we obtain $W_{TC2} = -4(K_{Q0}/X_0^3)w + 4(K_{Q0}/X_0^3)K_c(P_G/\delta X)w^2/\xi_0 = -(K_{Q0}/X_0^3)\delta X + (K_{Q0}/X_0^3)K_c(P_G/\delta X)\delta X^2/4\xi_0$, and the total expansion energy will be twice that, since there are two dimensions. Hence, $W_{QC3} = W_{QC1} + 2W_{QC2} = 2(K_{Q0}/X_0^3)\delta X - (K_{Q0}/X_0^3)K_cP_G\delta X/\xi_0 - 2(K_{Q0}/X_0^3)\delta X + (K_{Q0}/X_0^3)K_cP_G\delta X/2\xi_0 = -(K_{Q0}/X_0^3)K_cP_G\delta X/2\xi_0$. The total quantum-energy of an arbitrary analysis-cube outside the spherical region is the undisturbed energy plus the displacement energy plus the expansion energy,

$E_{QPGO} = K_{Q0}/X_0^2 - (2K_{Q0}K_c/X_0^2\xi_0)P_G + (2K_{Q0}K_c^2/X_0^2\xi_0^2)P_G^2 - (K_{Q0}/X_0^3)K_cP_G\delta X/2\xi_0$, or, $E_{QPGO} = (2K_{Q0}/X_0^2)[(1/2) - (K_c/\xi_0)P_G + (K_c^2/\xi_0^2)P_G^2 - K_cP_G\delta X/4X_0\xi_0]$. Here we have derived the force for positive P_G. For negative P_G (or $\mathbf{P_G}$ at any angle) the work done is still negative, since such a cube is still expanding outward in the direction of the quantum-force. Hence we will replace P_G by its absolute value, $|\mathbf{P_G}|$:

$$E_{QPGO} = (2K_{Q0}/X_0^2)[(1/2) - K_c|\mathbf{P_G}|/\xi_0 + K_c^2(|\mathbf{P_G}|/\xi_0)^2 - K_c|\mathbf{P_G}|\delta X/4X_0\xi_0] \tag{H4}$$

We now recall that the gamma-force will be given by Eq. (216), $\mathbf{F_\gamma} = -\mathbf{F_{TL}} - \mathbf{F_Q}$. With the tension directed inward and the quantum-force directed outward and using Eqs. (H1) and (H3), $F_{TP} = K_{T0}X_0[1 + 2K_cx/\xi_0]$ and $F_{QP} = [2K_{Q0}/X_0^3][1 - 2K_cx/\xi_0]$, respectively, we obtain $\mathbf{F_{P\gamma O}} = K_{T0}X_0[1 + 2K_cx/\xi_0]\hat{\mathbf{r}} - [2K_{Q0}/X_0^3][1 - 2K_cx/\xi_0]\hat{\mathbf{r}}$. And with Eq. (21) above, $2K_{Q0}/X_0^3 = K_{T0}X_0$, we get $\mathbf{F_{P\gamma O}} = K_{T0}X_0[1 + 2K_cx/\xi_0 - 1 + 2K_cx/\xi_0]\hat{\mathbf{r}}$, or,

$$\mathbf{F_{P\gamma O}} = 4K_cK_{T0}(X_0/\xi_0)x\hat{\mathbf{r}} \tag{H5}$$

Since the tension exceeds the quantum-force for the positive-attached-aether case, the gamma-force could be a negative tension, a positive quantum-force or some combination of gamma-tension and gamma-quantum-force summing to the net force field given in Eq. (H5). We now use the same coupling of gamma-force components that was employed in Eqs. (218) and (219)

$$\mathbf{F_{\gamma PT}} = 4K_c(1+ K_{GC})K_{T0}(X_0/\xi_0)x\hat{\mathbf{r}} \tag{218}$$

$$\mathbf{F_{\gamma PQ}} = -4K_cK_{GC}K_{T0}(X_0/\xi_0)x\hat{\mathbf{r}} \tag{219}$$

The gamma-tension-force displacement-work on the cube is $W_{\gamma PTD} = (X_0/\xi_0)\, K_c \int F_{\gamma PT}\, dx = -(X_0/\xi_0)\, K_c \int 4K_c(1+K_{GC})K_{T0}(X_0/\xi_0)x\ dx = -2K_c^2(1+K_{GC})K_{T0}(X_0/\xi_0)^2x^2 = -2K_c^2(1+K_{GC})K_{T0}(X_0/\xi_0)^2P_G^2$.

The gamma-quantum-force displacement-work on the cube is $W_{\gamma PQD} = (X_0/\xi_0)\, K_c \int F_{\gamma PQ}\, dx = -(X_0/\xi_0)\, K_c \int 4K_cK_{GC}K_{T0}(X_0/\xi_0)x\ dx = -2K_c^2K_{GC}K_{T0}(X_0/\xi_0)^2x^2 = -2K_c^2K_{GC}K_{T0}(X_0/\xi_0)^2P_G^2$. For both the tension and quantum-force components, work is done on the cube displacement, lowering the field energies leading to minus signs, as discussed in sections D.3.5 and H.2.

The compression energy is now calculated using Eq. (63), $W_{TC} = 2\int F_T\,dw$. From Eq. (218), $F_{\gamma PT} = 4K_c(1+K_{GC})K_{T0}(X_0/\xi_0)x$. We now use Eq. (62) with P_G for P, $x = (P_G/\delta X)w$, and for the tension component of the gamma-force we have $F_{\gamma PT} = 4K_c(1+ K_{GC})K_{T0}(X_0/\xi_0)(P_G/\delta X)w$. As specified in Eq. (63) we integrate from w=0 to w=$\delta X/2$. Hence $W_{\gamma PTC1} = 2\int 4K_c(1+K_{GC})K_{T0}(X_0/\xi_0)(P_G/\delta X)w\,dw = 4K_c(1+K_{GC})K_{T0}(X_0/\xi_0)(P_G/\delta X)w^2 = K_c(1+K_{GC})K_{T0}(X_0/\xi_0)P_G\delta X$. For the gamma-force quantum-force component from Eq. (219) we have $F_{\gamma PQ} = -4K_cK_{GC}K_{T0}(X_0/\xi_0)x$, and we again use $x = (P_G/\delta X)w$ to get $F_{\gamma PQ} = -4K_cK_{GC}K_{T0}(X_0/\xi_0)(P_G/\delta X)w$. As specified in Eq. (63) we integrate from w = 0 to w = $\delta X/2$. Hence $W_{\gamma PQC1} = -2\int 4K_cK_{GC}K_{T0}(X_0/\xi_0)(P_G/\delta X)w\,dw = -4K_cK_{GC}K_{T0}(X_0/\xi_0)(P_G/\delta X)w^2 = -K_cK_{GC}K_{T0}(X_0/\xi_0)P_G\delta X$.

Recall that Eq. (63) is for one dimension only. Eq. (73) informs that outside the sphere there is an expansion $\delta Y = \delta X/2$ for each of the other two dimensions and for those dimensions we integrate from 0 to $\delta Y/2$, which is equal to 0 to $\delta X/4$. The signs of the work change because these dimensions expand instead of compress. For each transverse dimension we obtain a gamma tension work of $W_{\gamma PTC2} = -4K_c(1+K_{GC})K_{T0}(X_0/\xi_0)(P_G/\delta X)w^2 = -K_c(1+K_{GC})K_{T0}(X_0/\xi_0)P_G\delta X/4$. Similarly we obtain a gamma quantum-force work of $W_{\gamma PQC2} = 4K_cK_{GC}K_{T0}(X_0/\xi_0)(P_G/\delta X)w^2 = K_cK_{GC}K_{T0}(X_0/\xi_0)P_G\delta X/4$.

Since there are two transverse dimensions, adding the radial compression energy to the transverse expansion energies we obtain $W_{\gamma PTC3} = W_{\gamma PTC1} + 2W_{\gamma PTC2} = K_c(1+K_{GC})K_{T0}(X_0/\xi_0)P_G\delta X - K_c(1+K_{GC})K_{T0}X_0P_G\delta X/2\xi_0 = K_c(1+K_{GC})K_{T0}X_0P_G\delta X/2\xi_0$ and $W_{\gamma PQC3} = W_{\gamma PQC1} + 2W_{\gamma PQC2} = -K_cK_{GC}K_{T0}(X_0/\xi_0)P_G\delta X + K_cK_{GC}K_{T0}X_0P_G\delta X/2\xi_0 = -K_cK_{GC}K_{T0}X_0P_G\delta X/2\xi_0$.

The total gamma-energy of an arbitrary analysis-cube outside the spherical region is the sum of the displacement energies ($W_{\gamma PTD}$ and $W_{\gamma PQD}$) and the compression energies ($W_{\gamma PTC3}$ and $W_{\gamma PQC3}$), $E_{\gamma PGO} = -2K_c^2(1+K_{GC})K_{T0}(X_0/\xi_0)^2P_G^2 - 2K_c^2K_{GC}K_{T0}(X_0/\xi_0)^2P_G^2 + K_c(1+K_{GC})K_{T0}X_0P_G\delta X/2\xi_0 - K_cK_{GC}K_{T0}X_0P_G\delta X/2\xi_0 = -2K_c^2(1+2K_{GC})K_{T0}(X_0/\xi_0)^2P_G^2 + K_cK_{T0}X_0P_G\delta X/2\xi_0$. Here we have derived the force for positive P_G. For $\mathbf{P_G}$ at any angle the work done is still the same, so we will replace P by its absolute value, $|\mathbf{P_G}|$, $E_{\gamma PGO} = K_{T0}X_0^2[-2K_c^2(1+2K_{GC})(|\mathbf{P_G}|/\xi_0)^2 + K_c|\mathbf{P_G}|\delta X/2X_0\xi_0]$, or

$$E_{\gamma PGO} = K_{T0}X_0^2[-2K_c^2(|\mathbf{P_G}|/\xi_0)^2 - 4K_c^2K_{GC}(|\mathbf{P_G}|/\xi_0)^2 + K_c|\mathbf{P_G}|\delta X/2X_0\xi_0] \quad \text{(H6)}$$

The total energy is found by summing the tension, quantum and gamma energies, Eqs. (H2), (H4) and (H6). $E_{PGO} = K_{T0}X_0^2[(1/2) + K_c|\mathbf{P_G}|/\xi_0 + K_c^2|\mathbf{P_G}|^2/\xi_0^2 - K_c|\mathbf{P_G}|\delta X/4X_0\xi_0] + (2K_{Q0}/X_0^2)[(1/2 - K_c|\mathbf{P_G}|/\xi_0 + K_c^2|\mathbf{P_G}|^2/\xi_0^2 - K_c|\mathbf{P_G}|\delta X/4X_0\xi_0] + K_{T0}X_0^2[-2K_c^2(|\mathbf{P_G}|/\xi_0)^2 - 4K_c^2K_{GC}(|\mathbf{P_G}|/\xi_0)^2 + K_c|\mathbf{P_G}|\delta X/2X_0\xi_0]$. Next, we use Eq. (22), $2K_{Q0}/X_0^2 = K_{T0}X_0^2$, to get $E_{PGO} = K_{T0}X_0^2(1 - 4K_c^2K_{GC}|\mathbf{P_G}|^2/\xi_0^2)$, which is Eq. (225). Thus we see that Eq. (225) holds both inside and outside the sphere of extrinsic-energy.

H.2. The Physical Nature of the Gamma-Force Outside a Spherical Positive-Attached-Aether Region Containing Extrinsic-Energy. Outside a sphere of immersed extrinsic-energy, the positive-attached-aether will be compressed in the radial direction rather than expanded. The radial compression leads to a change in the sign of the work done from what was considered in section D.3 where we considered a radial expansion. The other difference from section D.3 is that we now no longer have any extrinsic-energy within the analysis-cubes to initiate the gamma forces. Instead, the gamma forces will be given by Eqs. (218) and (219) because of the continuity of those forces from the region inside the sphere. Beyond those two changes, the discussion of section D.3.5 remains valid outside the sphere.

H.3. Negative-Aetherial-Displacement Outside a Spherical Attached-Aether Region Containing Extrinsic-Energy. Eq. (213), $\mathbf{P_{GOUT}} = P_0R_0^2\hat{\mathbf{r}}/r^2$, coupled with Eq. (211), $\mathbf{N_{GL}} = -\mathbf{P_{GL}}$, inform us that outside of the sphere of extrinsic-energy the negative-attached-aether cubes will be radially expanded rather than compressed. To see this, notice that a spherical shell originally located a distance r from the center of the sphere with thickness dr is pulled inward by a distance $N_R = N_0R_0^2\hat{\mathbf{r}}/r^2$. Since there is no extrinsic-energy outside of the sphere, the density is a constant ρ_0 there and hence the volume of the shell remains constant during its inward displacement. With dr_D defined as the shell thickness after it is displaced by N_R, volume invariance results in $4\pi r^2dr = 4\pi(r-N_R)^2dr_D$, or $dr_D/dr = r^2/(r-N_R)^2 = r^2/[r^2(1-N_R/r)^2] = (1-N_R/r)^{-2} \approx 1 + 2N_R/r$. Now Eq. (213) coupled with Eq. (211) give the magnitude $N_R = N_0R_0^2/r^2$, and therefore $dr_D/dr \approx 1 + 2N_0R_0^2/r^3$, or $dr_D \approx dr + 2drN_0R_0^2/r^3$. This results in the amount of expansion being $\delta r_D = dr_D - dr = 2drN_0R_0^2/r^3$, where dr can be any small radial distance. Assigning dr to the size of an analysis-cube, $dr = X_0$, we see that the analysis-cubes are expanded by $\delta X = 2X_0N_0R_0^2/r^3$.

In the directions perpendicular to **r** the cubes will compress. The area of each spherical shell decreases to $4\pi(r-N_R)^2$ from its original $4\pi r^2$. We have $(r-N_R)^2/r^2 = (1-N_R/r)^2 \approx 1-2N_R/r$, and we can see that the relative area compression is $\delta A/A = -2N_R/r$. Now we form $A = Y^2$, and we see $\delta A/\delta Y = 2Y$, leading to $(\delta A/\delta Y)/A = (2Y)/Y^2$, or $\delta A/A = 2\delta Y/Y$ and hence $\delta Y/Y = -N_R/r$. Therefore a cube of size $Y = X_0$ will compress by $\delta Y = X_0N_R/r = X_0N_0R_0^2/r^3$, or $\delta Y = \delta X/2$, which is Eq. (73).

Now consider cube-N_G, a cube that is displaced by a distance N_G. Integrating Eq. (32) with a negative sign since the displacement is in the direction of the force, $W_{TD} = -(X_0/\xi_0)\, K_c \int K_TX_0\, dx$, from 0 to $|\mathbf{N_G}|$, the first-order work done against the tension on cube-N_G from the displacement is $W_{TD} = -K_cK_{T0}X_0^2N_G/\xi_0$.

Setting the expansion effects aside, the tension-energy of cube-N_G is $E_T = E_{T0} + W_{TD} = (1/2)K_{T0}X_0^2 - K_cK_{T0}X_0^2N_G/\xi_0$. With $E_T = (1/2)K_TX_0^2$, we arrive at the expression $K_T = K_{T0}[1 - 2K_cN_G/\xi_0]$. The tension-force remains $F_T = K_TX_0$, but now K_T includes the next order correction. As extrinsic-energy is slowly injected, cube-N_G will move a distance N_G. At the beginning of the motion F_T is of course just $K_{T0}X_0$, and it is at the end that $F_{TN\_END} = K_{T0}X_0[1 - 2K_cN_G/\xi_0]$. With x again defined as the deviation of the cube center from its nominal center (which varies from 0 to N_G during the detached-aether injection) the full expression for tension-force in the analysis-cube is

$$F_{TN} = K_{T0}X_0[1 - 2K_cx/\xi_0] \tag{H7}$$

The second-order effect on the cube tension-energy is included by calculating the work done on the field due to the cube displacement which is Eq. (32), $W_{TND} = -K_c\,(X_0/\xi_0) \int F_{TN}\,dx = -(X_0/\xi_0) K_{T0}X_0\ K_c \int [1-2K_cx/\xi_0]\,dx = -K_c(X_0/\xi_0)K_{T0}X_0x + K_c^2(X_0/\xi_0)^2K_{T0}x^2 = -K_cX_0^2K_{T0}N_G/\xi_0 + K_c^2(X_0/\xi_0)^2K_{T0}N_G^2$.

The radial expansion energy is now calculated by Eq. (63), $W_{TC} = 2 \int K_TX_0\,dw$. From Eq. (H7), $K_TX_0 = K_{T0}X_0[1 - 2K_cx/\xi_0]$. We now use Eq. (62) with N_G instead of P, $x = (N_G/\delta X)w$, to obtain $K_TX_0 = K_{T0}X_0[1-2K_c(N_G/\delta X)w/\xi_0]$. As specified in Eq. (63) we integrate from $w = 0$ to $w = \delta X/2$. Hence $W_{TC1} = 2 \int K_TX_0\,dw = 2K_{T0}X_0 \int [1-2K_c(N_G/\delta X)w/\xi_0]\,dw = 2K_{T0}X_0w - 2K_{T0}X_0K_c(N_G/\delta X)w^2/\xi_0 = K_{T0}X_0\delta X - K_{T0}X_0K_cN_G\delta X/2\xi_0$.

Recall that Eq. (63) is for one dimension only. Eq. (73) informs that outside the sphere there is a compression $\delta Y = \delta X/2$ for each of the two transverse dimensions and for those dimensions we integrate from 0 to $\delta Y/2$, which is equal to 0 to $\delta X/4$. Also, since it is a compression we do work with the tension and our sign is negative. For each transverse dimension then we obtain $W_{TC2} = -2K_{T0}X_0w + 2K_{T0}X_0K_c(N_G/\delta X)w^2/\xi_0 = -K_{T0}X_0\delta X/2 + K_{T0}X_0K_c(N_G/\delta X)\delta X^2/8\xi_0$. The total compression energy is $W_{TC3} = W_{TC1} + 2W_{TC2} = K_{T0}X_0\delta X - K_{T0}X_0K_cN_G\delta X/2\xi_0 - K_{T0}X_0\delta X + K_{T0}X_0K_cN_G\delta X/4\xi_0 = -K_{T0}X_0K_cN_G\delta X/4\xi_0$. The total tension-energy of an arbitrary analysis-cube outside the spherical region is the undisturbed energy plus the displacement energy plus the expansion energy, $E_{TNGO} = (1/2)K_{T0}X_0^2 - K_cX_0^2K_{T0}N_G/\xi_0 + K_c^2(X_0/\xi_0)^2K_{T0}N_G^2 - K_{T0}X_0K_cN_G\delta X/4\xi_0$, and since the derivation will be the same for any angle we can replace N_G by $|\mathbf{N_G}|$, and $E_{TNGO} = (1/2)K_{T0}X_0^2 - K_cX_0^2K_{T0}|\mathbf{N_G}|/\xi_0 + K_c^2(X_0/\xi_0)^2K_{T0}|\mathbf{N_G}|^2 - K_{T0}X_0K_c|\mathbf{N_G}|\delta X/4\xi_0$, or,

$$E_{TNGO} = K_{T0}X_0^2[(1/2) - K_c|\mathbf{N_G}|/\xi_0 + K_c^2(|\mathbf{N_G}|/\xi_0)^2 - K_c|\mathbf{N_G}|\delta X/4X_0\xi_0] \qquad \text{(H8)}$$

Turning to the quantum-field, the energy increase as cube-N_G is displaced is given by Eq. (33), $W_{QD} = (X_0/\xi_0)K_c\int 2K_Q/X_0^3\,dx$, and the displacement is N_G. The work done against the first order quantum-force-field as cube-N_G is displaced is $W_{QD} = 2N_GK_cK_{Q0}/X_0^2\xi_0$. Setting the expansion effects aside to focus on the effect of displacement, the quantum-energy of cube-N_G is $E_Q = E_{Q0} + W_{QD} = K_{Q0}/X_0^2 + 2N_GK_cK_{Q0}/X_0^2\xi_0 = (K_{Q0}/X_0^2)(1 + 2K_cN_G/\xi_0)$. With $E_Q = K_Q/X_0^2$, we arrive at the expression $K_Q = K_{Q0}(1 + 2K_cN_G/\xi_0)$. The quantum-force remains $F_Q = 2K_Q/X_0^3$, but now K_Q includes the next order correction. As detached-aether is slowly injected, the center of cube-N_G will move a distance N_G. At the beginning of the motion F_{QN} is of course just $2K_{Q0}/X_0^3$, and it is at the end that $F_{QN} = (2K_{Q0}/X_0^3)[1 + 2K_cN_G/\xi_0]$. With x again defined as the deviation of the cube center from its nominal center (which varies from 0 to N_G during the detached-aether injection) the full expression for quantum-force in the analysis-cube is

$$F_{QN} = (2K_{Q0}/X_0^3)[1 + 2K_cx/\xi_0] \qquad \text{(H9)}$$

The second order effect on the cube quantum-energy due to the cube displacement is now included with $W_{QDN} = (X_0/\xi_0)K_c \int (2K_{Q0}/X_0^3)[1+2K_cx/\xi_0]\,dx = (2K_{Q0}K_c/X_0^2\xi_0)x + (2K_{Q0}K_c^2/X_0^2\xi_0^2)x^2 = (2K_{Q0}K_c/X_0^2\xi_0)N_G + (2K_{Q0}K_c^2/X_0^2\xi_0^2)N_G^2$.

The expansion energy is calculated using Eq. (64), $W_{QC} = -4 \int K_Q/X_0^3\, dw$. From Eq. (H9), $F_{QN}/2 = K_Q/X_0^3 = (K_{Q0}/X_0^3)[1 + 2K_c x/\xi_0]$. We now use Eq. (62) with N_G for P, $x = (N_G/\delta X)w$, to obtain $K_Q/X_0^3 = (K_{Q0}/X_0^3)[1+2K_c(N_G/\delta X)w/\xi_0]$. As specified in Eq. (64) we integrate from w = 0 to w = $\delta X/2$. Hence $W_{QC1} = -4 \int K_Q/X_0^3\, dw = -4(K_{Q0}/X_0^3) \int [1+2K_c(N_G/\delta X)w/\xi_0]\, dw = -4(K_{Q0}/X_0^3)w - 4(K_{Q0}/X_0^3)K_c(N_G/\delta X)w^2/\xi_0 = -2(K_{Q0}/X_0^3)\delta X - (K_{Q0}/X_0^3)K_cN_G\delta X/\xi_0$.

Recall that Eq. (64) is for one dimension only. Eq. (73) informs that outside the sphere there is a compression $\delta Y = \delta X/2$ for each of the other two dimensions and for those dimensions we integrate from 0 to $\delta Y/2$, which is equal to 0 to $\delta X/4$. Also, since it is a compression the quantum-pressure will increase and our signs are positive. For each transverse dimension then we obtain $W_{QC2} = 4(K_{Q0}/X_0^3)w + 4(K_{Q0}/X_0^3)K_c(N_G/\delta X)w^2/\xi_0 = (K_{Q0}/X_0^3)\delta X + (K_{Q0}/X_0^3)K_c(N_G/\delta X)\delta X^2/4\xi_0$, and the total compression energy will be twice that, since there are two dimensions. Hence, $W_{QC3} = W_{QC1} + 2W_{QC2} = -2(K_{Q0}/X_0^3)\delta X - (K_{Q0}/X_0^3)K_cN_G\delta X/\xi_0 + 2(K_{Q0}/X_0^3)\delta X + (K_{Q0}/X_0^3)K_cN_G\delta X/2\xi_0 = -(K_{Q0}/X_0^3)K_cN_G\delta X/2\xi_0$. The total quantum-energy of an arbitrary analysis-cube outside the spherical region is the undisturbed energy plus the displacement energy plus the expansion energy, $E_{QNGO} = K_{Q0}/X_0^2 + (2K_{Q0}K_c/X_0^2\xi_0)N_G + (2K_{Q0}K_c^2/X_0^2\xi_0^2)N_G^2 - (K_Q/X_0^3)K_cN_G\delta X/2\xi_0$, or, $E_{QNGO} = (2K_{Q0}/X_0^2)[(1/2) + (K_c/\xi_0)N_G + (K_c^2/\xi_0^2)N_G^2 - K_cN_G\delta X/4X_0\xi_0]$. Here we have derived the force for positive N_G. For $\mathbf{N_G}$ at any angle the work done is the same, so we will replace N_G by its absolute value, $|\mathbf{N_G}|$:

$$E_{QNGO} = (2K_{Q0}/X_0^2)[(1/2) + K_c|\mathbf{N_G}|/\xi_0 + K_c^2(|\mathbf{N_G}|/\xi_0)^2 - K_c|\mathbf{N_G}|\delta X/4X_0\xi_0] \tag{H10}$$

We now recall that the gamma-force will be given by Eq. (216), $\mathbf{F_\gamma} = -\mathbf{F_{TL}} - \mathbf{F_Q}$. With the tension directed inward and the quantum-force directed outward and using Eqs. (H7) and (H9), $F_{TN} = K_{T0}X_0[1 - 2K_cx/\xi_0]$ and $F_{QN} = [2K_{Q0}/X_0^3][1 + 2K_cx/\xi_0]$, respectively, we obtain $\mathbf{F_{\gamma NO}} = K_{T0}X_0[1 -$

$2K_c x/\xi_0]\hat{\mathbf{r}} - [2K_{Q0}/X_0^3][1 + 2K_c x/\xi_0]\hat{\mathbf{r}}$. And with Eq. (21) above, $2K_{Q0}/X_0^3 = K_{T0}X_0$, we get $\mathbf{F}_{\gamma\mathbf{NO}} = K_{T0}X_0[1 - 2K_c x/\xi_0 - 1 - 2K_c x/\xi_0]\hat{\mathbf{r}}$, or,

$$\mathbf{F}_{\gamma\mathbf{NO}} = -4K_c K_{T0}(X_0/\xi_0)x\hat{\mathbf{r}} \tag{H11}$$

Since the quantum-force exceeds the tension for the negative-attached-aether case, the gamma-force could be a positive tension, a negative quantum-force or some combination of gamma-tension and gamma-quantum-force summing to the net force field given in Eq. (H11). We now use the same coupling of gamma-force components that was employed in Eqs. (G6) and (G7)

$$\mathbf{F}_{\gamma\mathbf{NQ}} = -4K_c(1+K_{GC})K_{T0}(X_0/\xi_0)x\hat{\mathbf{r}} \tag{G6}$$

$$\mathbf{F}_{\gamma\mathbf{NT}} = 4K_c K_{GC} K_{T0}(X_0/\xi_0)x\hat{\mathbf{r}} \tag{G7}$$

The gamma-quantum-force displacement-work on the cube is Eq. (33), $W_{\gamma NQD} = (X_0/\xi_0)\ K_c \int F_{\gamma NQ}\ dx = -(X_0/\xi_0)\ K_c \int 4K_c(1+K_{GC})K_{T0}(X_0/\xi_0)x\ dx = -2K_c^2(1+K_{GC})K_{T0}(X_0/\xi_0)^2x^2 = -2K_c^2(1+K_{GC})K_{T0}(X_0/\xi_0)^2N_G^2$. The gamma-tension-force displacement-work on the cube is Eq. (32), $W_{\gamma NTD} = (X_0/\xi_0)\ K_c \int F_{\gamma NT}\ dx = -(X_0/\xi_0)\ K_c \int 4K_c K_{GC} K_{T0}(X_0/\xi_0)x\ dx = -2K_c^2 K_{GC} K_{T0}(X_0/\xi_0)^2x^2 = -2K_c^2 K_{GC} K_{T0}(X_0/\xi_0)^2N_G^2$. For both the tension and quantum-force components, work is done on the cube displacement, lowering the field energies, as discussed in sections D.3.5 and H.4.

The expansion energy is now calculated using Eq. (63), $W_{TC} = 2 \int F_T\ dw$. From Eq. (G6), $F_{\gamma NQ} = -4K_c(1+K_{GC})K_{T0}(X_0/\xi_0)x$. We now use Eq. (62) with N_G for P, $x = (N_G/\delta X)w$, and for the gamma-quantum-force we have $F_{\gamma NQ} = -4K_c(1+K_{GC})K_{T0}(X_0/\xi_0)(N_G/\delta X)w$. As specified in Eq. (63) we integrate from $w = 0$ to $w = \delta X/2$, and the work is positive for expansion against the negative quantum-force component, $W_{\gamma NQC1} = 2 \int 4K_c(1+K_{GC})K_{T0}(X_0/\xi_0)(N_G/\delta X)w\ dw =$

$4K_c(1+K_{GC})K_{T0}(X_0/\xi_0)(N_G/\delta X)w^2 = K_c(1+K_{GC})K_{T0}(X_0/\xi_0)N_G\delta X$. For the gamma-tension we have $F_{\gamma NT} = 4K_cK_{GC}K_{T0}(X_0/\xi_0)x$, and we again use $x = (N_G/\delta X)w$ to get $F_{\gamma NT} = 4K_cK_{GC}K_{T0}(X_0/\xi_0)(N_G/\delta X)w$. As specified in Eq. (63) we integrate from $w = 0$ to $w = \delta X/2$ and the work is negative for expansion against the negative gamma-tension. Hence $W_{\gamma NTC1} = -2 \int 4K_cK_{GC}K_{T0}(X_0/\xi_0)(N_G/\delta X)w\, dw = -4K_cK_{GC}K_{T0}(X_0/\xi_0)(N_G/\delta X)w^2 = -K_cK_{GC}K_{T0}(X_0/\xi_0)N_G\delta X$.

Recall that Eq. (63) is for one dimension only. Eq. (73) informs that outside the sphere there is a compression $\delta Y = \delta X/2$ for each of the other two dimensions and for those dimensions we integrate from 0 to $\delta Y/2$, which is equal to 0 to $\delta X/4$. Also, since it is a compression we will do negative work against the negative gamma-quantum-force and we will do positive work against the negative-gamma-tension. For each transverse dimension then we obtain a gamma quantum-force work of $W_{\gamma NQC2} = -4K_c(1+K_{GC})K_{T0}(X_0/\xi_0)(N_G/\delta X)w^2 = -K_c(1+K_{GC})K_{T0}(X_0/\xi_0)N_G\delta X/4$. Similarly we obtain a gamma tension work of $W_{\gamma NTC2} = 4K_cK_{GC}K_{T0}(X_0/\xi_0)(N_G/\delta X)w^2 = K_cK_{GC}K_{T0}(X_0/\xi_0)N_G\delta X/4$.

Adding the radial expansion energies to the transverse compression energies, and since there are two transverse dimensions we get $W_{\gamma NQC3} = W_{\gamma NQC1} + 2W_{\gamma NQC2} = K_c(1+K_{GC})K_{T0}(X_0/\xi_0)N_G\delta X - K_c(1+K_{GC})K_{T0}X_0N_G\delta X/2\xi_0 = K_c(1+K_{GC})K_{T0}X_0N_G\delta X/2\xi_0$ and $W_{\gamma NTC3} = W_{\gamma NTC1} + 2W_{\gamma NTC2} = -K_cK_{GC}K_{T0}(X_0/\xi_0)N_G\delta X + K_cK_{GC}K_{T0}X_0N_G\delta X/2\xi_0 = -K_cK_{GC}K_{T0}X_0N_G\delta X/2\xi_0$.

The total gamma-energy of an arbitrary analysis-cube outside the spherical region is the sum of the displacement energies ($W_{\gamma NTD}$ and $W_{\gamma NQD}$) and expansion energies ($W_{\gamma NTC3}$ and $W_{\gamma NQC3}$), $E_{\gamma NGO} = -2K_c^2(1+K_{GC})K_{T0}(X_0/\xi_0)^2N_G^2 - 2K_c^2K_{GC}K_{T0}(X_0/\xi_0)^2N_G^2 + K_c(1+K_{GC})K_{T0}X_0N_G\delta X/2\xi_0 - K_cK_{GC}K_{T0}X_0N_G\delta X/2\xi_0 = -2K_c^2(1+2K_{GC})K_{T0}(X_0/\xi_0)^2N_G^2 + K_cK_{T0}X_0N_G\delta X/2\xi_0$. Here we have derived the force for positive N_G. For $\mathbf{N_G}$ at any angle the work is the same, and we replace N_G by its absolute value, $|\mathbf{N_G}|$, $E_{\gamma NGO} = K_{T0}X_0^2[-2K_c^2(1+2K_{GC})(|\mathbf{N_G}|/\xi_0)^2 + K_c|\mathbf{N_G}|\delta X/2X_0\xi_0]$, or

$$E_{\gamma NGO} = K_{T0}X_0^2[-2K_c^2(|\mathbf{N_G}|/\xi_0)^2 - 4K_c^2K_{GC}(|\mathbf{N_G}|/\xi_0)^2 + K_c|\mathbf{N_G}|\delta X/2X_0\xi_0] \qquad \text{(H12)}$$

The total energy is found by summing the tension, quantum and gamma energies, Eqs. (H8), (H10) and (H12). $E_{NGO} = K_{T0}X_0^2[(1/2) - K_c|\mathbf{N_G}|/\xi_0 + K_c^2|\mathbf{N_G}|^2/\xi_0^2 - K_c|\mathbf{N_G}|\delta X/4X_0\xi_0] + (2K_{Q0}/X_0^2)[(1/2 + K_c|\mathbf{N_G}|/\xi_0 + K_c^2|\mathbf{N_G}|^2/\xi_0^2 - K_c|\mathbf{N_G}|\delta X/4X_0\xi_0] + K_{T0}X_0^2[-2K_c^2(|\mathbf{N_G}|/\xi_0)^2 - 4K_c^2K_{GC}(|\mathbf{N_G}|/\xi_0)^2 + K_c|\mathbf{N_G}|\delta X/2X_0\xi_0]$. Next, we use Eq. (22), $2K_{Q0}/X_0^2 = K_{T0}X_0^2$, to get $E_{NGO} = K_{T0}X_0^2(1 - 4K_c^2K_{GC}|\mathbf{N_G}|^2/\xi_0^2)$, which is Eq. (226). Thus we see that Eq. (226) holds both inside and outside the sphere of extrinsic-energy.

H.4. The Physical Nature of the Gamma-Force Outside a Spherical Negative-Attached-Aether Region Containing Extrinsic-Energy. Outside a sphere of immersed extrinsic-energy, the negative-attached-aether will be expanded in the radial direction rather than compressed. The radial expansion leads to a change in the sign of the work done from what was considered in section G.4 where there was a radial compression for the non-displacement work. The other difference from section G.4 is that we now no longer have any extrinsic-energy within the analysis-cubes to initiate the gamma forces. Instead, the gamma forces will be given by Eqs. (G6) and (G7) because of the continuity of those forces from the region inside the sphere. Beyond those two changes, the discussion of section G.4 remains valid outside the sphere.

Appendix I – A Sun in Motion Through the Aether

I.1. A Sun Not at Rest with the Aether

Up to now we have done our analysis from a frame of reference at rest with respect to the aether. Also, our perihelion and light bending calculations have assumed that the sun is at rest with respect to the aether. We will now investigate what happens when the sun moves with respect to the aether.

I.1.1. Lorentz invariance of electrodynamics. Due to the nature of the Lorentz Transformation of space and time, an analysis from the aetherial rest frame is all that is needed for electromagnetism, as it is well known that Maxwell's Equations and the Lorentz Force Equation are invariant in form under a Lorentz Transformation. In frames moving with respect to the aether, meter sticks shrink, clocks slow down, and the electromagnetic fields transform in just the right way so that the equations retain their form. A rest frame observer will say that the moving frame observers have the wrong length, time and field measurements, but the moving frame observers will say that it is the rest frame observers who are incorrect. This situation led to Poincare's proposal that there might be a relativity principle involved. However, herein we take the pre-relativistic viewpoint that the rest frame observer is the only one making correct measurements. Nonetheless, once the rest frame measurements are established, all observers will agree that the Lorentz transformations can be made from frame to frame, and therefore once the equations are established in the preferred frame they are established in all frames. Electrodynamic experiments in all frames are therefore understood, and a derivation of the electrodynamics equations is only needed from the single aetherial rest frame, as provided above.

I.1.2. Gravitation cannot assume Lorentz invariance. For the gravitational equations derived above we can no longer rely on Lorentz invariance, and we must now analyze what happens when entities and frames are moving with respect to the aether. As far as moving observers are concerned, the analysis is simple; we just apply the Lorentz transformations from frame to frame. However, for moving objects, other effects enter in and these effects will now be investigated.

I.2. Effect on the Sun and $\nabla\phi_{SUN}$ due to Motion Through the Aether. We now turn to a calculation of the extrinsic-energy-flow effect on the sun as it moves through the aether. We will make the assumption that the aetherial rest frame coincides with the frame wherein the cosmic background radiation is most isotropic; while there is no a priori reason that this must be true, it is a reasonable guess. This leads to a velocity of the sun with respect to the aether of $v/c \approx 1.23 \times 10^{-3}$ [I1]. We will also assume that the potential ϕ propagates along with the sun.

I.2.1. $|\nabla\phi_{SUN}(R_{SUN})|$ Near the Surface of a Solid Sun Moving Through the Aether. Consider now the effect of extrinsic-energy-flow on $\nabla\phi_{SUN}$. To calculate the flow effect, we refer again to Eqs. (302) and (303) for the expansion of the analysis-cubes:

$$\delta X_{3X}/X_0 = \delta X_{3Y}/X_0 = (X_3 - X_0)/X_0 \approx [K_{G1} - K_{G2}]\rho_E/4 + K_{F4}\rho_{FE}/4 \quad (302)$$

$$\delta X_{3Z}/X_0 = (X_1 - X_0)/X_0 \approx [K_{G1} - K_{G2}]\rho_E/4 \quad (303)$$

We start by assuming the sun to be a rigid, spherical, body. (This obviously incorrect assumption will be relaxed below, but it is a useful way to begin our analysis.) With every analysis-cube within the sun's boundary expanding via Eqs. (302) and (303), and recalling Eq. (242), $\mathbf{P}_{GL} = -\varepsilon_0\nabla\phi_G/2\rho_0$, and with $\mathbf{P}_{GL}$ the integral of δX (the cubes expand and their expansions add into the displacement $\mathbf{P}_{GL}$), we see that $\nabla\phi_{SUN}$ is proportional to the integral of Eqs. (302) and (303). Hence we see that $\nabla\phi_{SUN}$ becomes oblate; $\nabla\phi_{SUN}$ is larger in the X and Y directions than it is in the Z direction by the factor $([K_{G1} - K_{G2}]\rho_E/4 + K_{F4}\rho_{FE}/4)/([K_{G1} - K_{G2}]\rho_E/4) = 1 + K_{F4}\rho_{FE}/[K_{G1} - K_{G2}]\rho_E$.

Now, notice that without the $K_{F4}\rho_{FE}/4$ term of Eq. (302) $\nabla\phi_{SUN}$ is radially outward and equal in magnitude everywhere at the sun's surface. Since $\mathbf{P}_{GL} = -\varepsilon_0\nabla\phi_G/2\rho_0$ is the distance that the aether is displaced, we see that the aether originally at the sun's surface will be expanded to lie on the sphere $x^2 + y^2 + z^2 = a_0^2$ (if there is no flow effect) and here the radius a_0 is

$$a_0 = R_{SUN} + \mathbf{P_{GL}}(R_{SUN}) = R_{SUN} - \varepsilon_0|\nabla\phi_{SUN}(R_{SUN})|/2\rho_0 = R_{SUN}(1+\rho_G/6\rho_0) \quad \text{(no flow effect)} \quad \text{(I1)}$$

Eq. (I1) makes use of Eq. (213) for a spherical source of extrinsic-energy, $\mathbf{P_{GOUT}} = P_0R_0^2\hat{\mathbf{r}}/r^2 = (\rho_G/6\rho_0)R_0^3\hat{\mathbf{r}}/r^2$ ($r > R_0$), which at $R_0 = R_{SUN}$ is $\mathbf{P_{GL}} = (\rho_G/6\rho_0)R_{SUN}\hat{\mathbf{r}}$.

Assume the sun is centered on the z axis. Including the $K_{F4}\rho_{FE}/4$ term causes an increase in the x and y components of $\nabla\phi_{SUN}(R_{SUN})$ and no increase in the z component. We will next show that this leads to the sphere of radius a_0 being distorted into an oblate spheroid that has an intersection with the xz plane of

$$x = (1 + \varepsilon)(a_0^2 - z^2)^{1/2} \quad \text{(I2)}$$

In Eq. (I2), we define ε as

$$\varepsilon = K_{F4}\rho_{FE}/4 \quad \text{(I3)}$$

Squaring Eq. (I2), $x^2 = (1 + \varepsilon)^2(a_0^2 - z^2)$, or $x^2/(1 + \varepsilon)^2 = (a_0^2 - z^2)$, or $x^2/(1 + \varepsilon)^2 + z^2 = a_0^2$, which is the equation of an ellipse. Now, $x^2/(1 + \varepsilon)^2 = x^2/(1 + 2\varepsilon + \varepsilon^2) \approx x^2(1 - 2\varepsilon)$, to first order in ε. Making the substitutions $x = r\sin\theta$ and $z = r\cos\theta$ leaves $x^2(1 - 2\varepsilon) + z^2 = a_0^2 = r^2\sin^2\theta(1 - 2\varepsilon) + r^2\cos^2\theta = r^2 - 2\varepsilon r^2\sin^2\theta = a_0^2 = r^2(1 - 2\varepsilon\sin^2\theta)$, or,

$$r = a_0/(1 - 2\varepsilon\sin^2\theta)^{1/2} \approx a_0(1 + \varepsilon\sin^2\theta) = a_0(1 + \varepsilon - \varepsilon\cos^2\theta) \quad \text{(I4)}$$

Now set

$$a_0 = a - 2a\varepsilon/3 \quad \text{(I5)}$$

Eqs. (I4) and (I5) lead to $r = (a - 2a\varepsilon/3)(1 + \varepsilon - \varepsilon\cos^2\theta) = (a + a\varepsilon - a\varepsilon\cos^2\theta - 2a\varepsilon/3) = (a - a\varepsilon\cos^2\theta + a\varepsilon/3) = a(1 - \varepsilon\cos^2\theta + \varepsilon/3) = a[1 - (1/3)\varepsilon(3\cos^2\theta - 1)]$ to first order in ε. And using the second Legendre polynomial $P_2(x) = (1/2)(3x^2 - 1)$, we obtain

$$r = a[1 - (2/3)\varepsilon P_2(\cos\theta)] \tag{I6}$$

Eq. (I6) is the equation of the intersection of an oblate spheroid and the xz plane. Due to the symmetry of x and y found in Eq. (302) we see that Eq. (I6) can apply to any plane containing the z axis, and therefore Eq. (I6) applies more generally; Eq. (I6) is the equation for the boundary of an oblate-spheroid in spherical coordinates. Eq. (I6) is the expression for the position of the surface of the aether that would have been at the sun's surface were it not for the tension-modification caused by the extrinsic-energy and the extrinsic-energy-flow. Again recalling Eq. (242), $\mathbf{P_{GL}} = -\varepsilon_0\boldsymbol{\nabla}\phi_G/2\rho_0$, we obtain $-\varepsilon_0\boldsymbol{\nabla}_\mathbf{r}\phi_{SUN}/2\rho_0 = r - R_{SUN} = a[1 - (2/3)\varepsilon P_2(\cos\theta)] - R_{SUN} = a[1 - (1/3)\varepsilon(3\cos^2\theta - 1)] - R_{SUN} \approx (a_0 + 2a_0\varepsilon/3)[1 - (1/3)\varepsilon(3\cos^2\theta - 1)] - R_{SUN}$. (The last equality uses Eq. (I5), $a_0 = a - 2a\varepsilon/3 = a(1 - 2\varepsilon/3)$, or $a = a_0/(1 - 2\varepsilon/3) \approx a_0 + 2a_0\varepsilon/3$. $\boldsymbol{\nabla}_\mathbf{r}$ is the radial component of $\boldsymbol{\nabla}$.) With this, $-\varepsilon_0\boldsymbol{\nabla}_\mathbf{r}\phi_{SUN}(R_{SUN})/2\rho_0 = a_0 + 2a_0\varepsilon/3 - (a_0/3)\varepsilon(3\cos^2\theta - 1) - R_{SUN}$ to first order in ε. And now, from Eq. (I1) $a_0 - R_{SUN} = R_{SUN}\rho_G/6\rho_0$, leaving $-\varepsilon_0\boldsymbol{\nabla}_\mathbf{r}\phi_{SUN}(R_{SUN})/2\rho_0 = R_{SUN}\rho_G/6\rho_0 + 2a_0\varepsilon/3 - (a_0/3)\varepsilon(3\cos^2\theta - 1)$ or

$$-\boldsymbol{\nabla}_\mathbf{r}\phi_{SUN}(R_{SUN}) = R_{SUN}\rho_G/3\varepsilon_0 + 4\rho_0 a_0\varepsilon/3\varepsilon_0 - (2\rho_0 a_0/3\varepsilon_0)\varepsilon(3\cos^2\theta - 1) \tag{I7}$$

Eq. (I7) is the equation for the aetherial displacement, and it is what we'd expect. In the z direction $\cos\theta = 1$ and Eq. (I7) becomes $-\boldsymbol{\nabla}_\mathbf{r}\phi_{SUN}(R_{SUN}) = R_{SUN}\rho_G/3\varepsilon_0$, which is equal to the case with no flow, while in the x direction $\cos\theta = 0$ and for that case $-\boldsymbol{\nabla}_\mathbf{r}\phi_{SUN}(R_{SUN}) = R_{SUN}\rho_G/3\varepsilon_0 + 2\rho_0 a_0\varepsilon/\varepsilon_0$, an increase in the displacement of $a_0\varepsilon$ over the no flow case. (The displacement is $\mathbf{P_{GL}} = -\varepsilon_0\boldsymbol{\nabla}_\mathbf{r}\phi_{SUN}/2\rho_0 = \varepsilon_0 R_{SUN}\rho_G/6\varepsilon_0\rho_0 + a_0\varepsilon$.) From Eq. (302) we see $\delta X_{3X}/X_0$ from the flow effect is $K_{F4}\rho_{FE}/4 = \varepsilon$, and therefore each analysis cube will expand by εX_0, and the entire sphere of radius a_0 will expand by $a_0\varepsilon$. This shows that Eq. (I7) verifies our use of Eq. (I1).

I.2.2. Field and Potential of a Uniform-Mass-Density Oblate-Spheroid. Next, consider the potential outside of a uniform-mass-density oblate-spheroid (ϕ_{UMD}), [I2]

$\phi_{UMD} = –GM/r + [0.4GMa^2/r^3]\varepsilon_2P_2(\cos\theta) = –GM/r + [0.2GMa^2/r^3]\varepsilon_2(3\cos^2\theta – 1)$ (I8)

To find the field $\nabla\phi_{UMD}$ note that in spherical coordinates we have $\nabla\phi = \hat{\mathbf{r}}\partial\phi/\partial r + \hat{\mathbf{q}}(1/r)(\partial\phi/\partial\theta)$, where $\hat{\mathbf{r}}$ is a unit vector in the **r** direction and $\hat{\mathbf{q}}$ is a unit vector in the θ direction, and hence:

$\nabla\phi_{UMD} = \hat{\mathbf{r}}\partial\phi_{UMD}/\partial r + \hat{\mathbf{q}}(1/r)(\partial\phi_{UMD}/\partial\theta)$

$= \hat{\mathbf{r}}GM/r^2 – \hat{\mathbf{r}}[0.6GMa^2/r^4]\varepsilon_2(3\cos^2\theta – 1) – \hat{\mathbf{q}}[1.2GMa^2/r^4]\varepsilon_2\cos\theta\sin\theta$ (I9)

I.2.3. $\nabla\phi_{SUN}$(r) of a Solid Sun for r > R_{SUN}. Once outside of the extrinsic-energy and extrinsic-energy-flow of the sun, Laplace's Equation, $\nabla^2\phi_{SUN} = 0$, will apply for the primary gravitational effects. ($\nabla^2\phi_{SUN}$ represents the divergence of the displacements of the walls of an aetherial analysis-cube. Without additional forces inside the cube, the density will remain at its nominal value resulting in Laplace's Equation.) Based on Eq. (I9) and the fact that Eq. (I7) at the sun's boundary is an oblate spheroid, we will guess at a solution for ϕ_{SUN} and hence $\nabla\phi_{SUN}$ for the region $r > R_{SUN}$:

$\phi_{SUN} = –C_4/r + (C_5/r^3)[(1/2)(3\cos^2\theta – 1)]$ ($r > R_{SUN}$) (I10)

$\nabla\phi_{SUN} = \hat{\mathbf{r}}\partial\phi_{SUN}/\partial r + \hat{\mathbf{q}}(1/r)(\partial\phi_{SUN}/\partial\theta)$

$= \hat{\mathbf{r}}C_4/r^2 – \hat{\mathbf{r}}[3C_5/2r^4](3\cos^2\theta – 1) – \hat{\mathbf{q}}[3C_5/r^4]\cos\theta\sin\theta$ ($r > R_{SUN}$) (I11)

In the above, C_4 and C_5 are arbitrary constants we will used to fit our solution. We can now verify that Eq. (I10) satisfies $\nabla^2\phi_{SUN} = 0$:

$\nabla^2\phi_{SUN} = (1/r^2)\partial/\partial r(r^2\partial\phi/\partial r) + (1/r^2\sin\theta)\partial/\partial\theta(\sin\theta\partial\phi/\partial\theta)$

$= (1/r^2)\partial/\partial r(r^2[C_4/r^2 – (3C_5/2r^4)(3\cos^2\theta – 1)] + (1/r^2\sin\theta)\partial/\partial\theta[(\sin\theta)(C_5/r^3)(3\cos\theta)(–\sin\theta)$

$= (1/r^2)\partial/\partial r([C_4 – (3C_5/2r^2)(3\cos^2\theta – 1)] – (1/r^2\sin\theta)\partial/\partial\theta([3C_5/r^3]\cos\theta\sin^2\theta)$

$= (1/r^2)\{–(–2)(3C_5/2r^3)(3\cos^2\theta – 1)]\} – (1/r^2\sin\theta)([3C_5/r^3]\{–\sin^3\theta + 2\sin\theta\cos^2\theta)$

$= (3C_5/r^5)(3\cos^2\theta – 1) – ([3C_5/r^5)\{–\sin^2\theta + 2\cos^2\theta)$

$= (3C_5/r^5)(3\cos^2\theta – 1) – ([3C_5/r^5)\{–(1 – \cos^2\theta) + 2\cos^2\theta\}$

$= (3C_5/r^5)(3\cos^2\theta - 1) - (3C_5/r^5)\{3\cos^2\theta - 1\} =$

$$\nabla^2\phi_{SUN} = 0 \qquad \text{(I12)}$$

The values of the constants C_4 and C_5 are found by setting the magnitude of the radial component of Eq. (I11) consistent with Eq. (I7) at some value of r, $r = a_1$: $C_4/a_1^2 - [3C_5/2a_1^4](3\cos^2\theta - 1) = -R_{SUN}\rho_G/3\varepsilon_0 - 4\rho_0 a_0\varepsilon/3\varepsilon_0 + (2\rho_0 a_0/3\varepsilon_0)\varepsilon(3\cos^2\theta - 1)$. From this, we see that the settings $3C_5/2a_1^4 = -(2\rho_0 a_0/3\varepsilon_0)\varepsilon$ and $C_4/a_1^2 = -R_{SUN}\rho_G/3\varepsilon_0 - 4\rho_0 a_0\varepsilon/3\varepsilon_0$ do indeed result in agreement between the radial components of Eqs. (I7) and (I11) with the sign difference because of the conventions chosen, as can be seen by comparing the leading terms of Eqs. (245) with Eq. (I10). This setting of values is the application of the boundary condition set up by the extrinsic-energy and extrinsic-energy-flow. Only the radial component of (I11) is employed so far because there is no θ (polar angle) dependence in Eq. (I7). The polar component of (I11) is discussed next.

I.2.4. $\nabla\phi_{SUN}(r)$ of a Fluid Sun for $r > R_{SUN}$. A more realistic sun would include modeling of a plasma. Recall now Eq. (237), $\mathbf{F_G} = Q_G\nabla\phi_G$, and recall that Q_G is a quantity proportional to the extrinsic-energy of the body experiencing the force. If $\nabla\phi_G$ of Eq. (237) is determined by Eq. (I11) we see that there would be both an inward radial force as well as a polar force directed away from the poles. The radial force will be balanced by the outward plasma pressure (collisions will maintain the plasma density). In the absence of other effects (such as rotation) there will originally be no counterbalancing polar force. In this situation, the plasma particles will move away from the poles until a new hydrostatic equilibrium is obtained between the gravitational field and plasma pressure leading to an oblateness in the sun.

Note that the Lorentz contraction will also play a role in determining $\nabla\phi_{SUN}$. While $\nabla\phi_{SUN}$ is a disturbance in the fixed aether, the sun itself, which is the source for ϕ_{SUN}, is moving through the aether. And the sun is subject to the electromagnetic forces that determine its size, forces that obey the Lorentz Transformation. Therefore, the sun will become slightly oblate (as observed from the aetherial-rest-frame) in its direction through the aether. To leading order in v/c (which is $v^2/2c^2$) this Lorentz contraction oblateness is on the order of $7x10^{-7}$.

The sun's rotation also affects its oblateness. The measured value of the oblateness is a complicated topic and it involves an interplay between theory and experiment [I3].

For our model, we expect a gravitational oblateness of $\varepsilon = K_{F4}\rho_{FE}/4$ as given by Eq. (I3). We now recall Eqs. (385) and (386) for our two proposed possibilities for K_{F4}:

$$K_{F4KE} = (2.596x10^{-23}\ s^2/kg)\xi_0/K_{GC}K_c \tag{385}$$

$$K_{F4MO} = (2.596x10^{-23}\ s^2/kg)\xi_0/K_{GC}K_c c \tag{386}$$

As can be seen, Eqs. (385) and (386) involve our three free parameters ξ_0, K_{GC} and K_c. And so we conclude that the extrinsic-energy flow induced oblateness of a moving sun cannot rule out either the kinetic-energy or total-energy flow effects described in section D.16. (Although it will lead to constraints on the values of our three free parameters.) And we recall from section D.16 that other speculations for K_{F4} are possible as well.

I.3. Energy Flow Effect on the Advance of the Perihelions.

I.3.1. Evaluating $\nabla\phi_{CM}Q_{GPL}$ for the perihelion analysis. Eq. (331) introduces the energy-flow effect that can affect the advance of the perihelions. And also, the planetary and sun motion through the aether should now be included in the perihelion calculations. With the sun and the sun's secondary masses being defined as the central mass M_{CM}, Eq. (247) yields $\nabla\phi_{CM} = -(\rho_{GCM}R_{CM}^3/3r^2\varepsilon_0)\hat{\mathbf{r}}$ for $r > R_{CM}$. With Eq. (193), $\rho_G = 3[K_{G1}-K_{G2}]\rho_E\rho_0/2$, this becomes $\nabla\phi_{CM} = -([K_{G1}-K_{G2}]\rho_{ECM}\rho_0 R_{CM}^3/2r^2\varepsilon_0)\hat{\mathbf{r}}$. The extrinsic-energy of the central mass will be $E_{CM} = (4/3)\pi R_{CM}^3\rho_{ECM}$, or $R_{CM}^3\rho_{ECM} = 3E_{CM}/4\pi$, resulting in

$$\nabla\phi_{CM} = -(3[K_{G1}-K_{G2}]\rho_0 E_{CM}/8\pi r^2\varepsilon_0)\hat{\mathbf{r}} \tag{I13}$$

The central mass includes that of the sun, M_{SUN}, as well as the masses found in Eqs. (268) and (271), which are, respectively, $\Delta M_{1OUT} = 4K_{G5}\gamma_M Mr - 3K_{G5}\gamma_M MR$ and $\Delta M_{2OUT} = 5K_{G6}(\gamma_M M)^2/r - 6K_{G6}(\gamma_M M)^2/R$. Recalling Eq. (274), $M_{EFF} = \gamma_M M - 3K_{G5}\gamma_M MR - 6K_{G6}\gamma_M^2 M^2/R$, we can now express the central mass extrinsic-energy as $E_{CM} = M_{EFF}c^2 + 4K_{G5}\gamma_M Mc^2 r + 5K_{G6}\gamma_M^2 M^2 c^2/r$, leaving

$$\nabla\phi_{CM} = -(3[K_{G1}-K_{G2}]\rho_0/8\pi\varepsilon_0)(4K_{G5}\gamma_M Mc^2/r + M_{EFF}c^2/r^2 + 5K_{G6}\gamma_M^2 M^2 c^2/r^3)\hat{\mathbf{r}} \qquad \text{(I14)}$$

Next, the planet will be treated as a moving sphere of radius R_{PL} and mass M_{PL}. In this modeling, recalling Eq. (236), $Q_G = K_{GC}V_{sphere}\rho_G$, $Q_{GPL} = K_{GC}(4/3)\pi R_{PL}^3\rho_{GPL}$ and with Eq. (193), $\rho_G = 3[K_{G1}-K_{G2}]\rho_E\rho_0/2$, $Q_{GPL} = 2\pi K_{GC}R_{PL}^3[K_{G1}-K_{G2}]\rho_{EPL}\rho_0$. And with $\rho_{EPL} = \gamma_{PL}M_{PL}c^2/[(4/3)\pi R_{PL}^3]$,

$$Q_{GPL} = 3K_{GC}[K_{G1}-K_{G2}]\gamma_{PL}M_{PL}c^2\rho_0/2 \qquad \text{(I15)}$$

Eqs. (I14) and (I15) lead to $\nabla\phi_{CM}Q_{GPL} = -(9K_{GC}[K_{G1}-K_{G2}]^2\rho_0^2\gamma_{PL}M_{PL}c^2/16\pi\varepsilon_0)(4K_{G5}\gamma_M Mc^2/r + M_{EFF}c^2/r^2 + 5K_{G6}\gamma_M^2 M^2 c^2/r^3)\hat{\mathbf{r}}$. Recalling Eq. (251), $G_N = 9K_{GC}[K_{G1}-K_{G2}]^2\rho_0^2 c^4/16\pi\varepsilon_0$, we obtain

$$\nabla\phi_{CM}Q_{GPL} = -G_N\gamma_{PL}M_{PL}(4K_{G5}\gamma_M M/r + M_{EFF}/r^2 + 5K_{G6}\gamma_M^2 M^2/r^3)\hat{\mathbf{r}} \qquad \text{(I16)}$$

I.3.2. Perihelion Advance Including the Kinetic-Energy-Flow Effects. Consider again Eq. (331), $\mathbf{F}_{\mathbf{sphereGF}} = Q_G[(1+C_F/C_M)\nabla\phi_G - (C_F/C_M)(\hat{\mathbf{v}}\cdot\nabla\phi_G)\hat{\mathbf{v}}]$. Using Eq. (330), $C_F/C_M = K_{F4}\rho_{FE}/[K_{G1}-K_{G2}]\rho_E$, and Eq. (339), $K_{F4KE} = [K_{G1}-K_{G2}]$, we get $C_F/C_M = \rho_{FEKE}/\rho_E$. The planetary kinetic energy density ρ_{PLKE} is the kinetic energy divided by the volume, $\rho_{PLKE} = (\gamma_{PL} - 1)M_{PL}c^2/V$, while the extrinsic-energy density ρ_E is $\gamma_{PL}M_{PL}c^2/V$. Hence $C_F/C_M = \rho_{FEKE}/\rho_E = (1-1/\gamma_{PL})$ and this allows Eq. (331) to be expressed as $\mathbf{F}_{\mathbf{PL\_SUN\_KE}} = Q_G[(1+1-1/\gamma_{PL})\nabla\phi_G-(1-1/\gamma_{PL})(\hat{\mathbf{v}}\cdot\nabla\phi_G)\hat{\mathbf{v}}]$ or, since here we have $Q_G = Q_{GPL}$ and $\nabla\phi_G = \nabla\phi_{CM}$

$$\mathbf{F}_{\mathbf{PL\_SUN\_KE}} = (2 - 1/\gamma_{PL})Q_{GPL}\nabla\phi_{CM} - (1 - 1/\gamma_{PL})(\hat{\mathbf{v}}\cdot Q_{GPL}\nabla\phi_{CM})\hat{\mathbf{v}} \qquad \text{(I17)}$$

Now we can substitute (I16) into (I17) to get the equation for the force on a planet including the sun and planetary extrinsic-energies and extrinsic-energy flows

$\mathbf{F_{PL\_SUN\_KE}}$ =

$$-G_N\gamma_{PL}M_{PL}(4K_{G5}\gamma_M M/r+M_{EFF}/r^2+5K_{G6}\gamma_M{}^2M^2/r^3)[(2-1/\gamma_{PL})\hat{\mathbf{r}}-(1-1/\gamma_{PL})(\hat{\mathbf{v}}\cdot\hat{\mathbf{r}})\hat{\mathbf{v}}] \qquad \text{(I18)}$$

I.3.3. Third Numerical Integration Approach for Evaluating the Perihelion Advance. (Including Sun and Planetary Kinetic-Energy-Flow Effects.) Eq. (I18) can be used to calculate the advance of the perihelions. In our earlier numerical integrations of section D.13, the force in the x (y) direction is found by $F_X = [x/r]F$ ($F_Y = [y/r]F$) where F is given by Eq. (275) as $\mathbf{F_{GOUT}} = -G_N\gamma_m m(M_{EFF}/r^2 + 4K_{G5}\gamma_M M/r + 5K_{G6}\gamma_M{}^2M^2/r^3)\hat{\mathbf{r}}$. Eq. (I18) complicates things, as both the magnitude and the direction of the force changes. And, since the sun is now also moving through the aether, additional complications arise.

The third numerical integration program uses x and y as the plane of the ecliptic, and we assume the sun's velocity through the aether to be in the y direction for our work here, as we merely want an estimate. The motion of the sun and planets was integrated from the aether frame. Evaluating Eq. (274), $M_{EFF} = \gamma_M M - 3K_{G5}\gamma_M MR - 6K_{G6}\gamma_M{}^2M^2/R$, requires evaluation of $3K_{G5}\gamma_M MR$ and $6K_{G6}\gamma_M{}^2M^2/R$. From Eq. (276). $K_{G5} \approx 10^{-21}$ m^{-1} and with $R_{SUN} = 6.9634\text{x}10^8$ m, $3K_{G5}\gamma_M M_{SUN}R/M_{SUN} = 3K_{G5}R_{SUN} \approx 2.089\text{x}10^{-12}$. With Eq. (293) giving $K_{G6} = 8.9167\text{x}10^{-28}$ m/kg, and with $R_{SUN} = 6.9634\text{x}10^8$ m and $M_{SUN} = 1.9885\text{x}10^{30}$ kg, $6K_{G6}M_{SUN}/R_{SUN} = 1.5668\text{x}10^{-5}$. Hence, we can set $M_{EFF} = \gamma_M M_{SUN} = M_{SUN}$ to within the certainty of the sun's mass.

The results of the third numerical integration agree with the first and second numerical integrations through the fourth decimal place. With such a small difference, effect of the sun moving through the aether at $v/c \approx 1.23\text{x}10^{-3}$ is not presently detectable from data of the perihelion advances. (See section I.3.6 for comments on the calculation.)

I.3.4. Perihelion Advance Including the Total-Energy-Flow Effect. Again consider Eq. (331), $\mathbf{F_{sphereGF}} = Q_G[(1+C_F/C_M)\nabla\phi_G - (C_F/C_M)(\hat{\mathbf{v}} \cdot \nabla\phi_G)\hat{\mathbf{v}}]$. Again using Eq. (330), $C_F/C_M = K_{F4}\rho_{FE}/[K_{G1}-K_{G2}]\rho_E$, and now using Eq. (341), $K_{F4MO} = [K_{G1}-K_{G2}]/c$, we get $C_F/C_M = \rho_{FEMO}/c\rho_E$. The planetary total energy density ρ_{PLMO} flow is the total energy multiplied by the velocity divided by the volume, $\rho_{PLMO} = v\gamma_{PL}M_{PL}c^2/V$, while the extrinsic-energy density ρ_E is $\gamma_{PL}M_{PL}c^2/V$. Hence $C_F/C_M = \rho_{FEMO}/c\rho_E = v/c$ this allows Eq. (331) to be expressed as $\mathbf{F_{PL\_SUN\_MO}} = Q_G[(1+v/c)\nabla\phi_G - (v/c)(\hat{\mathbf{v}} \cdot \nabla\phi_G)\hat{\mathbf{v}}]$ or, since here we have $Q_G = Q_{GPL}$ and $\nabla\phi_G = \nabla\phi_{CM}$

$$\mathbf{F_{PL\_SUN\_MO}} = (1+v/c)Q_{GPL}\nabla\phi_{CM} - (v/c)(\hat{\mathbf{v}} \cdot Q_{GPL}\nabla\phi_{CM})\hat{\mathbf{v}} \tag{I19}$$

Now we can substitute (I16) into (I19) to get the equation for the force on a planet including the sun and planetary extrinsic-energies and extrinsic-energy flows

$$\mathbf{F_{PL\_SUN\_MO}} = -G_N\gamma_{PL}M_{PL}(4K_{G5}\gamma_M M/r + M_{EFF}/r^2 + 5K_{G6}\gamma_M^2M^2/r^3)[(1+v/c)\hat{\mathbf{r}} - (v/c)(\hat{\mathbf{v}} \cdot \hat{\mathbf{r}})\hat{\mathbf{v}}] \tag{I20}$$

I.3.5. Fourth Numerical Integration Approach for Evaluating the Perihelion Advance. (Including Sun and Planetary Total-Energy-Flow Effects.) Eq. (I20) can be used to calculate the advance of the perihelions. The fourth numerical integration program proceeds similarly to the third (see section I.3.3) only changing the factor of C_F/C_M to use a total energy flow assumption rather than a kinetic energy flow assumption. The results of the fourth numerical integration were not reliable, as convergence was not obtained when the integration step size was decreased.

I.3.6. Need for Improved Numerical Integration Evaluation of the Perihelion Advance. The numerical evaluations described in sections D.13.3, D.13.4, I.3.3 and I.3.5 are all quite rudimentary and in need of improvement. They were performed on a simple PC, using java BigDecimals. A series of runs were performed by decreasing the step size used in the numeric integration, and for the evaluations described in section D.13.3 the results converged nicely as the number of steps was increased through the orders of 100,000 to 1 million to 10 million to 100 million. Convergence of the results described in section D.13.4 nicely converged up to the order of 10 million steps, but showed degradation in improvement when increasing to the order of 100 million steps. The results described in section I.3.3 showed improvement degradation at the order of 10 million steps, and the results described in section I.3.5 showed little evidence of conversion.

Also, only the two-body problem was investigated, and it was done under very simple assumptions. The study of the ephemerides can become quite detailed, including thousands of objects, which are not purely spherical. Such studies can use computational resources and codes far superior to the admittedly rudimentary treatments described herein. A full calculation of the perihelion advance is outside the scope of this presentation.

I.4. References for Appendix I.

[I1] https://arxiv.org/abs/1807.06205

[I2] https://farside.ph.utexas.edu/teaching/celestial/Celestial/node19.html.

[I3] J. P. Rozelot and C. Damiani, Eur. Phys. J. H, **36**, 407-436 (2011).

Appendix J – Dense Stellar Objects

The celestial bodies discussed in the main text, such as the sun and planets, had densities that led to field-masses that were small compared to the physical mass of the object. Furthermore, in the main text we were concerned with interactions between celestial bodies and not their internal make up. (Since our concern was the forces between bodies we were free to approximate that the objects were constant density spheres, since the internal mass distribution did not affect the physics of interest.) However, we found in section E.2 that neutron star parameters led to conditions where our treatment broke down. In this Appendix J we will investigate situations where the second-field-mass can be a significant fraction of the physical mass, and we will also evaluate more realistic internal models of dense stellar objects.

As we look at different stellar objects within our universe, we find different ways that gravitational collapse is prevented. In stars like our sun, internal gravity is resisted by pressure gradients from ongoing nuclear fusion. In white dwarfs, the resisting pressure is supplied by electron degeneracy, and in neutron stars by neutron degeneracy. But in the theory proposed herein, we see that a new term must be included in our analysis, as we need to include the effects of the negative-field-mass. (Note that in what follows we will often use the term negative-field-mass for the second-field-mass, since it is a negative mass. This is in contrast to the first-field-mass which is a positive mass commonly known as dark matter.)

J.1. A Particles-in-a-Box Approach for Analyzing Dense Stellar Objects. Here we will develop a simple particles-in-a-box quantum mechanical model for dense stellar objects. Our first aim is to achieve a model that is in reasonable agreement with standard models of white dwarfs when negative-field-mass is ignored. From there we will be able to introduce and evaluate the negative-field-mass effects on white dwarfs, and once that is done we will extend our analysis to objects of even higher density. To keep things simple, some less important physical realities will be set aside for future study.

Our model will consider a cubic box as an analytical cell, and we will consider the dense stellar objects to be comprised of a large number of such cells. This model has a simplistic advantage over the completely degenerate and quantum-relativistic-thermodynamic approach of Chandrasekhar. Our simpler approach will aid our initial understanding of the problem, but another advantage is that it is a more realistic modeling. We postulate that upon a momentum exchange a quantum wave-function will collapse to a region whose size is $\hbar/2$ divided by the momentum exchanged. Such collapses happening within the dense stellar objects will essentially form boundaries for individual particles and we model these boundaries as the boundaries of our boxes. Rather than states occupying the entire object in a fully degenerate way, we instead will have states of significantly smaller physical size, each modelled as a particle-in-a-box. A further attribute of this treatment is that we can model heat within the system. While the thermal energies within dense stellar objects are often small enough that the large majority of lowest energy states are occupied, the thermal temperature is still far from zero. And, as seen below, smaller bounding boxes will lead to higher energies as compared to larger boxes, and hence this model could be used to model temperature within the system, although we will not complete those details here.

Our aim is to model dense stellar objects at their most elemental level, and since white dwarf stars are in a substantially electron degenerate state, and neutron stars are in a substantially neutron degenerate state, we have a situation wherein the elemental constituents of both are fermions. The most elemental level of each is thus a single fermion, and we can have two fermions in nearly identical states, each with a spin opposite of the other. Thus we will consider a model that builds up from a representative cubic box that contains two fermions, and a collection of those boxes will provide the degenerate pressure which supports the system against gravitational collapse. This approach requires definition of a representative analytical cell, and this is done below.

J.2. Lorentzian Solutions for Particles-in-a-Box. In an earlier work[J1] we have derived the following equation for particles wherein their energy becomes appreciable with respect to their rest-mass energy:

$$-\hbar^2 c^2 \nabla^2 \Psi = [E_n^2 - 2E_n V + V^2 - m^2 c^4]\Psi. \quad \text{(J1)}$$

(Reference [J1] uses the term "high velocity" instead of the term "relativistic" because we are now considering high energy particles within an absolute theory. Here we will use "Lorentzian" for such conditions.) In Eq. (J1) E_n is the energy of the n^{th} state, V is the potential energy of the system being analyzed, and mc^2 is the rest mass energy of the particle.

For the x dimension of particles-in-a-box, V equals zero between x=0 and x=ξ and V is infinite otherwise. With a similar condition in the other two dimensions Eq. (J1) becomes

$-\hbar^2c^2\nabla^2\Psi = [E_n^2 - m^2c^4]\Psi$ (for regions inside a cubic square well potential) (J2)

A solution to Eq. (J2) is

$\Psi = \sin(n_x\pi x/\xi)\sin(n_y\pi y/\xi)\sin(n_z\pi z/\xi)$ (J3)

Substituting Eq. (J3) into Eq. (J2) we obtain $(n_x^2 + n_y^2 + n_z^2)\hbar^2c^2\pi^2/\xi^2 = [E_n^2 - m^2c^4]$, or

$E_{nSO} = [(n_x^2+n_y^2+n_z^2)\hbar^2c^2\pi^2/\xi^2 + m^2c^4]^{1/2}$ (J4)

In Eq. (J4) the subscript n is now associated with the n_x, n_y, and n_z of the solution given in Eq. (J3); the subscript S indicates we have a box holding a single fermion, and the O subscript indicates we are dealing with one spin state. (Note that for $m^2c^4 >> \hbar^2c^2\pi^2/\xi^2$ we get $E_{nSO} = mc^2[(n_x^2+n_y^2+n_z^2)\hbar^2c^2\pi^2/\xi^2m^2c^4 + 1]^{1/2} \sim= mc^2 + (n_x^2+n_y^2+n_z^2)\hbar^2c^2\pi^2/2mc^2\xi^2 = mc^2 + (n_x^2+n_y^2+n_z^2)\hbar^2\pi^2/2m\xi^2$, which is correct in the low velocity limit. This agrees with Eq. (4), $E_{Q\xi} = K_{Q\xi 0}/\xi_Q^2$, if we set $K_{Q\xi 0} = (n_x^2+n_y^2+n_z^2)\hbar^2\pi^2/2m$ and recognize that Eq. (4) has $\xi = \xi_Q$, and that we now have three dimensions, and that Eq. (4) does not explicitly mention the mc^2 term.)

The simple model of single particles each in an individual box is not a very good approximation for substantially degenerate situations. In the fully degenerate case, the box becomes very large in size (it becomes the size of the whole star). And larger boxes will lead to lower average energies for the fermions within them than will a collection of small boxes of the same total volume. To see this, consider eight boxes wherein we store 16 fermions, and where we'll define ξ_C as the length of each side of the eight individual boxes. The total ground state energy for eight individual boxes will be $E = 16[3\hbar^2c^2\pi^2/\xi_C^2 + m^2c^4]^{1/2}$, since for each fermion we will have $n_x = n_y = n_z = 1$ (the 3) and we will have two spin states within each of our 8 boxes (the 16). But now consider the situation where we put all of our fermions into a single box with each side having a length $2\xi_C$. (This larger box has the same volume as the total volume of our original eight boxes.) For the larger box the lowest energy will have 2 fermions in the $n_x = n_y = n_z = 1$ state, each with an energy of $[3\hbar^2c^2\pi^2/4\xi_C^2 + m^2c^4]^{1/2}$. Six fermions will be in a state where one of the n values is 2 and the others 1, each with an energy of $[6\hbar^2c^2\pi^2/4\xi_C^2 + m^2c^4]^{1/2}$. Six fermions will be in a state where one of the n values is 1 and the others 2, each with an energy of $[9\hbar^2c^2\pi^2/4\xi_C^2 + m^2c^4]^{1/2}$. And two fermions will be in a state where one of the n values is 3 and the others 1, each with an energy of $[11\hbar^2c^2\pi^2/4\xi_C^2 + m^2c^4]^{1/2}$. Hence, the total energy in our larger box is $2[3\hbar^2c^2\pi^2/4\xi_C^2 + m^2c^4]^{1/2} + 6[6\hbar^2c^2\pi^2/4\xi_C^2 + m^2c^4]^{1/2} + 6[9\hbar^2c^2\pi^2/4\xi_C^2 + m^2c^4]^{1/2} + 2[11\hbar^2c^2\pi^2/4\xi_C^2 + m^2c^4]^{1/2}$. It can be seen that the total energy of the larger box is considerably smaller than the total energy of the 8 smaller boxes.

In the general case, the larger the box, the lower the energy will be as compared to a collection of individual small boxes of the same volume. For a box that has each edge equal to $N\xi_C$, where N is some integer value, the energy of any specific state is given by Eq. (J4) with $\xi = N\xi_C$ and the total energy within the box will be the sum of the energies of the individual states:

$$E_{nMO} = \Sigma_X \Sigma_Y \Sigma_Z [(n_x^2+n_y^2+n_z^2)\hbar^2c^2\pi^2/N^2\xi_C^2 + m^2c^4]^{1/2} \qquad \text{(J5)}$$

In Eq. (J5) the M subscript indicates that we now have multiple fermions in our bigger box, and the O subscript indicates we are still only considering one spin state. In Eq. (J5) n_x, n_y, and n_z can take on values from 1 to an arbitrarily high number. To find the ground state energy within the box, we have written a computer code (named QuantumStates1) to iterate through all possible values of n_x, n_y, and n_z up to an upper limit where each of n_x, n_y, and n_z are allowed to go to 1.8 times N, and then we sort by energy and keep the smallest N^3 energies. (QuantumStates1 finds the lowest total energy of a box containing N^3 particles, considering only a single spin state.) The multiplier limit of 1.8 arises because the state $n_x = 1.7N$, $n_y = 1$, $n_z = 1$ has an energy proportional to $2.89N^2 + 2$, while the state $n_x = N$, $n_y = N$, $n_z = N$ has an energy proportional to $3N^2$. Now $2.89N^2 + 2$ will be less than $3N^2$ for N >= 5. Hence, for N >= 5, we will achieve a lower energy if we fill the state $n_x = 1.7N$, $n_y = 1$, $n_z = 1$ than if we fill the state $n_x = N$, $n_y = N$, $n_z = N$, so we must allow for such possibilities when looking for the ground state. However, the state $n_x = 1.8N$, $n_y = 1$, $n_z = 1$ has an energy proportional to $3.24N^2 + 2$, which is always greater than $3N^2$, so we need not consider states with n_x, n_y or n_z >= 1.8N when looking for the energy of the ground state.

In QuantumStates1 we assume the kinetic energy is small compared to the rest mass energy. In this limit Eq. (J4) is $E_{nSO} = mc^2 + (n_x^2+n_y^2+n_z^2)\hbar^2c^2\pi^2/2mc^2\xi^2$. The kinetic energy is therefore $(n_x^2+n_y^2+n_z^2)\hbar^2c^2\pi^2/2mc^2\xi^2$. For an individual cell we have $\xi = \xi_C$ and for the big box we have $\xi = N\xi_C$. The ratio of the total kinetic energy in the big box to the kinetic energy of N^3 individual cells is thus $\Sigma_X \Sigma_Y \Sigma_Z (n_x^2+n_y^2+n_z^2)\hbar^2c^2\pi^2/2mc^2(N\xi_C)^2$ (the large box total kinetic energy) divided by $N^3 3\hbar^2c^2\pi^2/2mc^2\xi_C^2$ (the kinetic energy of N^3 individual cells). This gives $f_{QBTL} = [\Sigma_X \Sigma_Y \Sigma_Z (n_x^2+n_y^2+n_z^2)\hbar^2c^2\pi^2/2mc^2(N\xi_C)^2]/[N^3 3\hbar^2c^2\pi^2/2mc^2\xi_C^2]$, or,

$$f_{QBTL} = \Sigma_X \Sigma_Y \Sigma_Z (n_x^2+n_y^2+n_z^2)/3N^5 \qquad \text{(J6)}$$

In Eq. (J6), f_{QBTL} introduces the quantum-box total kinetic energy reduction factor in the low energy limit, and the sum is over all of the squared quantum numbers n_x^2, n_y^2, and n_z^2 for the lowest energy values contained in the big box.

The maximum kinetic energy divided by the ground state individual cell kinetic energy is $[(n_{xM}^2+n_{yM}^2+n_{zM}^2)\hbar^2c^2\pi^2/2mc^2(N\xi_C)^2]/[3\hbar^2c^2\pi^2/2mc^2\xi_C^2] = (n_{xM}^2+n_{yM}^2+n_{zM}^2)/3N^2$, or

$$f_{QBML} = (n_{xM}^2+n_{yM}^2+n_{zM}^2)/3N^2 \qquad \text{(J7)}$$

In Eq. (J7), f_{QBML} introduces the quantum-box maximum kinetic energy reduction factor in the low energy limit, and n_{xM}^2, n_{yM}^2, and n_{zM}^2 correspond to the values of the fermion with the highest energy within the ground state energy of the big box. (We fill all of the lowest energy states of the big box to find the ground state. The maximum kinetic energy of those lowest energy states is what we call the max kinetic energy.)

The expressions in Eqs. (J6) and (J7) are extremely simple to evaluate numerically, and some results from QuantumStates1 are shown in Table J1. It is readily apparent from Table J1 that putting all of the fermions in one big box results in a lower energy than if we confine them into individual small boxes within the same volume, and that the maximum energy of a fermion in the big box is less than that of the individually confined fermions.

Table J1 – Results of Numerical Evaluations of Various Large Box Sizes for Low Energies

N	N^3	Total Kinetic Energy/Σ Ground State Individual Cell Kinetic Energy, f_{QBTL}	Max Kinetic Energy / Ground State Individual Cell Kinetic Energy, f_{QBML}
1	1	1.0	1.0
2	8	0.7292	0.9167
10	1000	0.3575	0.5767
50	125000	0.3173	0.5256
250	15625000	0.3097	0.5156

When the kinetic energy is no longer small compared to the rest mass energy, additional complexity is involved. In that more general case the kinetic energy ratio is

$f_{QBT} =$ (J8)

$$\Sigma_X \Sigma_Y \Sigma_Z \{[(n_x^2+n_y^2+n_z^2)\hbar^2c^2\pi^2/(N\xi_C)^2+m^2c^4]^{1/2} - mc^2\}/N^3\{[3\hbar^2c^2\pi^2/\xi_C^2+m^2c^4]^{1/2} - mc^2\}$$

In Eq. (J8) f_{QBT} is the quantum-box total kinetic energy reduction factor in the general case. Continuing with the general case, the maximum ground state kinetic energy ratio is

$$f_{QBM} = \{[(n_{xM}^2+n_{yM}^2+n_{zM}^2)\hbar^2c^2\pi^2/(N\xi_C)^2+m^2c^4]^{1/2} - mc^2\}/\{[3\hbar^2c^2\pi^2/\xi_C^2 + m^2c^4]^{1/2} - mc^2\} \quad (J9)$$

In Eq. (J9) f_{QBM} is the quantum-box maximum kinetic energy reduction factor in the general case. The more general Eqs. (J8) and (J9) are still straightforward to evaluate numerically, although they are a bit more complex and they involve variable inputs for the cell size ξ_C and for the rest mass of the species being considered. The numerical evaluation is done by code QuantumStates2.

J.3. Defining A Representative Quantum Analytical Cell. As stated above in section J.1, our aim is to achieve a model that builds up from a cubic box containing two fermionic particles, and yet as just described there will be a large number of different quantum number sets for the fermions within dense stellar objects. Therefore to achieve our goal we will define a representative quantum analytical cell to contain one pair of fermions. We will assume that the size of our cell will be ξ_C in all three dimensions, and that the size of an arbitrarily larger box will have a size of $N\xi_C$ in all three dimensions. (The larger box will contain N^3 cells. The larger box will have its size determined by the quantum collapse caused by thermal collisions and it is one of many physical cubes making up the star. The smaller box is our representative cube containing just two fermions that we use for our numerical integrations.) We will define a representative quantum analytical cell such that the quantum-pressure at the walls of the representative cell is the same as the quantum-pressure on the walls of the larger box.

Our analysis-cubes will have an outward force resulting from the quantum-pressure. To find the pressure we form the quantity $dE_{nSO}/d\xi$, where $d\xi$ is a virtual increase of the cube dimensions during a virtual expansion of the cube. Recalling Eq. (J4), $E_{nSO} = [(n_x^2+n_y^2+n_z^2)\hbar^2c^2\pi^2/\xi^2 + m^2c^4]^{1/2}$ we now calculate $dE_{nSO}/d\xi$

$$dE_{nSO}/d\xi = -[(n_x^2+n_y^2+n_z^2)\hbar^2c^2\pi^2/\xi^2 + m^2c^4]^{-1/2}[(n_x^2+n_y^2+n_z^2)\hbar^2c^2\pi^2/\xi^3] \quad \text{(J10)}$$

Eq. (J10) expresses the change of energy of the entire cube divided by a small change in ξ.

To further clarify, consider the ground state energy of a cube when it is expanded by $d\xi$. In that case $E_{nSO}(\xi+d\xi) = [3\hbar^2c^2\pi^2/(\xi+d\xi)^2 + m^2c^4]^{1/2} \sim= [3\hbar^2c^2\pi^2/(\xi^2+2\xi d\xi) + m^2c^4]^{1/2} = [3\hbar^2c^2\pi^2/\xi^2(1+2d\xi/\xi) + m^2c^4]^{1/2} \sim= [(1-2d\xi/\xi)3\hbar^2c^2\pi^2/\xi^2 + m^2c^4]^{1/2} = [3\hbar^2c^2\pi^2/\xi^2 + m^2c^4 -$

$2(d\xi/\xi)3\hbar^2c^2\pi^2/\xi^2]^{1/2} = [3\hbar^2c^2\pi^2/\xi^2 + m^2c^4]^{1/2}[1 - \{2(d\xi/\xi)3\hbar^2c^2\pi^2/\xi^2\}/\{3\hbar^2c^2\pi^2/\xi^2 + m^2c^4\}]^{1/2} \sim= [3\hbar^2c^2\pi^2/\xi^2 + m^2c^4]^{1/2}[1 - \{(d\xi/\xi)3\hbar^2c^2\pi^2/\xi^2\}/\{3\hbar^2c^2\pi^2/\xi^2 + m^2c^4\}]$. And so we find that $dE_{nSO} = E_{nSO}(\xi+d\xi) - E_{nSO}(\xi) = -(d\xi/\xi)3\hbar^2c^2\pi^2/\xi^2\}/\{3\hbar^2c^2\pi^2/\xi^2 + m^2c^4\}^{1/2} = -d\xi[3\hbar^2c^2\pi^2/\xi^3]/[3\hbar^2c^2\pi^2/\xi^2 + m^2c^4]^{1/2}$, which, when rearranged, is the same as Eq. (J10) for the ground state.

As described near Eqs. (4) and (5), the magnitude of the force on each face of the cube will be

$F_{FACE} = (1/3)dE_{nSO}/d\xi$

$$F_{FACE} = [(n_x^2+n_y^2+n_z^2)\hbar^2c^2\pi^2/3\xi^3] / [(n_x^2+n_y^2+n_z^2)\hbar^2c^2\pi^2/\xi^2 + m^2c^4]^{1/2} \tag{J11}$$

(Note that for the ground state, where $n_x^2+n_y^2+n_z^2 = 3$, in the limit $3\hbar^2c^2\pi^2/\xi^2 << m^2c^4$ Eq. (J11) becomes $F_{FACE} = \hbar^2\pi^2/\xi^3m$. This agrees with Eq. (5), $F_{Q\xi} = dE_{Q\xi}/dX_Q = 2K_{Q\xi0}/\xi_Q^3$, if $K_{Q\xi0} = \hbar^2\pi^2/2m$. It also agrees with the $K_{Q\xi0}$ we calculated in the parenthetic note following Eq. (J4) in the case of a single dimension. The force specified in Eq. (5), like that in Eq. (J11), is the force on a single face.)

Since the pressure is the force divided by the area, the pressure on a single face is the force of Eq. (J11) divided by the area of a single face. And with the single face area equal to ξ^2, we obtain

$$P_{Q\xi S} = [(n_x^2+n_y^2+n_z^2)\hbar^2c^2\pi^2/3\xi^5] / [(n_x^2+n_y^2+n_z^2)\hbar^2c^2\pi^2/\xi^2 + m^2c^4]^{1/2} \tag{J12}$$

The subscript S in Eq. (J12) reminds us we are so far dealing with a single particle in a box. We must now include both of the two fermions in our analytical cell. The second fermion will contribute an amount of pressure equal to that of the first, leaving the pressure on the walls of our analytical cell as

$$P_{Q\xi} = 2[(n_x^2+n_y^2+n_z^2)\hbar^2c^2\pi^2/3\xi^5] / [(n_x^2+n_y^2+n_z^2)\hbar^2c^2\pi^2/\xi^2 + m^2c^4]^{1/2} \tag{J13}$$

Recalling Eq. (J4), $E_{nSO} = [(n_x^2+n_y^2+n_z^2)\hbar^2c^2\pi^2/\xi^2 + m^2c^4]^{1/2}$, Eq. (J13) becomes $P_{Q\xi} = 2[(n_x^2+n_y^2+n_z^2)\hbar^2c^2\pi^2/3\xi^5]/E_{nSO}$, which leads to

$$\xi = [2(n_x^2+n_y^2+n_z^2)\hbar^2c^2\pi^2 / 3P_{Q\xi}E_{nSO}]^{1/5} \tag{J14}$$

The pressure inside an arbitrary large cube with edge lengths of $N\xi_C$ is obtained by summing the individual pressures given by Eq. (J13) over all of the quantum states within that cube:

$$P_{QN\xi C} = \Sigma_X \Sigma_Y \Sigma_Z\, 2[(n_x^2+n_y^2+n_z^2)\hbar^2c^2\pi^2/3(N\xi_C)^5] / [(n_x^2+n_y^2+n_z^2)\hbar^2c^2\pi^2/(N\xi_C)^2 + m^2c^4]^{1/2} \quad \text{(J15)}$$

Evaluation of Eq. (J15) is done by a computational routine running through all values of n_x, n_y, and n_z between 1 and less than 1.8N and then selecting the lowest N^3 pressures and summing them. We will now define the representative cube as having all three edge lengths equal to ξ_C while also achieving the same pressure as that obtained from Eq. (J15). We do this by setting a factor f in the following expression:

$$P_{QN\xi C} = P_{REP} = 2f\hbar^2c^2\pi^2/3\xi_C^5 / [f\hbar^2c^2\pi^2/\xi_C^2 + m^2c^4]^{1/2} \sim= 2f\hbar^2c^2\pi^2/3mc^2\xi_C^5 \quad \text{(J16)}$$

In Eq. (J16) P_{REP} is the pressure of the representative cube and we replace $(n_x^2+n_y^2+n_z^2)$ by f and the final expression results because our representative cube will typically have $f\hbar^2c^2\pi^2/\xi_C^2 << m^2c^4$ for cases of interest. Solving (J16) for f

$$f = P_{QN\xi C}/(2\hbar^2c^2\pi^2/3mc^2\xi_C^5) = 3P_{QN\xi C}mc^2\xi_C^5/2\hbar^2c^2\pi^2 \quad \text{(J17)}$$

For our representative cube we obtain an expression for ξ_C by rearranging Eq. (J17):

$$\xi_C = [2f\hbar^2c^2\pi^2 / 3mc^2P_{QN\xi C}]^{1/5} \quad \text{(J18)}$$

f is an effective quantum number that gives a pressure with the same form as that given in Eq. (J13) while also giving the same value as the pressure from Eq. (J15). We will use f later in numerical integrations. Since f changes rather slowly during our integrations it allows us to speed up those integrations rather than evaluating Eq. (J15) at every step during the process. And notice that as long as f is changing slowly we can still use Eq. (J16) even when $f\hbar^2c^2\pi^2/\xi_C^2 << m^2c^4$ is no longer valid. This is because we are only using Eq. (J16) to get an expression for f which we use to avoid calls to a slow subroutine. (As long as f is varying slowly, when we call the subroutine later no significant error will have occurred in our calculation.)

J.4. White Dwarf Maximum Density. To analyze what happens within a white dwarf we will begin with a quantum cell containing two electrons, and add to it two protons and two neutrons. We will make the simplifying assumption that the white dwarf is comprised of carbon. (We could use any element, as our goal is just to find the mass of a two-electron, two-proton and two-neutron cell; carbon is our arbitrary choice.) A single carbon atom has a mass of 1.99×10^{-26} kg and the density of un-pressurized graphite is 2.26×10^{3} kg/m^3. The volume of a single carbon atom in the graphite state is thus 1.99×10^{-26} kg/2.26×10^{3} kg/m^3 = 8.805×10^{-30} m^3. For a cubic cell of material containing two electrons, two protons and two neutrons each edge of the cell will consist of 1/3rd of an atom, leading to an unpressurized length of

$$\xi_C = (8.8\times10^{-30}\ \mathrm{m}^3/3)^{1/3} = 1.432\times10^{-10}\ \mathrm{m}. \qquad \text{(J19)}$$

We will now calculate how much our cell needs to be crushed before it becomes energetically favorable for it to collapse into a pure neutron state. When the kinetic energy of the quantum state exceeds the rest-mass energy difference between that of one neutron (939.56 MeV) and that of a proton plus an electron (938.27 MeV+0.51 MeV), the proton and electron will collapse into a neutron. This occurs at the energy of

$$E_{COLLAPSE} = [939.56\ \mathrm{MeV} - (938.27\ \mathrm{MeV}+0.51\ \mathrm{MeV})] = 0.78\ \mathrm{MeV} \qquad \text{(J20)}$$

We now recall Eq. (J4), $E_{nS} = [(n_x^2+n_y^2+n_z^2)\hbar^2c^2\pi^2/\xi^2 + m^2c^4]^{1/2}$. The ground state ($n_x = n_y = n_z = 1$) kinetic energy of our cell of size ξ is thus $E_{CG} = [3\hbar^2c^2\pi^2/\xi^2 + m^2c^4]^{1/2} - mc^2$, where the subscripts are C for our cell and G for the ground state. We know from section J.2 that the maximum energy within a larger cube with edges equal to $N\xi$ will be reduced from E_{CG} by a factor of f_{QBM}, and it is this largest energy electron that will have the most energy, leading to the collapse. Therefore we will need an energy of $E_{CG} = E_{COLLAPSE}/f_{QBM}$ to collapse our cell. So we set $E_{COLLAPSE}/f_{QBM} = 0.78$

$\text{MeV}/f_{QBM} = E_{CG} = [3\hbar^2c^2\pi^2/\xi^2 + m^2c^4]^{1/2} - mc^2$, or $[3\hbar^2c^2\pi^2/\xi^2 + m^2c^4] = (0.78\ \text{MeV}/f_{QBM} + mc^2)^2$, or $3\hbar^2c^2\pi^2/\xi^2 = (0.78\ \text{MeV}/f_{QBM} + mc^2)^2 - m^2c^4$, or

$$\xi_{COLLAPSE} = \{3\hbar^2c^2\pi^2/[(0.78\ \text{MeV}/f_{QBM} + mc^2)^2 - m^2c^4]\}^{1/2} \qquad \text{(J21)}$$

An issue with evaluating Eq. (J21) is that f_{QBM} is numerically determined from QuantumStates2 (see the end of section J.2 above) with ξ_C as one if the inputs to the numerical calculation. Fortunately we can guess a value of $\xi_{COLLAPSE}$, check for consistency, and then update and iterate until we obtain a $\xi_{COLLAPSE}$ that agrees with Eq. (J21). We then find $\xi_{COLLAPSE} = 6.455\text{x}10^{-13}$ m which leads to $f_{QBM} = 0.65$. (QuantumStates2 is run with N = 50, $\xi = 6.455\text{x}10^{-13}$ m and m = $9.11\text{x}10^{-31}$ kg.) With this we can now calculate the maximum white dwarf density

$$\rho_{WD0} = 1.99\text{x}10^{-26}\ \text{kg}\ /3(6.455\text{x}10^{-13}\ \text{m})^3 = 2.466\text{x}10^{10}\ \text{kg/m}^3 \qquad \text{(J22)}$$

(In Eq. (J22) ρ_{WD0} is an estimated maximum density obtained under several assumptions and omissions. We will return to this issue later in section J.7 below.)

J.5. Hydrostatic Equilibrium in a White Dwarf. To determine the density profile within white dwarf stars we will need to equate the force of the outward pressure differential to the inward gravitational force on an analytical-cube. Consider a cube with two of its faces perpendicular to **r**, where **r** is the vector from the star's center to the center of our analytical cube. The mass of the cube is $\rho A dr$ where ρ is the mass density inside of the cube, $A = dr^2$ is the area of the cube on a face perpendicular to r, and dr is the thickness of the cube in the r direction. The gravitational force will be the mass of the cube, $\rho A dr$, times the mass of the star inside of r, M_{IN}, times the gravitational constant divided by the distance squared, G/r^2. (For a spherically symmetric stellar mass, the attraction of a small mass to the star's center only depends on the stellar mass inside of the radial position r of the small mass because the net force from regions greater than r is zero.) The gravitational force will be directed inward. The pressure on the face nearest to the star's center can be defined as P, and the pressure on the face farthest from the star's center will be defined as P – dP. The force on any cube face due to pressure is the pressure times the area, so the total force on the cube from the pressure will be PA on the inner face minus (P–dP)A on the outer face, with a net force of dPA, directed outward. Equating the inward gravitational force to the outward pressure force, $dPA = GM_{IN}\rho Adr/r^2$, or

$$dP = GM_{IN}\rho dr/r^2. \tag{J23}$$

Eq. (J23) is the condition of hydrostatic equilibrium.

J.6. SDO1: A Computer Code to Evaluate Hydrostatic Equilibrium in a White Dwarf. Below we will make use of our representative analytical cube cell density:

$$\rho_{WD} = M_{CELL}/\xi_C^3 \tag{J24}$$

As mentioned in the section J.4 above, we consider a single cell that is 1/3rd of a Carbon atom, so

$$M_{CELL} = 1.99\text{x}10^{-26}\text{ kg}/3 = 6.63\text{x}10^{-27}\text{ kg} \tag{J25}$$

We can now write a simple numerical integration code SDO1 (for Super Dense Objects) to determine the mass and radius of a white dwarf. The code will also find the star density at each radius, once we set its central density.

First we need to write a method that solves for f as given in Eq. (J17). (See section J.3 above. Eqs. (J4) and (J15) are also used.) Next we set up the initial conditions at the central location of the star, by assuming a small starting sphere. We set: r = dr (we start with a small sphere of radius dr); $\rho_{WD} = \rho_{WD0}$ (we set the density of the white dwarf to some input density ρ_{WD0}); $M_{IN} = \rho_{WD}*(4/3)\pi r^3$ (M_{IN} is the mass inside of r); $\xi_C = (M_{CELL}/\rho_{WD})^{1/3}$ (see Eqs. (J24) and (J25); M_{CELL} is an input to the program; ξ_C is the edge length of our representative analytical cube); then we call the method to get f (since the method uses ξ_C for the calculation of f we find ξ_C first); and then we get the pressure on the cell $P_{REP} = 2f\hbar^2c^2\pi^2/3mc^2\xi_C^5$ from Eq. (J16).

Then we integrate through subsequent spherical shells. Within a loop we iteratively set: $P = P - GM_{IN}\rho dr/(r+dr/2)^2$ (we subtract the pressure difference specified in Eq. (J23), using the center of the spherical shell r+dr/2 in the denominator); $\xi_C = [2f\hbar^2c^2\pi^2 / 3mc^2P_{WD}]^{1/5}$ from Eq. (J18); $\rho_{WD}=M_{CELL}/\xi_C^3$ (Eq. (J24) is used to find the density); $M_{IN} = M_{IN} + \rho_{WD}(4/3)\pi[(r+dr)^3-r^3]$ (the mass inside the new value of r is what it was before plus the mass of this shell of mass); and r = r + dr (we prepare to step into the next shell of thickness dr). We update f whenever we've been through the loop COUNT times since the last update (COUNT is an input value to the program). We terminate the program when ξ_C is greater than one angstrom or when the pressure drops to zero.

J.7. Results and Accuracy of Computer Code SDO1. The computer code SDO1 was written to perform the numerical integrations identified in section J.6. Inputs to the code are: 1) ρ_{WD0}, the assumed central density of the stellar object; 2) M_{CELL}, the mass of the representative cell; 3) M_{DF}, the rest mass of the fermion providing the degenerate pressure; 4) DR, the radial step size for the integration; 5) COUNT, the number of times iterations are done before updating f; 6) N, the ratio of the big box edge length to the representative cell edge length; and 7) F_{EA}, an empirical adjustment factor. (F_{EA} will be discussed below.)

For the white dwarf analysis, M_{CELL} is 1/3rd of a carbon atom, 6.63×10^{-27} kg. In the case of a white dwarf, electrons provide the degenerate pressure, and hence $M_{DF} = 9.11\times10^{-31}$ kg. The remainder of the inputs were varied depending on what was being investigated.

The first set of runs of the code were done with an initial density corresponding to what we found in Eq. (J22), $\rho_{WD0} = 2.466\times10^{10}$ kg/m^3. Within that set of runs we set $F_{EA} = 1.0$ and the remaining parameters were varied to inspect convergence of the numerical integration. Table J2 presents some results.

Table J2 Results of SDO1 for White Dwarves, ρ_{WD0} = 2.466x10^{10} kg/m^3

ROW	DR	COUNT	N	Radius (m)	Mass (Ms)
1	25	1000	50	5962225	1.010
2	25	100	50	5970175	1.013
3	5	1000	50	5969425	1.012
4	25	1000	250	5899175	0.9849
5	25	1000	10	6279550	1.143

Row 1 in Table J2 is our representative run. Row 2 shows that making a more frequent call to find f (from every 1000 iterations to every 100) doesn't affect the outcome of the program much. Row 3 shows that slicing the star into 5-meter-thick spherical shells doesn't change the outcome much as compared to using 25-meter-thick spherical shells. And row 4 shows that confining the electrons to a box with edges 250 times that of the representative cell leads to a 2% reduction in mass as compared to a box with edges 50 times that of the representative cell. Lastly, Row 5 shows that confining the electrons to a box with edges 10 times that of the representative cell leads to a 13% increase in mass as compared to a box with edges 50 times that of the representative cell.

We needn't be very concerned about the change in our output due to changes in N, as that is to be expected. The smaller the bounding box for our particles-in-a-box, the higher their energy states will be. This is a feature of the present modeling. So everything seems to be quite stable from a convergence point of view.

However there is one aspect of concern in Table J2, and that is the predicted masses of the White Dwarfs, which are about 1.0 Ms. (Here in Appendix J, Ms is the mass of the sun.) Recall that Table J2 is using an initial density parameter that should be the threshold value for collapse into a neutron star, and this should be as large as White Dwarfs can get. Yet observations show what are believed to be White Dwarfs with masses up to 1.33 Ms, and the Chandrasekhar limit is roughly 1.4 Ms.

In order to address the possibility of larger mass White Dwarfs, we now reflect on the simplicity of our model and what we are leaving out. There are several factors that could lead to a larger mass. Thermal pressure and centrifugal forces will both add outward pressure. The coulomb attraction between electrons and protons will provide an additional potential energy we did not include when we calculated the density at which it becomes energetically favorable for collapse into neutrons. For our purposes we will now lump all of these effects into a single empirical adjustment factor F_{EA}, and we will apply that factor to the outward quantum pressure calculated in our program by setting $f = F_{EA}*f$ within SDO1.

While a more detailed estimation of the effects that go into F_{EA} is of interest, as long as F_{EA} does not deviate too far from unity, a bulk F_{EA} will be useful for us to keep things simple as we go on to investigate the effect of the negative-field-mass in the following sections. Table J3 presents results using various factors of F_{EA} while continuing to use an initial density corresponding to what we found in Eq. (J22), $\rho_{WD0} = 2.466\text{x}10^{10}$ kg/m^3. For Table J3, we also used the representative values of DR = 25, COUNT = 1000 and N = 50.

Table J3 Results of SDO1 for White Dwarves, $\rho_{WD0} = 2.466\text{x}10^{10}$ kg/m^3, Various F_{EA}'s

ROW	F_{EA}	Radius (m)	Mass (Ms)
1	1.0	5962225	1.010
2	1.07	6167675	1.118
3	1.15	6394425	1.246
4	1.20	6532150	1.328
5	1.25	6667025	1.412

As seen in Table J3, an empirical adjustment of 20% will lead to a mass that is as about as large as any observed white dwarf. Therefore the quantum (degenerate) pressure and gravity are the dominant effects, with other effects being secondary.

It is of interest to evaluate the contributors to F_{EA}, which we've identified as thermal pressure, rotational effects and Coulomb forces. Eq. (J20) informs us that $E_{COLLAPSE}$ = 780 keV. As an initial comment, the rotational energy of the atoms in the center of the star are negligible. A central temperature of 100,000 degrees Kelvin results in a thermal energy of around 8.6 eV, which is also negligible in comparison to $E_{COLLAPSE}$. This leaves us with the Coulomb effects, and the Coulomb force scales as Q_1Q_2/r^2, so the potential energy scales as Q_1Q_2/r where Q_1 is the charge on an electron and Q_2 is the charge on an ion. Hydrogen has a radius of about 5x10^{-11} m, and with ξ_C = 6.455x10^{-13} m, we see that our expected compaction of the radius will increase Coulomb energies by about a factor of 100. Hydrogenic atoms have ground state energies proportional to Q_2^2, and hydrogen has a ground state energy of 13.6 eV. (Here, Q_2 is the atomic number of the nucleus.) Heavier material will fall toward the core of the star, and the star's core may be made of iron with Q_2 = 26. Squeezing the radius down by a factor of 100 and multiplying that by 26^2 leads to an energy of (13.6 eV)(100) 26^2 = 919 keV for a squeezed single electron orbiting an iron nucleus. The other electrons of the iron atom will screen the iron nucleus charge, significantly reducing this energy, and hence we see that qualitatively an F_{EA} of 1.2 is plausible. While the empirically determined F_{EA} of 1.2 is entirely sufficient for our future purposes, our qualitative analysis also leads to certain conclusions: 1) Coulomb effects dominate F_{EA}; 2) an F_{EA} of 1 should be applied to neutron stars where there are no Coulomb effects; and 3) the inner core composition of white dwarfs is a critical factor when determining when they will become a supernova.

A final check of our initial model is to calculate how the size and mass of white dwarfs scale with each other. Table J4 shows the results of SDO1 for various central densities. For all of the calculations, we use the values of DR = 25, COUNT = 1000, N = 50 and F_{EA} = 1.2. For the prevalent mass region of 0.6Ms to 0.8Ms we see that the quantity Radius*Mass$^{1/3}$ is approximately constant, and even the result at 0.42 Ms is not too far off from that scaling law. For the case near the point of collapse at 1.328 Ms deviation from the scaling value Radius*Mass$^{1/3}$ appears.

Table J4 Results of SDO1 for White Dwarves, Various Values of ρ_{WD0}

ROW	ρ_{WD0} (kg/m^3)	Radius (m)	Mass (Ms)	Radius*Mass$^{1/3}$
1	2.466x10^{10}	6532150	1.328	7,179,962
2	3x10^9	9955300	0.8052	9,261,662
3	2x10^9	10740000	0.7063	9,564,612
4	1.25x10^9	11707050	0.5987	9,866,972
5	5x10^8	13787350	0.4184	10,312,042

As can be seen, our simple particles-in-a-box model gets us a quite good agreement with present understanding of white dwarfs. While we have introduced an empirical adjustment factor to assist with the agreement, that adjustment is only a 20% increase in the prevailing pressures, and as mentioned above such an increase is plausible due to effects we have neglected from our simple approach. And lastly, it is also to be admitted that the present understanding itself may not be fully accurate. We cannot do controlled experiments with white dwarfs – we can only observe them from very far away. And of course, our understanding of our observations is further guided by prevailing theory, which itself may be lacking. So at this point we are very satisfied that our particles-in-a-box modeling serves our purposes and we can now turn to an analysis of the effects of field-mass on the super dense objects found in our universe.

J.8. Two Hypotheses for the Effect of Negative-Field-Mass on Hydrostatic Equilibrium. In deriving the equation for hydrostatic equilibrium above in section J.5 we make use of the gravitational attraction on a small cube of mass ρAdr to the center of the star; this leads to a force of $GM_{IN}\rho Adr/r^2$ where M_{IN} is the mass of the star within a radius of r. When including the effects of the negative-field-mass it is obvious that the attraction will now involve only the M_{NET} inside of r, where $M_{NET} = M_{POS} - M_{NEG}$ and where M_{POS} is the positive mass of things like hadronic and leptonic matter and M_{NEG} is the negative-field-mass. (This is obvious since it is consistent with our evaluation of the advance of the perihelions.) However it is not clear whether we should include the negative mass inside the small volume Adr in the gravitational attraction equation.

Perhaps the gravitational attraction involved in hydrostatic equilibrium is determined by the positive mass only, since only the positive mass (such as that of electrons, protons and neutrons) involves particles that are free to move about and collide with each other, and our ideas of pressure involve such matter. Or, perhaps the gravitational attraction involved in hydrostatic equilibrium involves the total mass inside the small volume Adr and in that case the negative field-mass will also participate. This leads us to two hypotheses to consider:

$dP = GM_{IN}\rho_{POS}dr/r^2$ (hypothesis one, exclusion of negative-field-mass within Adr) (J26)

$dP = GM_{IN}\rho_{NET}dr/r^2$ (hypothesis two, inclusion of negative-field-mass within Adr) (J27)

Since it is not obvious which equation to use, in what follows we will consider both hypotheses. Resolution of which one manifests itself in nature will be determined empirically.

J.9. Effects of the Negative-Field-Mass on White Dwarves, Codes SDO2 and SDO3. Using the crude approximation given in section E.2 above, we'll evaluate M_{EFF} for the densest white dwarf star listed in table J4, with a radius of $R_{WD} = 6.532x10^6$ m and a mass of $M_{WD} = 1.328\ M_S \approx 2.641x10^{30}$ kg. We get $M_{WDEFF} = M_{WD} – 3K_{G5}M_{WD}R_{WD} – 6K_{G6}M_{WD}^2/R_{WD} = M_{WD} – (3x10^{-21}\ m^{-1} x6.532x10^6\ m)M_{WD} – [(6x9x10^{-28}\ m/kgx2.641x10^{30}\ kg)/6.532x10^6\ m]M_{WD} = M_{WD}–(1.960x10^{-14}) M_{WD} – (2.183x10^{-3})M_{WD}$,

$M_{WDEFF} \approx M_{WD} – (2.183x10^{-3})M_{WD} = 0.9978M_{WD}$ (J28)

We see from Eq. (J28) that our crude approximation indicates that the field-mass effects are small, even on the densest white dwarf. Nonetheless, we would like to do a more accurate calculation of the negative-field-mass, as we will need such a calculation for heavier and denser stellar objects later.

The effect of the first-field-mass is negligible. (We have just calculated it to be $1.960\text{x}10^{-14}\ M_{WD}$ in the crude limit of a uniform sphere of mass. A finer calculation is unnecessary as we already know it is negligibly small.) For the second-field-mass, the mass density is given in Eq. (263):

$$\delta\rho_{M2} = -4K_c^2K_{T0}|\mathbf{P_G}|^2/X_0\xi_0^2c^2 = -K_{G4}P_G^2 \tag{263}$$

Also important is Eq. (249), which for r > R informs us that $\mathbf{P_{GLOUT}} = (3[K_{G1}-K_{G2}]\gamma_M Mc^2/16\pi r^2)\hat{\mathbf{r}}$. And we must include our knowledge that $\mathbf{P_{GL}}$ will be determined by the mass within r for a spherically symmetric mass distribution. (Any spherically symmetric mass distribution outside of r will have its gravitational force effect cancelled.)

The total second-field-mass ΔM_{2OUT} within a spherical shell of outer radius r+dr and inner radius r can now be found by integrating $\delta\rho_{M2}$ within that shell, $\Delta M_{2SHELL} = 4\pi \int \delta\rho_{M2}\ r^2dr = -4\pi \int K_{G4}(3[K_{G1}-K_{G2}]\gamma_M M_{NET}c^2/16\pi r^2)^2r^2dr = 4\pi K_{G4}(3[K_{G1}-K_{G2}]\gamma_M M_{NET}c^2/16\pi)^2/r = 5K_{G6}(\gamma_M M_{NET})^2/r$ evaluated between r and r+dr. Here, we've recalled Eq. (270), $K_{G6} = (4\pi K_{G4}/5)(3[K_{G1}-K_{G2}]c^2/16\pi)^2$. With $\gamma_M = 1$, the mass of the shell will be $5K_{G6}M_{NET}^2/(r+dr) - 5K_{G6}M_{NET}^2/r = 5K_{G6}M_{NET}^2/r(1+dr/r) - 5K_{G6}M_{NET}^2/r \sim= (5K_{G6}M_{NET}^2/r)(1-dr/r) - 5K_{G6}M_{NET}^2/r = -(5K_{G6}M_{NET}^2/r)(dr/r) = -5K_{G6}M_{NET}^2dr/r^2$,

$$\Delta M_{2SHELL} = 4\pi \int \delta\rho_{M2}\ r^2dr = -5K_{G6}M_{NET}^2dr/r^2 \tag{J29}$$

In Eq. (J29) we use M_{NET} instead of M because the second-field-mass provides a negative mass, and the displacement $\mathbf{P_{GLOUT}}$ will be sourced by the net mass $M_{NET} = M_{POS} - \Delta M_2$.

We can now update our SDO1 program to new programs SDO2 and SDO3 that include the effects from the second-field-mass. As we integrate from the center of the dense stellar object outward we will numerically integrate both the positive mass as well as the negative mass (the second-field-mass). The net mass is the positive mass minus the magnitude of the negative mass, $M_{NET} = M_{POS} - 5K_{G6}M_{NET}^2dr/r^2$.

For the new programs we'll start with a calculation of a central sphere of positive mass (M_{POS}) just as we did for program SDO1. We then set the net mass inside the sphere (M_{IN}) equal to the positive mass, since $\mathbf{P_{GLOUT}}$ is zero at the sphere center, and we set the remaining quantities just as we did in SDO1 as described in section J.6. Then we integrate through subsequent spherical shells. We find the shell volume, shellVolume = $(4/3)\pi[(r+dr)^3 - r^3]$. For SDO2 we define $\rho = \rho_{POS}$ and set $P = P - GM_{IN}\rho_{POS}dr/(r+dr/2)^2$ subtracting the pressure difference specified in Eq. (J26) and using the center of the spherical shell r+dr/2 in the denominator. For SDO3 we set $P = P - GM_{IN}\rho_{NET}dr/(r+dr/2)^2$ using Eq. (J27) instead of Eq. (J26). Next, $\xi_C = [2f\hbar^2c^2\pi^2 / 3mc^2P_{REP}]^{1/5}$ from Eq. (J18). Note that Eq. (J18) is not dependent upon the density of the negative mass, as it results from the quantum pressure which is a function of the density of the fermions and their rest masses; the negative-field-mass plays no role. We set $\rho_{POS}=M_{CELL}/\xi_C^3$ from Eq. (J24) to find the density, and here we are determining the positive mass density and M_{CELL} is the positive mass of the cell only. Next we set $M_{POS} = M_{POS}$ + shellVolume x ρ_{POS}. M_{NEG}, the absolute value of the negative mass, is determined through Eq. (J29), $M_{NEG} = M_{NEG} + 5K_{G6}M_{NET}^2dr/(r+dr/2)^2$. The net mass is $M_{NET} = M_{POS} - M_{NEG}$. And $\rho_{NET} = \rho_{POS} - \{5K_{G6}M_{NET}^2dr/(r+dr)^2\text{shellVolume}\}$ where the second term is the negative mass in the shell divided by the volume of the shell, which is of course the negative mass density. Finally, we update r to r = r + dr (we prepare to step into the next shell of thickness dr in the next iteration). The codes again terminate when ξ_C is greater than one angstrom or when the pressure drops to zero. (The one angstrom value is passed in to the program.)

Table J5 presents a comparison between the code SDO1 which does not include the effects of the negative-field-mass and codes SDO2 and SDO3 which include the effects of the negative-field-mass. The codes were run with the representative values of DR = 25, COUNT = 1000, N = 50 and F_{EA} = 1.2.

Table J5 Results of SDO1 vs SDO2 vs SDO3 for White Dwarves, ρ_{WD0} = 2.466x10^{10}

Code	Radius (m)	Positive Mass (Ms)	Negative Mass (Ms)	Net Mass (Ms)
SDO1	6532150	1.328	N/A	N/A
SDO2	6536950	1.330	0.002	1.327
SDO3	24746300	1.335	0.004	1.331

In table J5, the results show that the magnitude of the mass difference between the codes is consistent with what is calculated in Eq. (J28), as the net mass calculated in codes SDO2 and SDO3 (which include the effects of the negative-field-mass) are of the order 10^{-3} different than the mass calculated in code SDO1 (which excludes the effects of the negative-field-mass). However it may seem odd that the calculation of positive mass increases as we go from SDO1 to SDO2 and SDO3, since we are including a negative mass in SDO2 and SDO3. The reason for the mass increase is because the change in pressure is less as we move outward from the center for SDO2, and this leads to a smaller increase in the cell size, leaving a denser star and it also takes longer to reach the termination of the program, leading to a slightly larger radius for the case of SDO2. For the case of SDO3 we have a significantly larger radius but not much larger mass. The radius calculated in SDO3 is much larger than that calculated in SDO2 because the use of ρ_{NET} in the hydrostatic equilibrium leads to a much smaller fall-off of pressure in the outer regions, which in turn reduces the decrease in ξ_C in those outer regions, taking it significantly longer to reach the one angstrom point of code termination. Since the density in the outer regions is so much smaller than that of its core, the net and positive masses calculated by SDO3 differ little from SDO2, despite the large difference in radius.

J.10. State Changes and Radius Definitions in White Dwarfs. When dealing with dense objects there can be various state changes as the pressure drops from the center of the body outward. The situation is familiar to us on earth, where there are state changes between the core, the mantle, the solids that make up the earth's crust, and the atmosphere above it. Eventually, as we move far enough outward, the density of the gas is so small that we can consider it to be part of the solar system medium and not part of the earth. Finding what we call "the radius" of heavenly objects thus involves some definition of what we mean. One useful definition may be that radius where some relevant state change occurs. In that definition, the radius of the earth can be defined as the radius within which atoms are closely packed into solids or liquids. On earth, the total mass of the atmosphere is a very small fraction of the mass of the solid and liquid matter.

For white dwarfs there is an additional state change, as the majority of the white dwarf is in an electron-degenerate state, while regions far from the center can become what we consider normal (although compressed) solids and liquids, and further out there can be an atmosphere. Here, we've made an arbitrary definition of the electron-degenerate matter surface to be the point where our representative cell size ξ_C grows to one angstrom in all three dimensions. To explore the ramifications of different termination sizes, calculations were done using SDO2 and SDO3 with termination points of 10^{-10} m, 1.432×10^{-10} m (the value for graphite on earth), and 10^{-11} m. The results of these calculations are shown in Tables J6 and J7. The codes were run with the representative values of $\rho_{WD0} = 2.466 \times 10^{10}$, DR = 25, COUNT = 1000, N = 50 and $F_{EA} = 1.2$.

Table J6 Results of SDO2 for White Dwarves for Various Termination Conditions

Code	Termination (Angstroms)	Radius (m)	Positive Mass (Ms)	Negative Mass (Ms)	Net Mass (Ms)
SDO2	1.0	6536950	1.329692	0.002219	1.327472
SDO2	1.432	6537350	1.329692	0.002220	1.327472
SDO2	0.1	6455525	1.329634	0.002189	1.327445

Table J6 shows that different choices of program termination do not significantly affect the mass and radius calculations for any of the conditions evaluated by SDO2. This is because SDO2 uses the positive mass density of Eq. (J26) in the pressure drop which results in a rapid convergence toward a vacuum pressure.

Table J7 Results of SDO3 for White Dwarves for Various Termination Conditions

Code	Termination (Angstroms)	Radius (m)	Positive Mass (Ms)	Negative Mass (Ms)	Net Mass (Ms)
SDO3	1.0	24746300	1.334743	0.003990	1.330752
SDO3	1.432	32390425	1.334893	0.004140	1.330753
SDO3	0.1	6494875	1.332895	0.002208	1.330687

Table J7 shows that different choices of program termination do not significantly affect the mass calculations evaluated by SDO3. However, the SDO3 radius calculations are rather strongly affected by the termination condition. This is not surprising. Since SDO3 uses the net mass density of Eq. (J27) in the pressure drop it has a slow convergence to a vacuum pressure. The core of the white dwarf has a cell size of $\xi = 6.455 \times 10^{-13}$ m while a one angstrom cell has a density close to four million times less, and a 0.1 angstrom cell has a density close to four thousand times less. Therefore, the total mass in the outer regions of the white dwarf as calculated by SDO3 do not contribute significantly to the total mass, but this relatively low-density material can lead to a large radius.

The results shown in tables J6 and J7 show that it is not important what termination point we use for the electron-degeneracy region if our aim is to determine the mass of the white dwarf. The termination point is however highly relevant for a calculation of the star radius if the hypothesis of Eq. (J27) is used.

The results summarized in Tables J6 and J7 also show that empirical mass observations are unlikely to differentiate between the hypotheses put forward as Eqs. (J26) and (J27). Furthermore, determination of the radius may prove difficult to observe due to the magnitude of the distance to the white dwarf and the relatively small amount of material in its outer regions. We will consider both of our hypotheses to remain viable so far.

Recall that our primary goal is to show the importance of negative-field-mass to the hydrostatic equilibrium of dense objects. Also recall that the thermal, centrifugal force and Coulomb effects will all influence our calculations, as already mentioned when we arrived at the empirical adjustment factor (F_{EA}) introduced in section J.7. A more thorough modeling of white dwarfs would include separate layers of solid, liquid and gaseous atomic matter outside of the electron-degenerate matter, and such an analysis should also include the effects we've simply rolled into F_{EA}. However, we will not consider these issues further here. Therefore, use of a one angstrom termination condition for the electron-degeneracy layer is acceptable toward our goal and we shall use that going forward.

J.11. A First Look at the Hydrostatics of Neutron Stars. Codes SDO2 and SDO4. In section E.2 we evaluated M_{EFF} for a neutron star under overly simplistic assumptions. We used a radius of $R_N = 10^4$ m and a mass of $M_N = 1.5\ M_S \approx 3 \times 10^{30}$ kg. In that case, $M_{NEFF} = M_N - 3K_{G5}M_NR_N - 6K_{G6}M_N^2/R_N = M_N - (3\text{x}10^{-21}\ \text{m}^{-1}\text{x}10^4\ \text{m})M_N - [(6\text{x}9\text{x}10^{-28}\ \text{m/kg}\ \text{x}3\text{x}10^{30}\ \text{kg})/10^4\ \text{m}]M_N = M_N - (3\text{x}10^{-17})M_N - 1.62M_N$, or

$M_{NEFF} \approx M_N - 1.62M_N = -0.62M_N$ (Under Overly Simplistic Assumptions) (395)

From Eq. (395) we see that the negative-field-mass is likely to be a significant contributor in the case of neutron stars.

We can now use the treatment we've developed for white dwarfs and apply it to neutron stars. Our computer code SDO2 has been written with the fermion mass, cell mass and termination condition being input values, so all we need to do at this point is to run SDO2 with appropriate values for these quantities to investigate neutron stars. The fermion mass is now the mass of the neutron, $M_{DF} = 1.675\text{x}10^{-27}$ kg, and in this case the cell mass is equal to twice the mass of the neutron, $M_{CELL} = 3.35\text{x}10^{-27}$ kg. We will now terminate the program when $\xi_C = 6.455\text{x}10^{-13}$ m, as that is the point where the neutrons will decay back into protons and electrons. (See discussion above Eq. (J22).) We will use a contemporary estimate for the neutron star central mass density, $\rho_{NS0} = 1\text{x}10^{18}$ kg/m^3, and we'll use DR = 1, COUNT = 1000, N = 50, and $F_{EA} = 1$ as our representative run. We'll then compare that run against runs with parameter modifications to investigate the numerical stability of our approach. Results are shown in Table J8. We see that SDO2 has good convergence, as changing various inputs leaves the results significantly the same. We also see from Table J8 that the negative-field-mass is a significant contributor to the hydrostatics of neutron stars under the assumption of the Eq. (J26) hypothesis used in code SDO2.

Table J8 Results of SDO2 for a Neutron Star, $\rho_{WD0} = 1\text{x}10^{18}$ kg/m^3, $F_{EA} = 1$

DR	COUNT	N	Radius (m)	Positive Mass (Ms)	Negative Mass (Ms)	Net Mass (Ms)
1	1000	50	17209	1.3191	0.4414	0.8777
0.1	1000	50	17253	1.3246	0.4438	0.8808
1	100	50	17246	1.3246	0.4437	0.8810
1	1000	200	16965	1.2713	0.4189	0.8524

When moving on to look at the assumption of the Eq. (J27) hypothesis we must now recognize that neutron stars may involve two states of matter. As discussed in section J.4, it is energetically favorable for a carbon atom to be crushed into a state of pure neutron material once a certain pressure is applied. And yet, since pressure drops as distance from the star's center, we will eventually reach a place where the pressure is not sufficient to maintain pure neutron matter. Hence, neutron stars may have several different states of matter. The central region will be in a neutron degenerate state. At more outward regions there will be an electron-degenerate region, then atomic liquids and/or solids, and finally an atmosphere.

An interesting effect arises in the transition from neutron-degenerate matter to electron-degenerate matter in the presence of the negative-field-mass. Since the density of the neutron-degenerate matter is considerably greater than that of the electron-degenerate matter, the inner portion of the electron-degenerate region can obtain a negative net mass. This result is allowed because the negative-field-mass depends on P_G. (Eq. (263) gives $\delta\rho_{M2} = -4K_c^2K_{T0}|\mathbf{P_G}|^2/X_0\xi_0^2c^2 = -K_{G4}P_G^2$.) In the highly dense neutron-degenerate material P_G can obtain a higher value than what is possible in the less dense electron-degenerate material. (P_G can typically only grow until the negative-field-mass approximately equals the positive mass density, as anything beyond that will lead to a negative net mass density which will in turn reduce P_G. Hence, higher density material will allow higher values of P_G than lower density materials will.) However, P_G is a continuous field (it is the positive-aetherial displacement due to gravity.) Therefore, a negative net-mass-density can exist in the electron-degenerate region near the boundary of the neutron-degenerate region, since P_G can be large enough in the nearby neutron-degenerate region to result in a negative-field-mass density larger than the positive mass density in the nearby electron-degenerate region.

When a negative net-mass-density occurs, the electron-degenerate density increases with radius within the star: dP is negative due to the negative net mass, increasing the pressure and decreasing the cell size. As radial distance increases away from the star center, this increase in density will continue until the electron-degenerate positive mass density is large enough that it exceeds the (now reduced) negative-field-mass density and the net mass density is again positive. From that point on, the electron-degenerate density will decrease with radius.

If the original expansion in size caused by the neutron decay leads to an electron-degenerate cell size that is not large enough, the density increase in the negative net mass region would then cause the electron-degenerate mass to collapse back into neutron-degenerate matter. This then provides us with the value at which the transition occurs between neutron-degenerate matter and electron-degenerate matter: the transition occurs when the subsequent compression in the negative net mass region leads to a minimum cell size that is just greater than the electron-degenerate collapse limit of $\xi_C = 6.455\text{x}10^{-13}$ m.

A new program was written to evaluate the hydrostatic equilibrium of neutron stars containing both neutron-degenerate matter and electron-degenerate matter. SDO4 uses a flag to determine whether we use the hypothesis of Eq. (J26) or the hypothesis of Eq. (J27). In the program, the calculation begins in the center of the star and the numerical integration proceeds outward, just as in SDO2 and SDO3. For SDO4 the representative cell in the central region consists of two neutrons, with a corresponding cell mass and rest mass, and F_{EA} is set to 1. A parameter, neutronChiLim, is passed into the program. Once the cell size reaches neutronChiLim, the neutron-degenerate portion of the integration is stopped and an electron-degenerate portion of the integration begins. (neutronChiLim is the value of ξ_{CN} at the boundary between the electron-degenerate and neutron-degenerate matter.) Just before the numerical integration enters the electron-degenerate portion of the code, the cell mass is updated to be one third of a Carbon atom, the rest mass is updated to that of the electron, and F_{EA} is set to a passed-in value. From there the numerical integration is done as it is in the programs SDO2 and SDO3. The minimum size of any electron-degenerate representative cell is observed in the program output and the program is then run again and again with an updated neutronChiLim input until this minimum size is just large enough (ξ_C just greater than $6.455\text{x}10^{-13}$ m) so that the material in the electron-degenerate region does not collapse back into pure neutron matter. The electron-degenerate portion of the code terminates when the electron-degenerate cell size is one angstrom. A final section of the code tracks the net mass in vacuum regions radially outside of the neutron star. Table J9 shows results from the code SDO4 under the conditions of central mass density, $\rho_{NS0} = 3.5\text{x}10^{17}$ kg/m^3, DR = 10, COUNT = 1000, N = 50, $F_{EA} = 1.2$ and the Eq. (J27) hypothesis. The boundary between the neutron-degenerate region and the electron-degenerate region (neutronChiLim) was set to 4.044×10^{-13} m.

Table J9 Results of SDO4 for a Neutron Star, $\rho_{WD0} = 3.5\times10^{17}$ kg/m^3, Eq. (J27) Hypothesis

Evaluation Point	Radius (m)	Positive Mass (Ms)	Negative Mass (Ms)	Net Mass (Ms)
Neutron-Degeneracy/Electron-Degeneracy boundary	372020	1.3065	0.4754	0.8311
Electron-Degeneracy/Free Space Boundary	19,662,810	1.3317	0.4917	0.8400
10 Times Greater than the Electron-Degeneracy/Free Space Boundary	196,628,100	1.3317	0.4920	0.8397

Table J10 shows results from the code SDO4 under the conditions of central mass density, $\rho_{NS0} = 1\times10^{18}$ kg/m^3, DR = 1, COUNT = 1000, N = 50, $F_{EA} = 1.2$ and the Eq. (J26) hypothesis. The boundary between the neutron-degenerate region and the electron-degenerate region was again set to 4.044×10^{-13} m. However, in SDO4 no calculation is done in the electron-degenerate region in this case; rather, the calculation goes from the neutron degenerate region directly into the free space region since the cell size expansion is calculated to be extremely rapid toward the star surface.

Table J10 Results of SDO4 for a Neutron Star, $\rho_{WD0} = 1\times10^{18}$ kg/m^3, Eq. (J26) Hypothesis

Evaluation Point	Radius (m)	Positive Mass (Ms)	Negative Mass (Ms)	Net Mass (Ms)
Neutron-Degeneracy/Free Space Boundary	17209	1.3191	0.4414	0.8777
~10 Times Greater than the Electron-Degeneracy/Free Space Boundary	173,811	1.3191	0.6980	0.6211

Tables J9 and J10 show that the hypothesis given by Eq. (J26) is preferred. This judgement is based on the prevailing theory of pulsars coupled with the observation of millisecond pulsars. For the neutron star results of Table J10, the surface of the pulsar would be rotating at $2\pi\times1.72\times10^7$ m/s = 1.081×10^8 m/s for a rotation period of 1 ms. On the other hand, for the neutron star results of Table J9, the surface of the star would need to be moving at substantially greater than the speed of light. Even the neutron-degenerate/electron-degenerate boundary surface would need to be moving at substantially greater than the speed of light for the neutron star results of Table J9, and this will eliminate the hypothesis of Eq. (J27) from further consideration.

Notice that the net mass calculated in Table J10 falls from 0.8777 Ms to 0.6211 Ms as we go from the star surface to ten times further out into space. This result follows because the high density of the neutron star leads to a high value of P_G at the star's surface, with a large negative-field-mass density nearly canceling out the large positive mass density there. Just outside of the surface, the positive mass density goes quickly to zero, yet due to continuity of P_G, the negative-field-mass remains large, leaving a negative net mass. This negative net mass reduces P_G until eventually the fall-off in P_G becomes small and the net mass no longer varies significantly. (Recall that there is always some fall-off in P_G, and indeed that is what leads to the Newtonian gravitational formula plus the advance of the perihelions.) By the time we are 10 times further into space than the star radius, this drop off has slowed substantially and 0.6211 Ms is a good estimate for the far-field net mass (the mass we observe) of the neutron star for the parameters and assumptions used here.

Neutron stars can come in a variety of masses, and these masses will be determined by their central core densities. Table J11 Shows the results of several runs of program SDO4 at different core densities. Inputs were DR = 1, COUNT = 1000, N = 50 and the Eq. (J26) hypothesis. Although DR = 0.1 was used for the final two rows.

Table J11 Results of SDO4 for a Neutron Star, Various ρ_{WD0} Values, Eq. (J26) Hypothesis

Evaluation Point	ρ_{WD0} (kg/m^3)	Radius (m)	Positive Mass (Ms)	Negative Mass (Ms)	Net Mass (Ms)
Neutron-Degeneracy/Free Space Boundary	$1x10^{17}$	22567	0.3762	0.0363	0.3400
10 Times Greater than the Electron-Degeneracy/Free Space Boundary	$1x10^{17}$	225670	0.3762	0.0729	0.3034
Neutron-Degeneracy/Free Space Boundary	$1x10^{18}$	17209	1.3191	0.4414	0.8777
~10 Times Greater than the Electron-Degeneracy/Free Space Boundary	$1x10^{18}$	173810	1.3191	0.6980	0.6211
Neutron-Degeneracy/Free Space Boundary	$1x10^{19}$	16234	5.6251	3.8744	1.7508
~10 Times Greater than the Electron-Degeneracy/Free Space Boundary	$1x10^{19}$	163963	5.6251	4.6975	0.9277
Neutron-Degeneracy/Free Space Boundary	$1x10^{21}$	19600	99.455	95.285	4.169
10 Times Greater than the Electron-Degeneracy/Free Space Boundary	$1x10^{21}$	196000	99.455	97.962	1.493
Neutron-Degeneracy/Free Space Boundary	$1x10^{24}$	22300	2274	2268	6.19
10 Times Greater than the Electron-Degeneracy/Free Space Boundary	$1x10^{24}$	223000	2274	2272	1.84
Neutron-Degeneracy/Free Space Boundary	$1x10^{27}$	26009	118571	118562	9.20
~10 Times Greater than the Electron-Degeneracy/Free Space Boundary	$1x10^{27}$	262692	118571	118569	2.26

Table J11 is enlightening concerning several aspects of our initial neutron star model. First we see the substantial effect that the negative-field-mass has on the eventual observed net mass. (Recall that the positive mass is the mass that the material in the star would have if not for the negative mass, the negative mass is due to the field-mass, and the net mass is the positive mass minus the absolute value of the negative mass.) The positive mass to net mass is 0.34 Ms to 0.3034 Ms for $\rho_{WD0} = 10^{17}$ kg/m^3, and 118571 Ms to 2.26 Ms for $\rho_{WD0} = 10^{27}$ kg/m^3. For the relatively light neutron star the ratio of the net mass to the positive mass is 89%, but for the heaviest one the net mass becomes only 0.002% of the positive mass. This effect comes about because a lot of very dense matter will lead to a high value of P_G, and a high negative mass density. The negative mass density will itself lower P_G, and the final result shown by the program is that for very high densities the negative mass nearly cancels the positive mass within much of the heavier neutron stars. As the central density is increased further, the net mass of the star only increases marginally.

A second observation concerns large stellar objects such as Sgr A* at the center of the Milky Way. Sgr A* has a reported mass of about 4 million Ms. Our model of the aether has no singularity, and Table J11 shows that it takes an increase in core density of a factor of 1000 just to increase the net mass from 1.84 Ms to 2.26 Ms. Observation of the code output shows that at high density the cell size near the center expands rapidly (to a lower density) and hence large changes in the central density do not result in much change to the eventual stellar mass. Obtaining a mass of 4 million Ms via the physics used in SDO4 will apparently require a truly gargantuan central density. And some large objects have reported masses vastly in excess of Sgr A*. The density we use for our last row of Table J11 is already about a billion times larger than that found in the smaller neutron stars, and yet the mass has only increased to 2.26 Ms. Of course it is entirely plausible that the underlying physics may change at extreme conditions, and we will turn to that possibility next.

J.12. Speculations on the Hydrostatics of Super Massive Objects; Code SDO5. We have just seen that program SDO4 does not easily lead to an understanding of super massive objects such as Sgr A*. To overcome this issue, we can explore what happens when we change certain conditions. Two obvious possibilities present themselves: 1) it is possible that high pressure will crush the neutrons into some new form of matter; and 2) the gravitational force relation may change in the presence of extreme conditions. The purpose of this section J.12 is to present speculative examples for those two possibilities in a way that is consistent with the foundations of this work. Also, here in section J.12 we will assume the correctness of presently prevailing mass estimates for certain observed objects, and in section J.13 we'll comment on the validity of that assumption. We'll allow our section J.12 speculations to evolve all the way to reflections on the origins of the universe; but in section J.13 we'll return to commentary that once again reminds us that we've only fleshed out a few speculative models out of many that may exist.

J.12.1. Speculative Example 1; Exotic Particle Formation. First we consider the possibility that high pressure will crush the neutron star into some new exotic material. If we use a lower rest mass and lower cell mass for the new exotic matter, from Eq. (J24) $\rho = M_{CELL}/\xi_C{}^3$, we see that for a given central density, our central ξ_C will be proportional to $M_{CELL}{}^{1/3}$. By Eq. (J16), $P_{REP} \sim= 2f\hbar^2c^2\pi^2/3mc^2\xi_C{}^5$ we see that P_{REP} is proportional to $1/\xi_C{}^5$ (neglecting any effect from f) and hence our central P_{REP} will be larger for smaller M_{CELL}. The fall-off in pressure is given from the equation of hydrostatic equilibrium, Eq. (J23), $dP = GM_{IN}\rho dr/r^2$. Now our central ρ is an assumed starting value, and so the pressure drop will be the same while the pressure itself is larger for a smaller M_{CELL}. (M_{IN} is a function of ρ, so the central dP will be unchanged.) Since the relative pressure drop is smaller, the density will take more distance to fall off. Hence we expect that a lower cell mass (from lower mass exotic particles) will lead to a larger stellar mass. The new code SDO5 extends SD04 to allow us to pass in the exotic particle mass and the cell mass as inputs. SDO5 does not include an electron-degenerate layer, as we have learned from SDO4 that such a layer is so thin as to not be relevant to our analysis. Table J12 presents results of the code SDO5 run with different M_{CELL} values. The values chosen are for the neutron as well as exotics that would have the mass of the pion, the mass of the electron and one tenth the mass of the electron. These are just examples based on familiarity with such masses; exotics are expected to be fermionic hadronic matter and be neither electrons nor pions. For Table J12, $\rho_{NS0} = 1x10^{21}$ kg/m^3, DR = 1, COUNT = 1000, N = 50 and $F_{EA} = 1$. We also set $M_{REST} = M_{CELL}/2$, as we assume the cell will contain two simple exotic particles each of rest mass M_{REST}. (We assume two spin states per cell. We use a large value of ρ_{NS0} because we are presently interested in super massive objects.) To aid in convergence, SDO5 increases DR and COUNT as it runs.

Table J12 Results of SDO5 for a Simple Exotic Particle Star, $\rho_{NS0} = 1x10^{21}$ kg/m^3, Various Fermion mass and M_{CELL} Values, Eq. (J26) Hypothesis

Evaluation Point	M_{CELL} (kg) (= 2 mc^2)	Radius (m)	Positive Mass (Ms)	Negative Mass (Ms)	Net Mass (Ms)
Neutron-Degeneracy/Free Space Boundary	$3.35x10^{-27}$ (Neutron)	19527	99.452	95.256	4.197
10 Times Greater than the Neutron-Degeneracy/Free Space Boundary	$3.35x10^{-27}$ (Neutron)	195270	99.452	97.906	1.546
Exotic-Degeneracy/Free Space Boundary	$4.82x10^{-28}$	1,136,249	312703	312353	350.3
10 Times Greater than the Pion-Degeneracy/Free Space Boundary	$4.82x10^{-28}$	11,362,490	312703	312602	101.3
Exotic-Degeneracy/Free Space Boundary	$1.82x10^{-30}$	$8.402x10^{10}$	$1.535x10^{15}$	$1.535x10^{15}$	$2.839x10^{7}$
10 Times Greater than the Exotic-Degeneracy/Free Space Boundary	$1.82x10^{-30}$	$8.402x10^{11}$	$1.535x10^{15}$	$1.535x10^{15}$	$1.599x10^{7}$
Exotic-Degeneracy/Free Space Boundary	$1.82x10^{-31}$	$5.905x10^{12}$	$1.928x10^{19}$	$1.928x10^{19}$	$1.082x10^{10}$
10 Times Greater than the Exotic-Degeneracy/Free Space Boundary	$1.82x10^{-31}$	$5.905x10^{13}$	$1.928x10^{19}$	$1.928x10^{19}$	$9.379x10^{9}$

From Table J12 we see that collapse from a neutron star into one made of simple exotic particles can indeed result in super massive stars. It is then of interest to see if a variation in the central density results in a wide variety of masses for such objects. Running SDO5 with an input density of $1x10^{17}$ kg/m^3 and assuming a collapse to a simple exotic particle with the mass of the electron leads to a predicted net mass of $7.794x10^{6}$ Ms at a radius of $8.374x10^{11}$ m, ten times the star's radius. As was the case with the neutron star, the final mass is quite insensitive to changes in the central density. (A change in central density by a factor of 10,000 results in a change in net mass of only a factor of $1.599 \times 10^7 / 7.794 \times 10^6 \sim 2$.) Additionally, the radius is considerably larger than that of a neutron star. For these reasons, collapse to a simple exotic particle does not appear to be a good model for super massive objects, at least for the cases we've investigated. It is possible that a theory of exotic particles with masses as a function of pressure could map to known super massive objects, but such an ad hoc proposal has no other support from existing empirical observations and so we will not look into it further.

J.12.2. Speculative Example 2; Extending Our First-Order Theory. Our second possibility of achieving large mass objects is to consider the possibility of altering our first-order derivations in section D for regions of high density, pressure, or gravitational fields. Looking back, we find that the original founding equations for our understanding of gravity are Eqs. (183) through (186).

$$K_{TP} = K_{T0}(1 - K_{G1}\rho_E) \tag{183}$$

$$K_{TN} = K_{T0}(1 - K_{G2}\rho_E) \tag{184}$$

$$K_{QP} = K_{Q0}(1 - K_{G2}\rho_E) \tag{185}$$

$$K_{QN} = K_{Q0}(1 - K_{G1}\rho_E) \tag{186}$$

Eqs. (183) through (186) already inform us that the aetherial quantum constants K_T and K_Q are reduced by the presence of an energy density ρ_E. In section D, we have assumed that the reductions have the specific linear form as specified by ***The Extrinsic-Energy Force-Reduction Law*.** We then use Eqs. (183) through (186) to arrive at Eqs. (249), (250), (251) and (263):

$$\mathbf{P_{GLOUT}} = -\mathbf{N_{GLOUT}} = (3[K_{G1} - K_{G2}]\gamma M_S c^2/16\pi r^2)\hat{\mathbf{r}} \tag{249}$$

$$\mathbf{F_{ME1ME2}} = \mathbf{F_{GNEWTON}} =$$

$$-(9\gamma_2 M_2 c^2 K_{GC}[K_{G1} - K_{G2}]^2 \rho_0^2 \gamma_1 M_1 c^2/16\pi r^2 \varepsilon_0)\hat{\mathbf{r}} = -(G_N \gamma_1 M_1 \gamma_2 M_2 / r^2)\hat{\mathbf{r}} \tag{250}$$

$G_N = 9K_{GC}[K_{G1}–K_{G2}]^2\rho_0^2c^4/16\pi\varepsilon_0 = 6.6743x10^{-11}$ N m^2/kg^2 (251)

$\delta\rho_{M2} = –4K_c^2K_{T0}|\mathbf{P_G}|^2/X_0\xi_0^2c^2 = –K_{G4}P_G^2$ (263)

Since $\mathbf{P_{GLOUT}}$ is proportional to $[K_{G1}–K_{G2}]$, we see that both $\mathbf{F_{GNEWTON}}$ and $\delta\rho_{M2}$ are proportional to $[K_{G1}–K_{G2}]^2$. We have seen that the derivations in section D work excellently to calculate observed phenomena in the presence of typical (non-extreme) values of ρ_E, pressure and gravitational field. Yet it is entirely possible that in extreme conditions the expressions for the quantum and tension constants will change, and that our founding equations are just the first order limits. Specifically, rather than $[K_{G1}–K_{G2}]^2$ being equal to some constant K_{G12}^2, we can hypothesize that under extreme conditions $[K_{G1}–K_{G2}]$ changes by some function, g, becoming

$[K_{G1}–K_{G2}] = g \times K_{G12}$ (J30)

In Eq. (J30) K_{G12} is the first order limit used in section D and the function g should converge to unity whenever densities, pressures and gravitational fields are not extreme. Such a hypothesis leads to modified equations for the gravitational force and negative-field-mass in extreme field, pressure or gravitational regions:

$\mathbf{F_{GNEWTON\_EXTREME}} = –[g^2G_N\gamma_1M_1\gamma_2M_2/r^2]\hat{\mathbf{r}}$ (J31)

$\Delta M_{2SHELL\_EXTREME} = 4\pi \int \delta\rho_{M2}\, r^2dr = – 5g^2K_{G6}M_{NET}^2dr/r^2$ (J32)

By reviewing the derivation in section D, it can be seen that Eqs. (J31) and (J32) are obtained once we accept Eq. (J30) as a new hypothesis in many cases. (Each proposed function g should be evaluated to ensure the derivations of section D hold. While sections D.3.2 and D.3.3 call out the linearity of $\rho_G = 3[K_{G1}–K_{G2}]\rho_E\rho_0/2$ from Eq. (193) to arrive at a linear dependence of the quantum and tension constants as the cubes move, notice that we can still obtain that linear dependence by arranging for ρ_e to follow an injection model such that linearity results. The linear portion of the quantum and tension constants is derived in section D from the motion of the cubes through the tension and quantum fields, and our injection models are only for heuristic purposes. To derive Eq. (192) we use $K_{G1}\rho_E << 1$ and $K_{G2}\rho_E << 1$, and those conditions should continue to hold.)

With Eq. (J31) now established we can replace Eq. (J23) with

$$dP_{EXT} = g^2 GM_{IN}\rho dr/r^2 \tag{J33}$$

A simple example function g to consider is one that leads to $\mathbf{P_{GLOUT}}$ saturating; $\mathbf{P_{GLOUT}}$ becomes a constant once a certain condition is met. This can be achieved if

$$g = 1 \text{ for } \gamma M_{NET}/r^2 <= \sigma_{LIM} \text{ ; } g = r^2\sigma_{LIM}/\gamma M_{NET} \text{ for } \gamma M_{NET}/r^2 > \sigma_{LIM} \tag{J34}$$

In Eq. (J34) σ_{LIM} is some limiting value. Since $\mathbf{P_{GLOUT}}$ is proportional to $[K_{G1}–K_{G2}]$, by Eq. (J30) we need to add a factor of g into Eq. (249) when determining $\mathbf{P_{GLOUT}}$ in extreme conditions. When $\gamma M_{NET}/r^2 <= \sigma_{LIM}$ we retain Eq. (249), $\mathbf{P_{GLOUT}} = (3[K_{G1}–K_{G2}]\gamma M_{NET}c^2/16\pi r^2)\hat{\mathbf{r}}$, while when $\gamma M_{NET}/r^2 > \sigma_{LIM}$ we get $\mathbf{P_{GLOUT}} = (3[K_{G1}–K_{G2}]\sigma_{LIM}c^2/16\pi)\hat{\mathbf{r}}$, showing the saturation. In this case of a simple saturation, the derivations of section D remain intact for $\gamma M_{NET}/r^2 <= \sigma_{LIM}$ while for $\gamma M_{NET}/r^2 > \sigma_{LIM}$ we simply saturate $\mathbf{P_{GLOUT}}$.

A starting guess for σ_{LIM} will be M_{NET}/r^2 at the surface of a solar mass neutron star, as we will guess that saturation begins to take place in such an environment. From Table J11 we estimate

$$\sigma_{LIM0} = 2\text{x}10^{30} \text{ kg}/(17{,}000 \text{ m})^2 = 6.920\text{x}10^{21} \text{ kg/m}^2 \tag{J35}$$

We can then use that guess as a starting point and pass an arbitrary multiplier g_{MULT} into SDO5 to investigate various values of $\sigma_{LIM} = g_{MULT} \text{ x } \sigma_{LIM0}$.

Eqs. (J32) through (J35) were encoded into the program SDO5. SDO5 also assumes $\gamma = 1$. Tables J13, J14 and J15 present the results of various runs of SDO5 under different central density and g_{MULT} assumptions. All runs assumed that the star consists of pure neutron matter, and the studies used the inputs DR = 1, COUNT = 1000, N = 50 and $F_{EA} = 1$. The tables report the radius of the star, with the negative mass and net mass reported at ten times that radius.

A notable aspect of our models is the large amount of negative and positive mass associated with the dense objects. In Table J13 we find that the observed net mass of a Type 1A supernova remnant is only 56% of its positive mass, and Sgr A* has a positive mass about $5\text{x}10^8$ greater than the net mass that we observe. We see in Table J15 that a change in g_{MULT} (which lowers the maximum P_G allowed) results in less positive mass, but it is still quite large. The lower g_{MULT} value also increases the radii and lowers the central densities of the massive stellar objects. If we postulate that millisecond pulsars are fast-spinning remnants of type 1A supernovas, then the radius given in Table J15 is already close to a limiting value. Therefore, our speculative model continues to lead to the conclusion that there is substantial hidden mass within super massive objects.

Table J13 Results of SDO5 for Various Initial Densities, gMult = 1 and Neutron Matter Using the Eqs. (J32), (J33) and (J34) Hypothesis

Description	ρ_{NS0} (kg/m^3)	Radius (m)	Positive Mass (Ms)	Negative Mass (Ms)	Net Mass (Ms)
Type 1A SN Remnant	$4.1\text{x}10^{17}$	20877	1.345	0.5901	0.7548
Neutron Star	$5.0\text{x}10^{17}$	30639	4.765	2.861	1.904
Neutron Star	$8.0\text{x}10^{17}$	244681	2422	2377	45.92
Neutron Star	$1.0\text{x}10^{18}$	$1.511\text{x}10^6$	570,184	569,852	331.8
Sgr A*	$1.62\text{x}10^{18}$	$2.297\text{x}10^9$	$2.005\text{x}10^{15}$	$2.005\text{x}10^{15}$	$3.807\text{x}10^6$
Andromeda Central Star	$1.74\text{x}10^{18}$	$1.220\text{x}10^{10}$	$3.001\text{x}10^{17}$	$3.001\text{x}10^{17}$	$9.076\text{x}10^7$
TON 618	$1.96\text{x}10^{18}$	$3.138\text{x}10^{11}$	$5.110\text{x}10^{21}$	$5.110\text{x}10^{21}$	$5.758\text{x}10^{10}$

Table J14 Results of SDO5 for Various Initial Densities, gMult = 0.3 and Neutron Matter Using the Eqs. (J32), (J33) and (J34) Hypothesis

Description	ρ_{NS0} (kg/m^3)	Radius (m)	Positive Mass (Ms)	Negative Mass (Ms)	Net Mass (Ms)
Type 1A SN Remnant	$1.05\text{x}10^{17}$	33757	1.405	0.4199	0.9849
Sgr A*	$3.6\text{x}10^{17}$	$2.778\text{x}10^9$	$7.823\text{x}10^{14}$	$7.823\text{x}10^{14}$	$4.230\text{x}10^6$
TON 618	$4.35\text{x}10^{17}$	$3.977\text{x}10^{11}$	$2.294\text{x}10^{21}$	$2.294\text{x}10^{21}$	$6.184\text{x}10^{10}$

Table J15 Results of SDO5 for Various Initial Densities, gMult = 0.1 and Neutron Matter Using the Eqs. (J32), (J33) and (J34) Hypothesis

Description	ρ_{NS0} (kg/m^3)	Radius (m)	Positive Mass (Ms)	Negative Mass (Ms)	Net Mass (Ms)
Type 1A SN Remnant	$3.0\text{x}10^{16}$	52279	1.362	0.261	1.101
Sgr A*	$9.41\text{x}10^{16}$	$3.352\text{x}10^{9}$	$3.589\text{x}10^{14}$	$3.589\text{x}10^{14}$	$3.677\text{x}10^{6}$
TON 618	$1.14\text{x}10^{17}$	$5.396\text{x}10^{11}$	$1.497\text{x}10^{21}$	$1.497\text{x}10^{21}$	$6.779\text{x}10^{10}$

J.12.3. Hidden Mass in Our Speculative Models. Tables J12 through J15 reveal that the mass we observe is far less than the positive mass within the heaviest objects. Here we will discuss qualitatively why this situation occurs for our $\mathbf{P_G}$ saturation speculative example. First consider a small star with positive mass; gravity will confine it to some radial distribution and there will be little negative mass. As we add mass, the gravity grows, density grows, and P_G grows, and therefore negative mass grows. When the star grows to where P_G hits the saturation point the negative density will reach its limit, and there will be a positive net density. That net density is the one that leads to the saturation P_G. When we add more positive mass, the size of the star must grow, since the positive density is only contained by gravity, and since gravity has now saturated the gravity cannot contain a higher density. As we continue to add mass to the star, any new net mass must be only enough to keep the saturation level of P_G. If there isn't enough net mass, P_G and the associated negative mass will fall, and this will lead back to the required increased net mass. If there is too much net mass, the positive mass density will grow (since the negative mass density is fixed in saturation) and since gravity is fixed by saturation, the positive mass will expand, reducing the positive density, and in turn, reducing the net density until it reaches the required value. Prior to saturation Eq. (249) gives $P_G = (3[K_{G1}–K_{G2}]\gamma M_{NET}c^2/16\pi r^2)\hat{\mathbf{r}}$. Hence the mass we need to add to M_{NET} to keep P_G constant grows as r^2 and yet the negative mass grows as r^3 (it has a constant density due to the saturation) and the positive mass grows as the negative mass minus the net mass (by definition) so the growth of the net mass is slower than the growth of the negative and positive masses, and as we get to larger and larger stars the negative mass tends to equalize with the positive mass. Indeed, this is what is seen in the results of Tables J13 to J15.

J.12.4. A Review of Results from Our Speculative Models. Section J.7 has described how our particle-in-a-box model used in code SDO1 provides a good match to the present status quo for white dwarf modeling. For the neutron stars that are remnants of type 1A supernovae we see a departure from contemporary calculations, and when it comes to extremely large objects our treatment is vastly different from the presently prevailing idea of black holes.

Tables J13 through J15 show that type 1A supernova remnants will have a negative-field-mass close to half of its raw positive mass for $\sigma_{LIM} = \sigma_{LIM0}$ and less than a quarter of its raw positive mass when $\sigma_{LIM} = \sigma_{LIM0}/10$. For larger mass objects, Tables J13 through J15 show values for radii and mass considerably different from prevailing theory. Clearly any finite radius is a significant departure from the General Relativity concept of a singularity. Additionally, Tables J13 through J15 show that what we observe (called the net mass) is frequently only a very small fraction of the raw positive mass present in super massive bodies.

The extension of our first-order theory (speculative example 2; section J.12.2) is a better fit to observations than proposing exotic particles (speculative example 1; section J.12.1). A recent measurement[J2] gives a diameter of 51.8 μarc-seconds (6.18×10^{10} m) for the central ring around Sgr A*. (Sgr A* is 26,000 light-years from earth, and a circumference of a circle with that radius is $2\pi \times 26{,}000$ light-years = 163,400 light years (ly). A light year is 3×10^{8} m/s x 365 days x 24 hours/day x 3600 s/hour = 9.46×10^{15} m. An arc-degree is 1/360 of the circumference, and a micro-arc-second is $(1/3600) \times 10^{-6}$ of that, resulting in a diameter of 163,400 ly x 9.46×10^{15} m/ly x 51.8 x $10^{-6}/(360 \times 3600) = 6.18 \times 10^{10}$ m.) From Table J12, we find a radius of 8.402×10^{10} m and a far-field net mass of 1.599×10^7 Ms for the case where an exotic hadron has the mass of an electron. From Table J13 we find radius of 2.297×10^9 m and a far-field net mass of 3.807×10^6 Ms for the case where g has a saturation at σ_{LIM0}, and from Table J15 we find radius of 3.352×10^9 m and a far-field net mass of 3.677×10^6 Ms for the case where g has a saturation of $\sigma_{LIM0}/10$. From these results, with the observed Sgr A* ring radius of 3.1×10^{10} m, we can conclude that our electron-mass exotic particle hypothesis leads to results outside of the recent measurement, while the two results associated with an extension of our first-order theory are well within the observed value. For this reason, we'll now set the exotic particle speculation aside.

J.12.5. Neglecting the First-field-mass Inside of Super Massive Objects. In this section J.12 we have shown how the hidden mass from the second-field-mass is nearly equal to the normal mass inside of super massive objects, and we have neglected any mass effect from the first-field-mass. To see why this neglect is permissible it is of interest to calculate the first-field-mass of the largest known stellar object, TON 618. From Eq. (266) we see that the first-field-mass inside of the radius of the star is $\Delta M_{1IN} = K_{G5}(\gamma_M M/R^3)r^4 = K_{G5}MR$, where r = R and γ_M=1. Eq. (276) gives $K_{G5} \approx 10^{-21}$ m^{-1} and Table J15 gives M = 6.779×10^{10} Ms and R = 5.396×10^{11} m for TON 618 giving a first-field-mass of $\Delta M_{1IN} = (10^{-21}\ m^{-1})(6.779 \times 10^{10}\ Ms)(5.396 \times 10^{11}\ m) = 36.58$ Ms. We see that the first-field-mass of TON 618 is negligible with respect to its observed mass, and since TON 618 has a larger mass and radius than any other object we consider it is permissible to neglect the dark mass here in section J.12. (Note that the second-field-mass affects P_G, which is why we use the net mass, and not the positive mass, when we calculate ΔM_{1IN} here.)

J.12.6. Ramifications of Our Speculative Models. Our speculative models have some rather substantial differences with respect to prevailing theory. The first difference is that our speculative models predict that an extremely high percentage of the universe's hadronic mass exists in the super massive objects near galactic cores. Instead of having a mass of about $4\text{x}10^6$ Ms, Table J15 reports that Sgr A* has a mass closer to $4\text{x}10^{14}$ Ms. With a present mass estimate of the Milky Way of $1.5\text{x}10^{12}$ Ms, this says that the hadronic hidden mass within Sgr A* is about 300 times greater than the sum of all of the other mass within the galaxy. With other galaxies having similar central objects, it follows that our speculative model predicts an amount of hadronic matter in the universe far greater than what is presently believed.

The hidden mass changes our view of the nature of the central galactic objects. In the prevailing dogma, the central object of the Milky Way contains only about $3\text{x}10^{-6}$ of the hadronic mass of the galaxy. But when we include the hidden mass, we see that our speculative model calculates the central galactic object to contain something like 99.6% of the galaxy's mass. This is a significant promotion. Rather than just being the biggest object among a trillion others, the central object now becomes the dominant object, with all other objects becoming a mere faint halo in comparison.

Our speculative model may lead to a better understanding of quasars and supernovas. Quasars may be the result of a large positive mass being accreted into what will eventually become a central object of far more modest mass, releasing tremendous energy during the process. Some of the larger objects presently being found may indeed be super massive neutron stars in the process of forming, rather than fully formed central objects. Prior to their formation they will have substantially more observed mass than they will after they have been fully formed, as much of the mass will become hidden by negative-field-mass once they compress into their final high-density state. (Before compression their larger size leads to lower values of P_G, and lower negative mass, than after they are compressed.) Since our speculative model predicts far more hadronic matter than presently believed, this means there is more fuel for quasars that presently believed, although note that there may be primordial super massive neutron stars at their center as well, as we'll describe below. The explosive power of supernovas may also be explained via a speculation that the formation of negative-mass during collapse of a star results in a considerable release of energy. And finally, our speculative model leads to further speculation concerning the origin of the universe. Were gravity to lead to a coalescing of the existing super massive neutron stars into one giant object, the mass of such a body would be far larger than the individual super massive objects we observe today. As we have speculated, existing super massive objects may lead to a saturated P_G and a modified gravitational attraction. Should an object become vastly more massive, it is possible that the stress on the aether might cause the aether itself to rip apart. After all, the aether is also a substance; it is capable of being torn apart. And recall that it is the aetherial disturbance that is responsible for the gravity holding the super massive object together. If the aether itself is no longer there, this would result in no more inward gravitational force at all. Yet the neutrons would still have enormous outward pressure due to their compressed state. In such a condition there would be a gigantic outward explosion. The origin of such an explosion would not be the big bang singularity of present theorizing; instead the origin of the explosion would be a single, highly dense ball of finite radius. After such an explosion we can further speculate that the torn aether will recombine in time, but the explosion of matter would continue outward, which is what we see today. (Such an explosion may also contain primordial neutron stars.)

J.12.7. A Replacement for the Concept of Black Holes. Our speculative example 2 fits the observational data of reference J2 quite well. Rather than the black hole model of a singularity surrounded by an event horizon further surrounded by an accretion disk, our new model is that of a large neutron star, surrounded by a negative mass density region just outside its surface, further surrounded by an accretion disk. Recall that a saturated P_G will occur right up to the surface of the neutron star, creating a negative mass density comparable to the positive mass density there. Just beyond the surface P_G and the negative mass density will still be large, but there will be no positive mass in that region, leaving only negative mass. This negative mass density then leads to a rapid drop off of P_G until conditions become that of a conventional gravitational field.

In our speculative example, instead of an event horizon that allows nothing, not even light, to escape, the negative mass region may also act to suppress photon emission. In the latter case this is because the quantum and tension constants undergo saturation effects that may severely affect photon traversal through such regions. (Recall that light is an aetherial oscillation depending upon aetherial tension as a restoring force opposing displacement.)

It is noteworthy that the gravitational radius specified by Table 1 in reference J2 is 4.8 μas, or $2\pi \times 26{,}000$ ly $\times 9.46\times10^{15}$ m/ly $\times 4.8 \times 10^{-6}/(360\times3600) = 5.72\times10^{9}$ m. We see that the calculated radii of Tables J13 and J15 of 2.297×10^{9} m and 3.352×10^{9} m, respectively, are reasonably close to and within that value. As a result, our speculative example is an excellent fit to the observational data, indicating that super massive, negative-mass-encased, neutron stars are a viable alternative to black holes.

J.13. Concluding Comments on Our Speculations and Other Possibilities. The primary important result of our second speculative example in section J.12.2 is that we have shown that a model for super massive objects can be achieved consistent with the axioms and hypotheses of the quantum luminiferous aether. This demonstration required only a single, simple speculation of a saturated P_G. However, it is repeatedly emphasized that the specifics of any model are speculative at this point. Other speculative models are certainly possible.

Even within our speculative model, variations could be proposed. It is unlikely that P_G will grow according to Eq. (249) until a single threshold value is reached and then discontinuously transition into a constant and remain that way thereafter. More likely, any physical saturation onset would occur in a continuous fashion and its value after saturation might deviate from an exact constant. We also don't know the precise value of where the threshold for saturation occurs.

And there are many possibilities beyond the simple examples we have proposed above. In section J.12 we have presented several exotic star possibilities along with a single proposal for an extension of the work of section D. There are an enormous number of other possibilities, especially when combinations of models are considered. In addition to the several exotic masses chosen, there are an infinite number of other choices, and one could also consider looking into compound exotics (with a lighter mass exotic bound to a heavier one in an exotic atom). Many other choices for extending the work of section D could be proposed. And it is entirely possible that super massive objects consisting of exotic particles will interact via forces that are extensions of the physics of section D, combining the approaches of both of our earlier speculative examples. Super massive objects could also have layers, such as a core, a mantle, a crust and an atmosphere. However this level of complexity has been avoided here as there are no supporting observations for any of this.

Recall also that we have assumed that present data associated with super massive objects is correct when we developed our speculative examples. However, as mentioned above in section J.12.6, quasars might be large dense clouds in the process of forming what will eventually become a lower mass object later on. There may be no super dense object with a mass as large as TON 618, as it might now be a less-dense object in the process of collapsing to a single high-density object, and its eventual mass may be much less once the hidden mass appears. What we consider to be "data" often is informed by what we believe to be true theoretically, and if our beliefs change (by a new fundamental physics) our understanding of the data may change as well. In fundamental physics, care and thoroughness is always needed.

One point that should cause pause and reflection is the prediction of large amounts of hidden mass. Section 12.6 notes that perhaps 99.6% of the hadronic mass of the universe is now hidden. While such a big prediction is possible, and while our advances in knowledge of the universe have often involved large deviations from what was thought before, it is also quite possible that any new speculation is simply wrong. Much further work should be done before we remove the word "speculation" from the models discussed in section J.12.

The important point is that we really don't know with any specificity what is going on inside of the super massive objects and that any modeling is of course quite speculative at this point. Perhaps a model can be found that greatly reduces the predicted levels of hidden mass while remaining consistent with the axioms herein. Of course, additional observational data, properly understood, is always helpful in advancing our understanding. Ideally we should somehow experiment with the aether itself, by isolating portions of it and then subjecting it to tests. Then perhaps we can get more specific about its properties under extreme conditions. (Admittedly, isolating the aether and experimenting with it could be very challenging to accomplish.)

Nonetheless, it is important that our theory provide some model to account for all observed gravitational phenomena including those within super massive objects, and our speculative modeling of section J.12 achieves that goal. Additionally, a maximum limit of P_G is certainly plausible. The shrinkage of the aetherial quanta will have a limit (it can't go less than zero), and the interaction between the two species of aether could easily lead to the maximum limit of P_G speculated in section J.12.2. For these reasons, our speculative example of section J.12.2 is likely as good or better than many others, and so we will now close this Appendix J with no further speculations, leaving the important work of refined speculations and continued observations for future efforts.

References for Appendix J.

[J1] "An Empirical and Classical approach for Non-Perturbative, High Velocity, Quantum Mechanics", D.J. Larson, Physics Essays, Volume 30: Pages 264-268, 2017.

[J2] The Event Horizon Telescope Collaboration, The Astrophysical Journal Letters, 930:L12 (21pp), 2022 May 10.

Appendix K – Final Remarks

K.1. A Return to a Physical Theory. Ever since the dawn of relativity and quantum mechanics, the prevailing physics theory has been rather divorced from physical models. This work has aimed to revert to the earlier way of doing physics, and we have now derived all of the equations governing electrodynamics and gravitation from a simple underlying physical model. As such, we also now have a physical interpretation of many physical entities, and this section will now discuss some of those physical interpretations.

K.1.1. Identifying Electric Charges as Free Aether. The first physical identification comes from Poisson's Equation, shown above as Eq. (55), $\nabla^2\phi = -\rho_D/\varepsilon_0$. In Eq. (55) and herein ρ_D is identified as free, detached-aether. Conventionally ρ_D is identified as electric charge. From this observation it is now understood that electric charge is in actuality an amount of aether that has become detached from the predominant attached-aether. Furthermore, positive charge is one kind of detached-aether, while negative charge is the other kind of detached-aether. This identification shows that the aether itself can be considered to be two infinite seas of charge - one negative, the other positive, which are internally attached and in the state of a solid. And since electric current is understood as the motion of electric charge, electric currents are now identified as moving detached-aether.

K.1.2. Identifying the Static Electric Field as a Longitudinal Aetherial Displacement

The scalar potential ϕ satisfies Eq. (56) above:

$$\mathbf{P_L} - \mathbf{N_L} = \nabla(\Psi_P - \Psi_N) = -\varepsilon_0 \nabla\phi/\rho_0 \tag{56}$$

With the static electric field being the gradient of ϕ, this reveals that static electric fields are simply the longitudinal separation of the positive and negative aetherial components. To illustrate, Figs. K.1 and K.2 show two depictions of the positive-attached-aether. Fig. K.1 shows undisturbed aether, while the Fig. K.2 shows the case where a cylinder of detached-positive-aether has been inserted. (Here we fill the cylinder with detached-positive-aether to illustrate the effect. As mentioned above, it is usually the case that the detached-aether density will be much less than the attached-aether density.) The attached-aether is green and the detached-aether is blue in the figures. Since the aether is incompressible, when detached-aether is inserted it must push the attached-aether out of the way. Arrows in Fig. K.2 show some such displacements. As can be seen, the further away the original attached-aether is from the center the less it needs to move out of the way. Several rings are drawn in the figures such that each cylindrical cross section contains the same area. Since the area is $2\pi r\delta r$, it is easy to see that δr decreases as 1/r. Hence the displacement, $\mathbf{P_L}$, caused by inserting a cylinder of detached-aether, decreases as 1/r. We also can employ Eq. (58), $\mathbf{N_L} = -\mathbf{P_L}$, to see that the static electric field is proportional to $\mathbf{P_L}$. In this aether model charge has been identified as detached-aether, and the static electric field is identified as being proportional to the displacement of the attached-aether. Hence, Figs. K.1 and K.2 show the familiar result that for an infinite cylinder of charge the electric field falls off as 1/r.

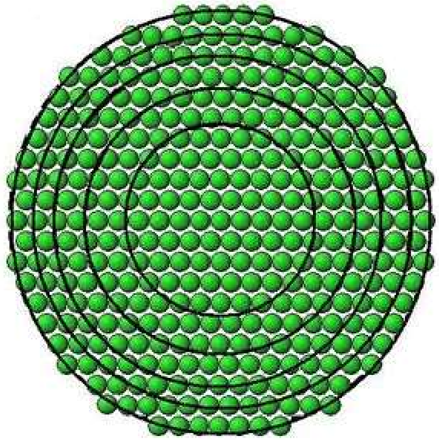

Figure K.1. Undisturbed, attached-positive-aether. *Cylindrical boundaries are arbitrarily assigned so that each cylindrical cross section has an equal area.*

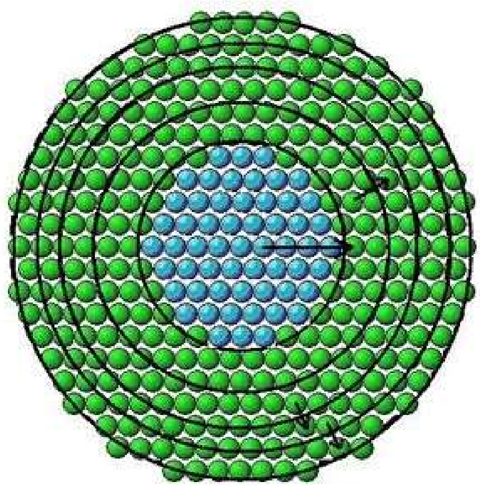

Figure K.2. Detached-positive-aether (blue) is injected into the attached-positive-aether (green). *Each cylinder of attached-aether is moved radially outward by one position as a result of the injection of detached-aether.*

The aether model gives us an answer for what the static electric field is in physical terms: it is a quantity that is proportional to the longitudinal separation of the positive-attached-aether from the negative-attached-aether.

K.1.3. Identifying the Vector Potential as Aetherial Displacement

Next, we look at Eq. (114)

$$\mathbf{P_T} = -\mathbf{N_T} = -K_{F3}\mathbf{A}/\mu_0 T_0 \tag{114}$$

We now see that the vector potential is proportional to the transverse separation of the positive-attached-aether from the negative-attached-aether.

K.1.4. Identifying Light as an Aetherial Wave

Light can now be identified as an aetherial wave - just as the classical theory anticipated in the time of Maxwell.

K.1.5. Why Magnetic Monopoles Do Not Exist

When written in terms of the electric and magnetic fields, Maxwell's equations have a certain symmetry that has led to speculation that a magnetic charge should exist alongside the known electric charge. However here we see that the physical entities underlying Maxwell's equations are the scalar and vector potentials in the Coulomb gauge, and in that form the symmetry argument for the existence of a magnetic monopole no longer exists. Positive electric charge is identified as positive-detached-aether, and negative charge as negative-detached-aether. There is no similar substance to form magnetic monopoles.

K.1.6. Why Dark Matter and Dark Energy Particles Do Not Exist. As described above, the observed gravitational effects being attributed to dark matter and dark energy (before publication of this work) are explained well by the gravitational-mass of the tension, quantum and gamma fields. No extra particles are required.

K.2. Future efforts. As science advances, improvements to our understanding are generally contributed to by many over time. The initial work on any subject is typically extended into areas unforeseen at the time of initial publication, and it is likely the same dynamic will hold with the work herein. This section specifies some of the possible future efforts that are already apparent.

The first future effort concerns electromagnetism at small (millimeter or less) scales as well as black holes where the assumptions that disturbances are small may break down. (See section E.3.3 for a start on one such effort.)

In this work we have shown that Maxwell's Equations, the Lorentz Force Equation, and the Equations of Gravity all result from an aetherial model wherein we have kept terms to the first order in non-vanishing quantities. Of note in this regard is the keeping the $\partial \mathbf{P}_T/\partial t$ and $\partial \mathbf{N}_T/\partial t$ terms in section C.14, but discarding them in sections C.6 and C.15. (In section C.14 they are the dominant surviving term.) Future efforts may involve analyses including the discarded terms.

It is possible that the delta-force leads to field-masses similar to the gamma-force. However, since the delta-force has both negative and positive sources it tends to neutralize while the gamma-force tends to accumulate, and we only observe the effect of the gamma field-mass so far. Future efforts may include the delta-force contribution to gravitational effects.

Future efforts may also include those investigation discussed in section C.17.1 concerning the empirical choices made herein concerning the direction of forces on the detached-aether and which components of the flow apply in certain situations.

Ephemerides calculations should be done using the gravitational equation described herein while including planetary and non-planetary objects as well a stellar oblateness. Advanced computational efforts should be employed on far superior computing resources than what has been described herein.

Of course, further experimental and observational efforts should be made to test the theory and to improve upon our knowledge of the fundamental constant values. It is especially of interest to attempt to isolate and manipulate the aether. (See section E.3.5.)

There is lastly the issue of the nuclear force. By the ABC Preon Model [17, 18] there is a single preonic force responsible for nuclear and preonic binding, and that force is carried by the neutrino. It is possible that such a force also has an aetherial underpinning. For this work we have kept the scope to just electromagnetism and gravity. Nuclear force evaluations are another candidate for future efforts.

K.3. The Limits of What We Know. Herein we have assumed that a solid aether occupies all of space. We must however keep in mind that our perspective is quite limited. As large as it is, our galaxy occupies only an infinitesimal volume compared to the boundaries of the known universe, and the true boundary (it there is one) could be much beyond that of which we are presently aware. Perhaps the aether is not a single stationary solid everywhere. Distant portions of it may be in motion with respect to other portions. It may be expanding or contracting at extremely large scales. The aether may be finite in size and have a boundary. This work has shown that if we assume a local solid aether moving with respect to us at $v \ll c$ that we arrive at all the known equations of electrodynamic and gravitational physics as observed by our limited local measurements. Care and humility should be taken when extrapolating these conclusions to regions far from what we can measure locally.